Optimal Disturbances and Receptivity of 3D Boundary Layers

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KTH Mechanics

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Telfona
Testing for Laminar Flow on New Aircraft

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Outline

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- Background
  - Motivation
  - Previous Work
- Method
  - Governing Equations
- Results
  - Verification
  - Parametric Study
- Conclusions

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Motivation

- Work is part of EU-project TELFONA
  - Objective to demonstrate ability to predict aircraft performance based on wind tunnel tests and CFD-results

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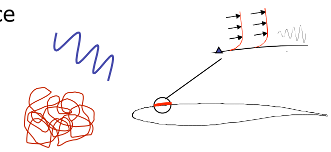
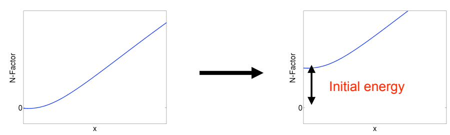
- Investigate receptivity in 3D-boundary layers
  - Different levels of free stream turbulence in wind tunnel compared to free flight
  - How do external perturbations enter the boundary layer



- Compute optimal disturbances
  - First step towards a receptivity model
  - Can later be used to compute receptivity coefficients



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Previous Work

- Optimal disturbances in 2D boundary layer
  - Andersson et. al. (1999) and Luchini (2000)
  - Corbett & Bottaro (2000) and Levin & Henningson (2003)
  - Streamwise vortices that develop into streamwise streaks
  - Very different from Tollmien-Schlichting waves
- Optimal growth in swept boundary layers
  - Corbett & Bottaro (2001)
  - Temporal approach using Orr-Sommerfeld eq.
  - **Transient and exponential growth are complementary**
- Receptivity in 3D boundary layers
  - Pralits & Luchini (Ercoftac 2007)
  - Compute amplification factor of crossflow modes
  - Projection of disturbances on adjoint modes

Governing Equations

- Objective: Describe algebraically and exponentially growing 3D-disturbances using **parabolic equations**

- Why not use PSE?
  - PSE follows **one** exponentially growing mode
  - Transient growth results from superposition of modes

- PSE can not describe transient growth

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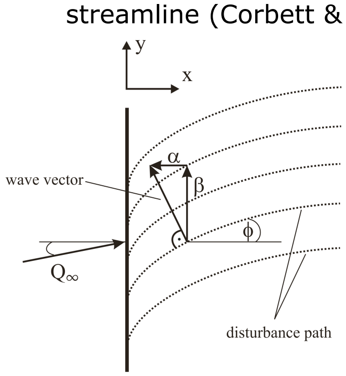
Governing Equations

- Disturbance Ansatz

$$\mathbf{q}'(x,y,z,t) = \hat{\mathbf{q}}(x,z)\exp i\Theta(x,y,t)$$
$$\Theta(x,y,t) = \int_{x_0}^{x_1} \alpha(x)dx + \beta y - \omega t$$

- Optimal disturbances almost aligned with the external streamline (Corbett & Bottaro 2001)



→ $\alpha = -\tan(\phi)\beta$

→ Prescribing α removes ambiguity

→ Oscillations absorbed by exp-function

→ Growth absorbed by shape function

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Governing Equations

- Assume slow variation of shape function along x
  - Restriction in terms of exponential growth
- How should terms be scaled?
  - Transient growth associated with boundary layer scaling
  - PSE-scaling used for exponential growth

	x	y,z	$\bar{u},\bar{v}$	$\bar{w}$	$\hat{u},\hat{v}$	$\hat{w}$	β	p	t	ω
BL.-scaling	l	δ	Q_∞	$Q_\infty\frac{\delta}{l}$	Q_∞	$Q_\infty\frac{\delta}{l}$	$\frac{1}{\delta}$	$\rho Q_\infty^2\frac{\delta}{l}$	$\frac{l}{Q_\infty}$	$\frac{Q_\infty}{l}$
PSE-scaling	l	δ	Q_∞	$Q_\infty\frac{\delta}{l}$	Q_∞	Q_∞	$\frac{1}{\delta}$	ρQ_∞^2	$\frac{\delta}{Q_\infty}$	$\frac{Q_\infty}{\delta}$

