Linear Stability of a Streamwise Corner Flow.

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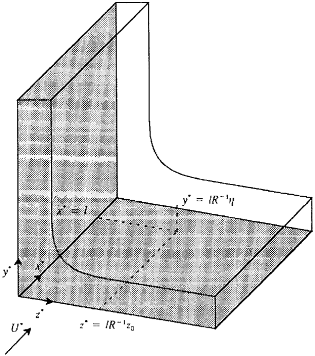
WORKSHOP ERCOFTAC 2008.

- Technological applications:
    - Wing body junctions.
    - Roots of blades.
    - Side walls in wind tunnels.
- Many interrogations remain:
    - Discrepancies remain between theories and experiments ref 1, ref 2.
    - Modification of the Tollmien-Schlichting mechanism ?
    - New mechanism associated with the corner flow ?
- Necessity to develop stability tools for 3D flows.



M. Zamir.
Similarity and stability of the laminar boundary layer in a streamwise corner.
Proc. R. Soc. Lond., 377:269–288, 1981.

S.J. Parker and S. Balachandar.
Viscous and Inviscid Instabilities of Flow Along a Streamwise Corner.
Theoret. Comput. Fluid Dynamics, 13:231–270, 1999.

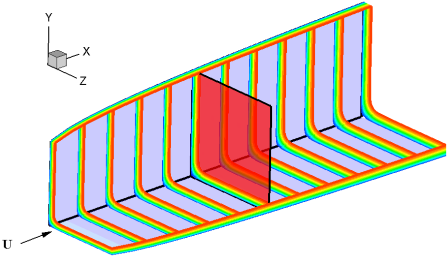
- 3D boundary layer equations in self similar form (η, ϵ) [1].

$$\begin{cases} \eta = \sqrt{\dfrac{\mathrm{Re}}{2x}}\, y \\ \epsilon = \sqrt{\dfrac{\mathrm{Re}}{2x}}\, z \end{cases}$$

- Dimensionless velocities

$$\begin{cases} \tilde{u}(\eta, \epsilon) = u \\ \tilde{v}(\eta, \epsilon) = v\sqrt{2\mathrm{Re}_x} \\ \tilde{w}(\eta, \epsilon) = w\sqrt{2\mathrm{Re}_x} \end{cases}$$

$\mathrm{Re} = U_e \tilde{\eta}/\nu$, $\tilde{\eta} = \delta_1 \sqrt{2}/1.7208$



S.G. Rubin.
Incompressible flow along a corner.
J. Fluid Mech., 26:97–110, 1966.

S.G. Rubin and B. Grossman.
Viscous flow along a corner: numerical solution of the corner layer equations.
Q. Appl. Math., 29:169–186, 1971.

- Elliptical form

$$\begin{cases} \phi = \eta\tilde{u} - \tilde{v} \\ \psi = \epsilon\tilde{u} - \tilde{w} \\ \theta = \partial\phi/\partial\eta - \partial\psi/\partial\epsilon \end{cases}$$

- Poisson-like equations system.

$$\begin{cases} \nabla^2\tilde{u} = -\partial\tilde{u}/\partial\eta\phi - \partial\tilde{u}/\partial\epsilon\psi \\ \nabla^2\tilde{\phi} = 2\partial\tilde{u}/\partial\eta - \partial\theta/\partial\epsilon \\ \nabla^2\tilde{\psi} = 2\partial\tilde{u}/\partial\epsilon + \partial\theta/\partial\eta \\ \nabla^2\tilde{\theta} = -\partial\theta/\partial\eta\phi - \\ \partial\theta/\partial\epsilon\psi + 2\tilde{u}\left(\theta - 2\eta\partial\tilde{u}/\partial\epsilon + 2\epsilon\frac{\partial\tilde{u}}{\partial\epsilon}\right) \end{cases}$$

- BC: Asymptotic matching solutions
- Spectral discretization: Chebyshev/Chebyshev. Symmetry conditions along the bissector s.
- Large non linear system: NEWTON NITSOL solver.

