Direct-Adjoint eigensolutions of boundary layer flows

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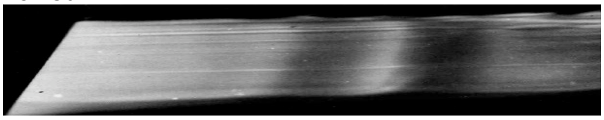
Introduction

Instability mechanisms in boundary layers

Zero-Pressure-Gradient:
Tollmien-Schlichting waves



Werlé, in "Album of fluid motion", Van Dyke

Adverse-Pressure-Gradient, Separation bubbles:
Kelvin-Helmholtz instability

Bradshaw, in "Album of fluid motion", Van Dyke

"Global" **non wave-like** instability modes

Gallaire, Marquillie & Ehrenstein, 2007

Introduction (II)

Modal linear instability approaches

Orr-Sommerfeld

- **Parallel flow assumption**

  1D basic flow

- **Ansatz:**

  $\hat{\mathbf{q}}(y)\exp(\mathrm{i}(\alpha x + \beta z - \omega t))$

- **Temporal / Spatial EVPs**

- **"Local" Analysis**

Non-parallel corrections

PSE

…

BiGlobal

- **Non-parallel** 2D basic flow

- **Ansatz:**

  $\hat{\mathbf{q}}(x, y)\exp(\mathrm{i}(\beta z - \omega t))$

- **Temporal EVP**

  Spatial growth recovered in the amplitude functions

- **"Global" Analysis**

Theory

Direct & Adjoint BiGlobal EVP

Introducing into the **direct** linearized Navier-Stokes equations the decomposition

$$(\hat{\mathbf{v}}^*, \hat{p}^*) = (\hat{\mathbf{v}}(x, y), \hat{p}(x, y))\mathrm{e}^{+\mathrm{i}(\beta z - \omega t)}$$

and into the **adjoint** linearized Navier-Stokes equations the decomposition

$$(\tilde{\mathbf{v}}^*, \tilde{p}^*) = (\tilde{\mathbf{v}}(x, y), \tilde{p}(x, y))\mathrm{e}^{-\mathrm{i}(\beta z - \omega t)}$$

one obtains, respectively,

Theory (II)

The Direct BiGlobal EVP

$$
\begin{aligned}
\hat{u}_x + \hat{v}_y + i\beta\hat{w} &= 0 \\
(\mathcal{L} - \bar{u}_x + \mathrm{i}\omega)\,\hat{u} - \bar{u}_y\hat{v} - \hat{p}_x &= 0 \\
-\bar{v}_x\hat{u} + (\mathcal{L} - \bar{v}_y + \mathrm{i}\omega)\,\hat{v} - \hat{p}_y &= 0 \\
(\mathcal{L} + i\omega)\,\hat{w} - i\beta\hat{p} &= 0
\end{aligned}
$$

where

$$
\mathcal{L} = \frac{1}{Re}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} - \beta^2\right) - \bar{u}\frac{\partial}{\partial x} - \bar{v}\frac{\partial}{\partial y}
$$

and

Theory (III)

The Adjoint BiGlobal EVP

$$
\begin{aligned}
\tilde{u}_x + \tilde{v}_y - \mathrm{i}\beta\tilde{w} &= 0 \\
\left(\mathcal{L}^\dagger - \bar{u}_x + \mathrm{i}\omega\right)\tilde{u} - \bar{v}_x\tilde{v} - \tilde{p}_x &= 0 \\
-\bar{u}_y\tilde{u} + \left(\mathcal{L}^\dagger - \bar{v}_y + \mathrm{i}\omega\right)\tilde{v} - \tilde{p}_y &= 0 \\
\left(\mathcal{L}^\dagger + \mathrm{i}\omega\right)\tilde{w} + \mathrm{i}\beta\tilde{p} &= 0
\end{aligned}
$$

where

$$
\mathcal{L}^\dagger = \frac{1}{Re}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} - \beta^2\right) + \bar{u}\frac{\partial}{\partial x} + \bar{v}\frac{\partial}{\partial y}
$$

Theory (IV)