- Apply both scalings, keep terms of order lower than Re_δ^{-2}
  - One extra term resulting from B.L.-scaling compared to PSE: $\bar{W}_x u$
  - $\mathbf{A}\hat{\mathbf{q}} + \mathbf{B}\hat{\mathbf{q}}_z + \mathbf{C}\hat{\mathbf{q}}_{zz} + \mathbf{D}\hat{\mathbf{q}}_x = 0$
- Optimise using method of Lagrangian multipliers

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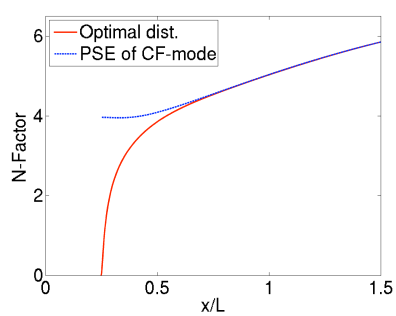
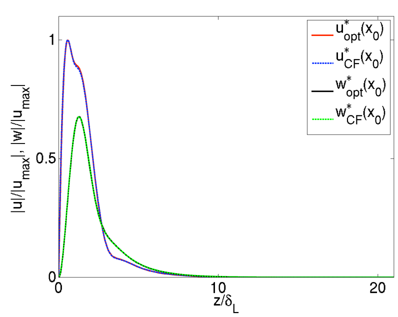
Verification

- Does optimal disturbance develop into crossflow mode?
  - Compare downstream development of optimal disturbance with that of corresponding cross flow mode (same β)
  - Compare solution of adjoint equation at initial position to that of the corresponding adjoint PSE solution





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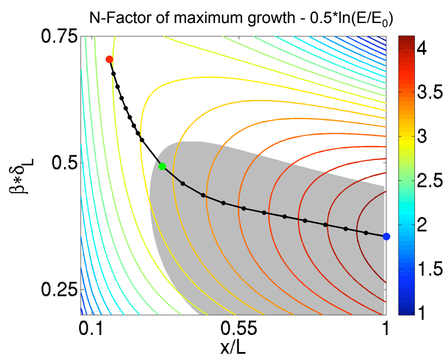
Results

- Baseflow: Falkner-Skan-Cooke boundary layer
  - Sweep angle: 45°
  - Adverse (β_H = -0.05) and favourable (β_H = 0.1) pressure gradient
  - $Re = U_L L/\nu = 0.7 \cdot 10^6$
- Optimal Growth for β_H = 0.1, x_0/L = 0.005 and ω = 0



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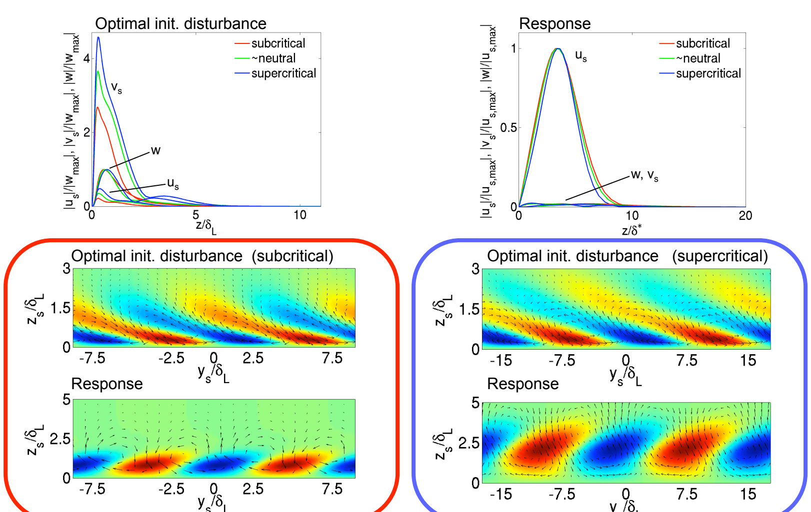
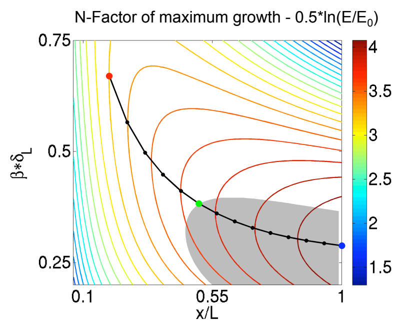
Results