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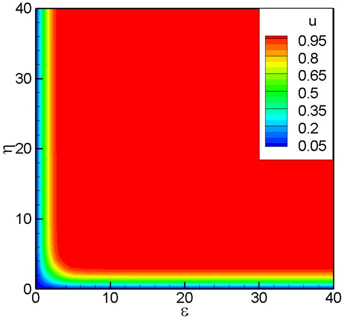

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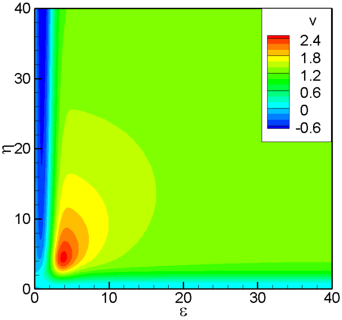

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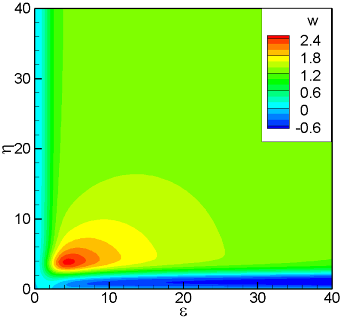
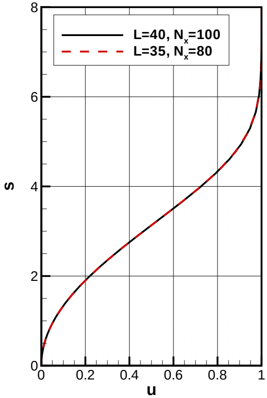
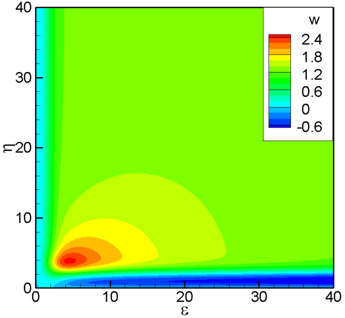


Strong three dimensionality ⇒ 3D Stability.



Inflectional profile along the bissector $\Rightarrow$ inviscid instability.

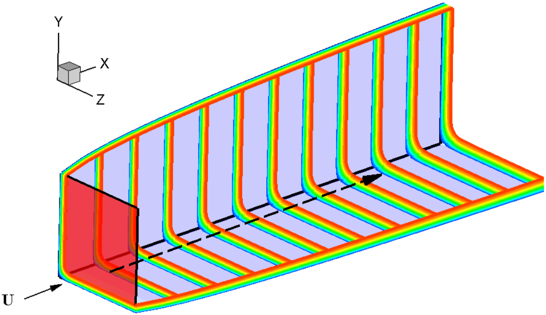


Strong three dimensionality $\Rightarrow$ 3D Stability.

- Spatial theory: from a parallel approach to the PSE applied to 3D Flows.
- Instantaneous flow: $\mathcal{Q} = \overline{\mathcal{Q}} + \epsilon\tilde{\mathbf{q}}$ with $\epsilon \ll 1$.
- Slow variation along x:

$$\begin{cases} \tilde{\mathbf{q}}\,(x, y, z, t) =^t [\tilde{\mathbf{u}}, \tilde{p}]\,(x, y, z, t) = \hat{\mathbf{q}}\,(X, y, z)\, e^{i(\mathcal{F}-\Omega t)} \\ \text{where } X = \varepsilon_x x \text{ with } \varepsilon_x \ll 1 \text{ and } \partial\mathcal{F}/\partial x = \alpha \end{cases}$$

- Space and time behaviour: $\mathcal{L}_1\hat{\mathbf{q}} + \mathcal{L}_2\partial\hat{\mathbf{q}}/\partial x = 0$
- Initialization: zeroth order in ϵ_x
- Integration along x of 3D PSE equations.