The Boundary Conditions for the Adjoint EVP

The solvability condition for the direct/adjoint EVPs is:

$$\nabla \cdot j(\hat{\mathbf{q}}, \tilde{\mathbf{q}}) = 0$$

Looking to the "bilinear concommitant":

$$j(\hat{\mathbf{q}}, \tilde{\mathbf{q}}) = \bar{\mathbf{v}}(\hat{\mathbf{v}} \cdot \tilde{\mathbf{v}}) + \frac{1}{Re}(\tilde{\mathbf{v}} \cdot \nabla \hat{\mathbf{v}} - \hat{\mathbf{v}} \cdot \nabla \tilde{\mathbf{v}}) + \hat{\mathbf{v}}\tilde{p} + \tilde{\mathbf{v}}\hat{p}$$

The condition **is trivially accomplished** if:

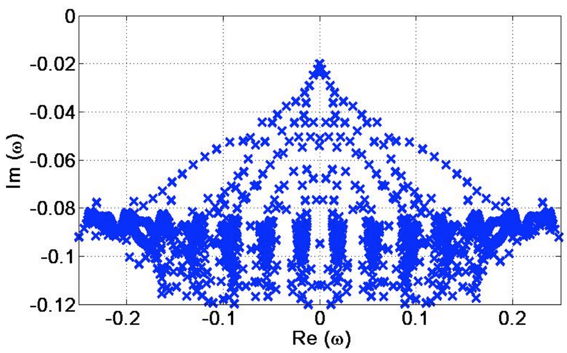
- Amplitude functions for (at least) one of them vanish:

  Dirichlet boundary conditions $\hat{\mathbf{q}} = 0$, $\tilde{\mathbf{q}} = 0$
- Periodicity is imposed to the domain

But hardly accomplished otherwise !!!

The BiGlobal spectrum

- Great number $O(10^2 - 10^3)$ of **temporal** eigenvalues
- **Families of eigenmodes** (where exist) not easy to identify
- Physical mechanisms not determined



How to classify the eigenmodes ?

Parallel flow analysis

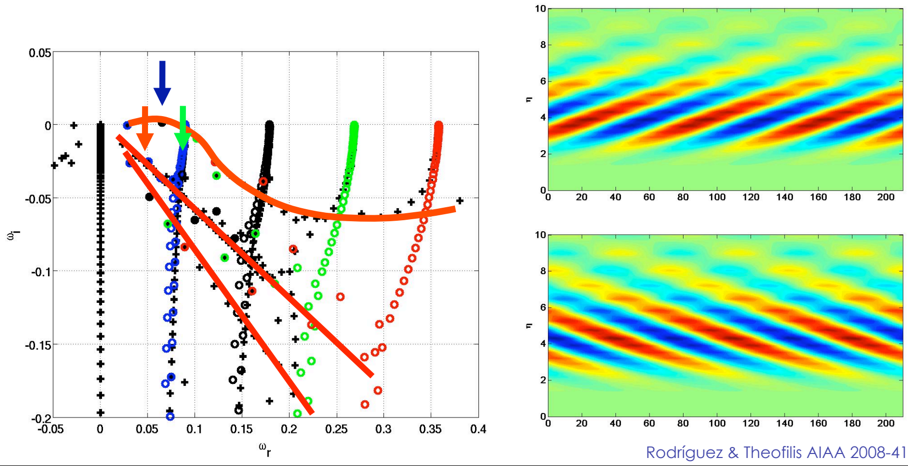
<u>**Comparison with temporal OSE**</u>

Basic Flow: Artificial parallel Blasius

Mack's Case: Re = 580, α = 0.179 (Mack JFM 1976)

Streamwise extension of the domain: $L_x = 4\,\pi / 0.179$

Periodic boundary conditions



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Towards non-parallelism

Case: $Re_{\delta *}$=450 at inflow, $Re_{\delta *}$=700 at outflow

Analysis 1:

- **Basic Flow:** Artificial parallel Blasius
- **Boundary conditions:** Robin* at inflow & outflow

Analysis 2:

- **Basic Flow:** Artificial parallel Blasius
- **Boundary conditions:** Robin* at inflow & extrapolation at outflow

Analysis 3:

- **Basic Flow:** Real Blasius boundary layer
- **Boundary conditions:** Robin* at inflow & extrapolation at outflow