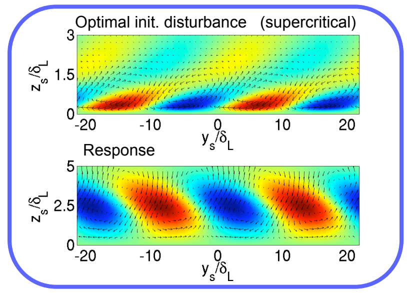
- Compare optimal disturbances and their downstream response for selected points in x-β plane

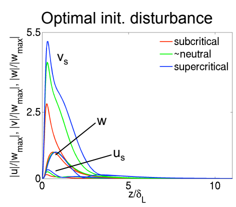


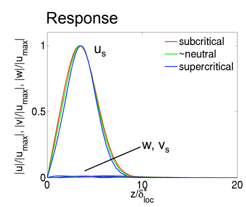
Results

- Adverse pressure gradient case (β_H=-0.05)









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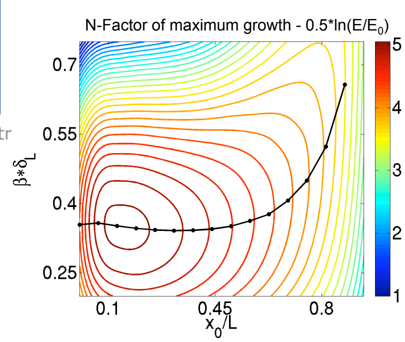
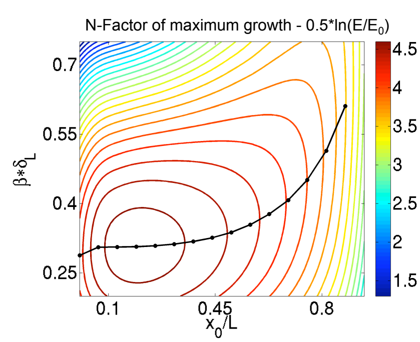
Results

- Dependence of the optimal growth at x_{opt} = 1 on the initial position x_0
- Leading edge is not the optimal initial position



β_H = 0.1



β_H = -0.05

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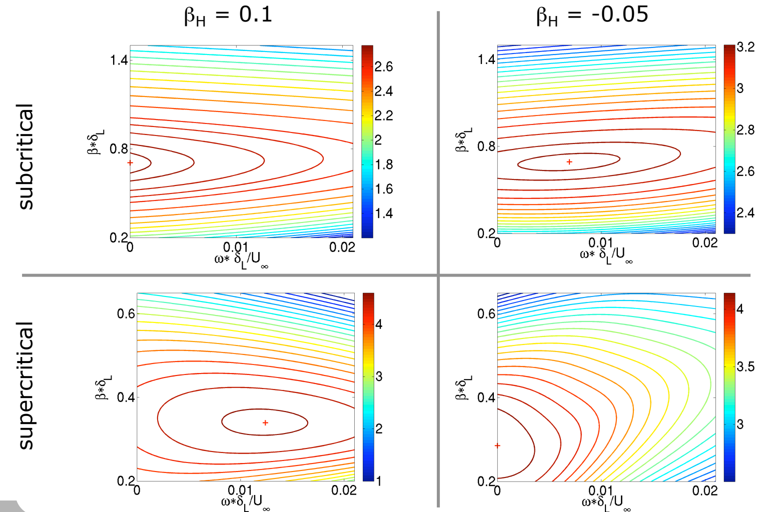
Results

- Dependence of optimal growth on frequency
- N-Factors of maximum growth, x_0 = 0.005



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Conclusions / Outlook

- Set of parabolic equations describes algebraically and exponentially growing disturbances
- Restricted in terms of exponential growth
- Transient and exponential growth generate disturbances of similar structure
- Use optimal disturbances for receptivity calculations
  - Project onto free stream turbulence