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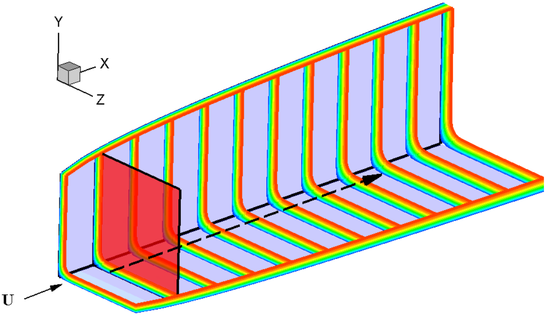

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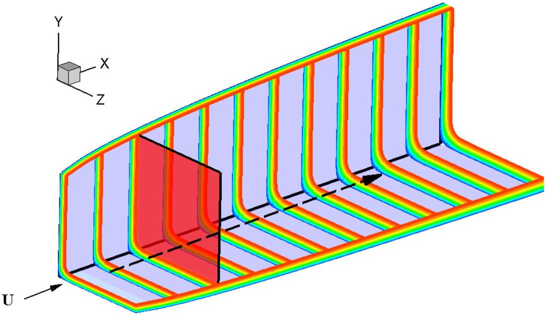

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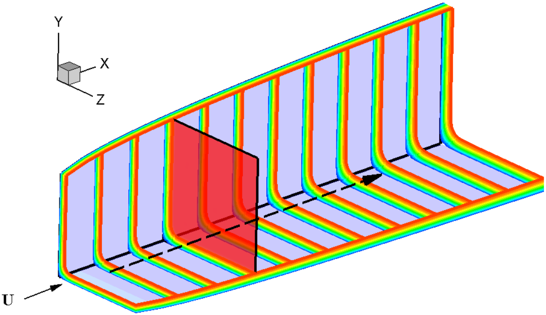

- Parallel flow assumption: the stability problem is rewritten as:
  $\left[\mathcal{A}_2(\mathrm{Re},\Omega)\,\alpha^2 + \mathcal{A}_1(\mathrm{Re},\Omega)\,\alpha + \mathcal{A}_0(\mathrm{Re},\Omega)\right]\hat{\mathbf{q}} = \mathcal{L}_{OS2D}\hat{\mathbf{q}} = 0$
  with $\alpha \in \mathcal{C}$ and $\Omega \in \Re$
- Spectral discretization: Chebyshev/Chebyshev. Symmetry conditions along s.
- Companion matrices combined with an Arnoldi Shift/Invert algorithm
- TS branch even/odd modes. Corner mode (Inviscid nature [1])

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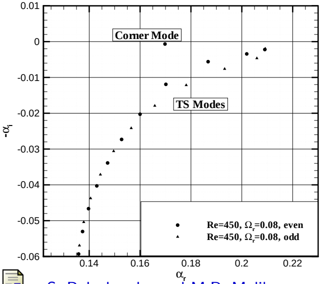

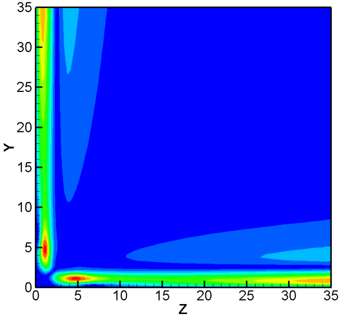

$|\hat{u}|$ TS odd 1

S. Balachandar and M.R. Malik.
Inviscid instability of streamwise corner flow.
J. Fluid mech., 282:187–201, 1995.

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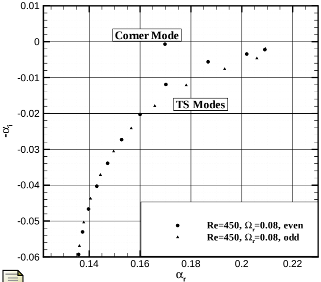

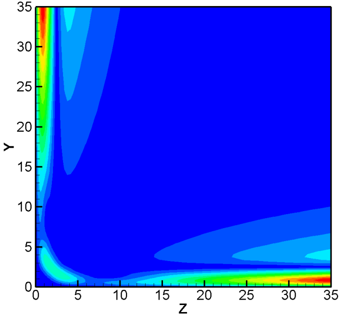

$|\hat{u}|$ TS even 1

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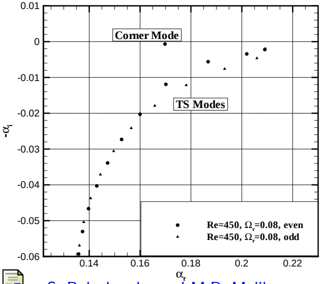

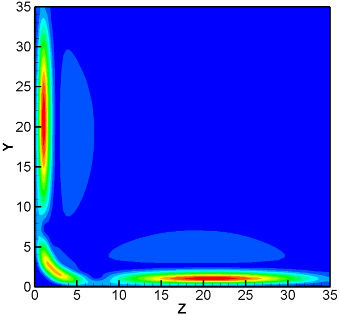

$|\hat{u}|$ TS even 2

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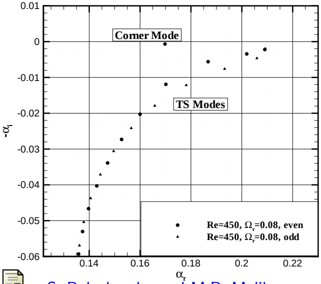

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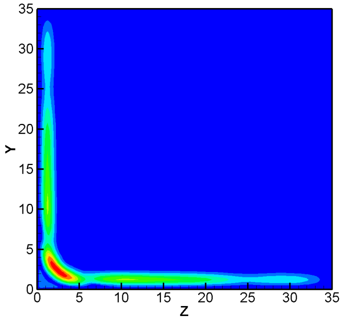

$|\hat{u}|$ Corner even

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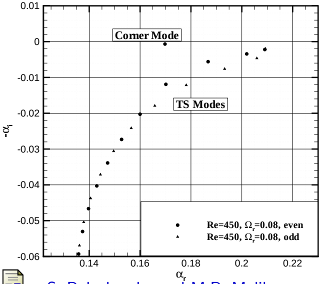

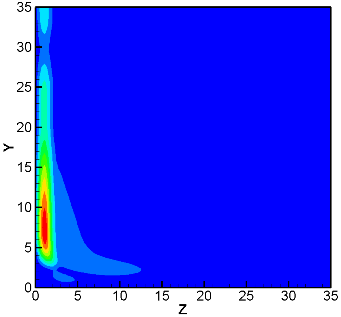

$|\hat{v}|$ Corner even

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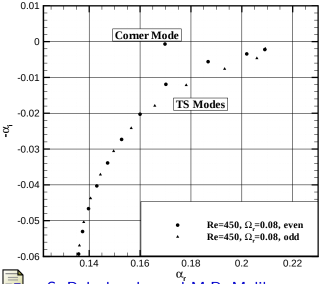

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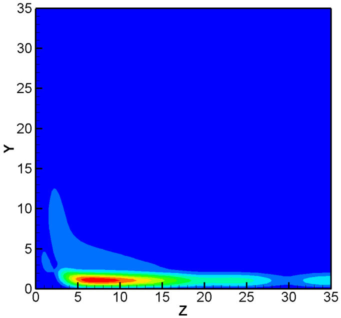

$|\hat{w}|$ Corner even

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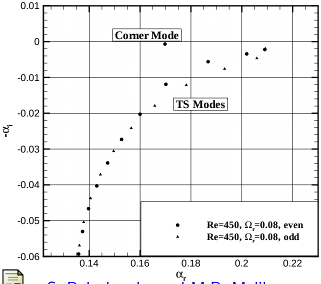

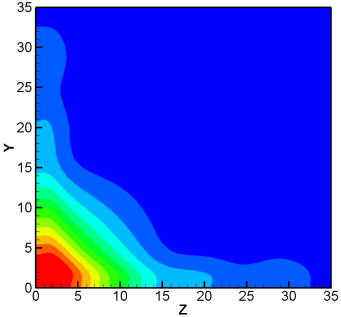

$|\hat{p}|$ Corner even

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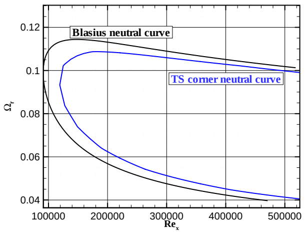

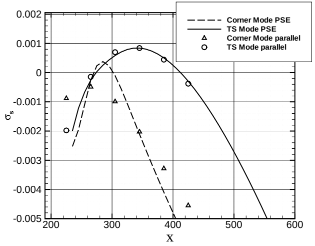

Re=450

- <u>Neutral curve TS</u>
- Influence of the corner: to damp TS waves.
- Corner mode: marginally stable.
- Similar results as Parker & Balachandar.

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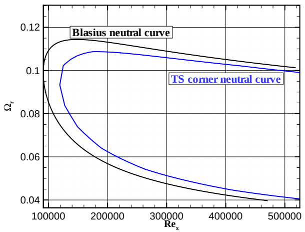
- <u>PSE 3D</u>
- Backward Euler along x.
- Normalization condition:

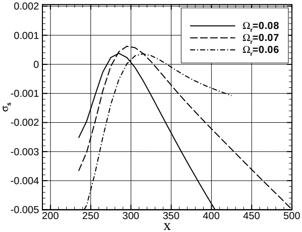
$$\int_0^{L_y}\int_0^{L_z} {}^t\hat{\mathbf{u}}^* \frac{\partial \hat{\mathbf{u}}}{\partial x}\, dz\, dy = 0$$

- Non-parallel correction:

$$\sigma_s = -\alpha_i + \Re\left(\frac{1}{C}\frac{\partial C}{\partial x}\right)$$

M. S. Broadhurst and S. J. Sherwin.
The Parabolized Stability Equations for 3D-Flows: Implementation and Numerical Stability.
Applied Num. Math., 2006.





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- Backward Euler along x.
- Normalization condition:

$$\int_0^{L_y}\int_0^{L_z} {}^t\hat{\mathbf{u}}^* \frac{\partial \hat{\mathbf{u}}}{\partial x}\,\mathrm{d}z\,\mathrm{d}y = 0$$

- Non-parallel correction:

$$\sigma_s = -\alpha_i + \Re\left(\frac{1}{C}\frac{\partial C}{\partial x}\right)$$

M. S. Broadhurst and S. J. Sherwin.
The Parabolized Stability Equations for 3D-Flows: Implementation and Numerical Stability.
Applied Num. Math., 2006.

Summary:

- PSE 3D: to analyze convective waves with respect to 3D flows.
- TS mechanism more stable.
- The non-parallel effects provide a correction of the spatial amplification rate of the corner mode: unstable area.
- Remains less amplified than the TS mode. The theory can not explain the experimental results.

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Hypothesis:

- Strong sensitivity to base flow modifications around the corner [1, 2, 3] ?

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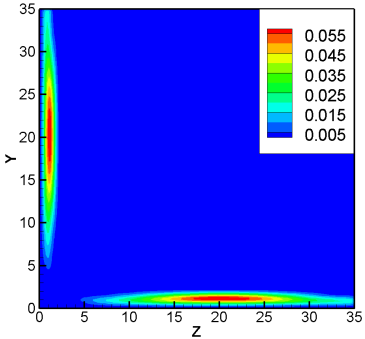
- Small perturbation of the base flow.

$$\begin{cases} \overline{U} \leftarrow \overline{U} + \delta U \\ \overline{V} \leftarrow \overline{V} + \delta V \\ \overline{W} \leftarrow \overline{W} + \delta W \end{cases} \Longrightarrow \begin{cases} \hat{u} \leftarrow \hat{u} + \delta \hat{u} \\ \hat{v} \leftarrow \hat{v} + \delta \hat{v} \\ \hat{w} \leftarrow \hat{w} + \delta \hat{w} \\ \Omega \leftarrow \delta \Omega \end{cases}$$

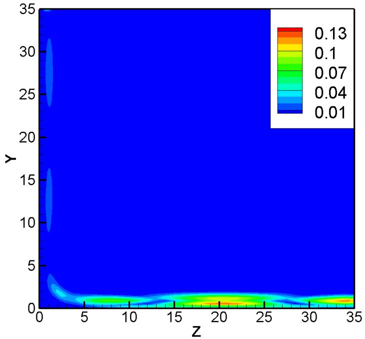
- Introducing a scalar product $<,>= \int_0^{L_y} \int_0^{L_z} {}^t\mathbf{q}^{+*}\mathbf{B}\mathbf{q}\ dzdy$
- $< \mathbf{q}^+, \mathcal{L}_{OS2D}\left(\mathbf{U} + \delta\mathbf{U}, \hat{\mathbf{q}} + \delta\mathbf{q}, \Omega + \delta\Omega\right) >= 0$
- $\delta\Omega = \int_0^{L_y} \int_0^{L_z} {}^t\mathbf{G_u}\delta\mathbf{U}\ dzdy$
- $\mathbf{G_u}$ a sensitivity function with respect to 3D flows.

- $Re = 500$, $\alpha = 0.25$.



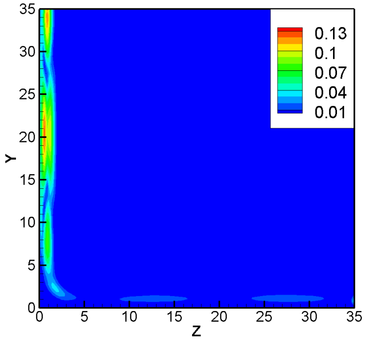
G_u TS mode

- $Re = 500$, $\alpha = 0.25$.



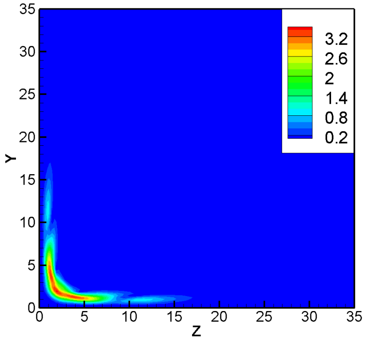
G_v TS mode

- $Re = 500$, $\alpha = 0.25$.



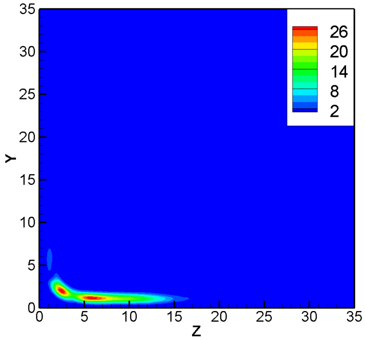
G_w TS mode

- $Re = 500$, $\alpha = 0.25$.



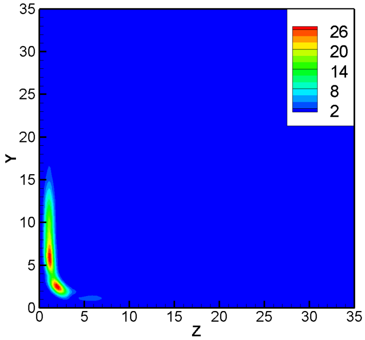
G_u Corner mode

- $Re = 500$, $\alpha = 0.25$.



G_v Corner mode

- $Re = 500$, $\alpha = 0.25$.
- Sensitivity functions are stronger for the corner mode.
- Most influence on the eigenvalue in the cross section.



G_w Corner mode

Prospects:

- To complete 3D PSE analyses. Critical Reynolds number associated with the corner mode and comparisons with DNS.
- Further explore the sensitivity functions with (Re, α).
- Modification of the critical Reynolds number with respect to r quantifying the deviation of the base flow $\overline{Q}$ [1] ?
- Extension of the analysis in compressible regime.

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