*** Robin boundary condition of Ehrenstein & Gallaire JFM 2005:**

$$\frac{\partial \hat{\mathbf{q}}}{\partial x} = \mathrm{i}\left(\alpha_{r,0} + \frac{\partial \alpha_r}{\partial \omega_r}(\omega_0)\cdot(\omega - \omega_0)\right)\hat{\mathbf{q}}$$

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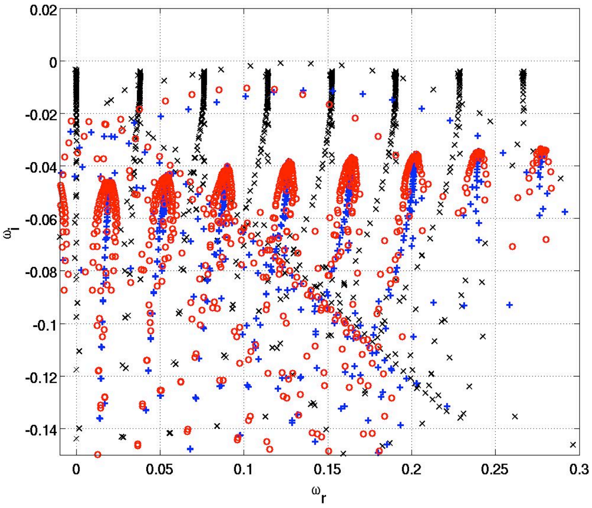
Towards non-parallelism

Case: Re_{δ^*}=450 at inflow, Re_{δ^*}=700 at outflow
Robin boundary condition evaluated at: ω_0 = 0.13



Analysis:

1 **X** Parallel Blasius, Robin + Robin

2 **+** Parallel Blasius, Robin + Extrapolation

3 **O** Real Blasius, Robin + Extrapolation

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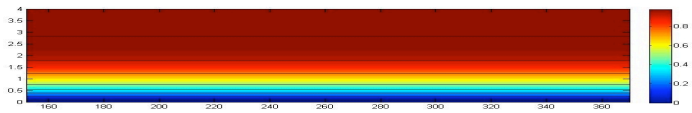
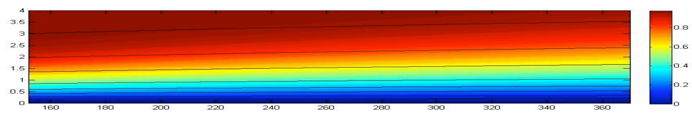
Non-parallel basic flow

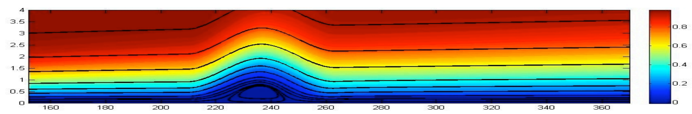
Parallel basic flow: (Artificial parallel Blasius BL)
- The **OSE eigenvalues** are recovered



Cuasi-parallel basic flow: (Real Blasius BL)
- The same **families of eigenvalues** are recovered



Non-parallel basic flow: (Separation bubble in BL)
- **What is to be recovered?**



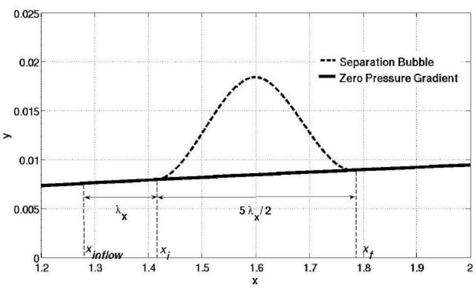
Non-parallel basic flow

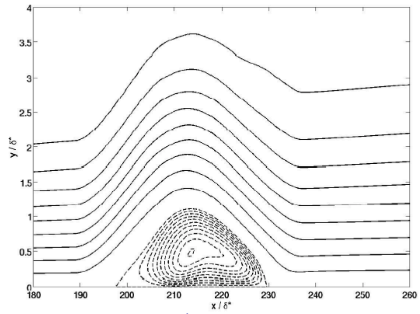
Boundary Layer transformation:

$$\xi = \frac{x}{L} \qquad \eta = y\sqrt{\frac{U_e}{\nu x}} \quad \rightarrow \quad \Psi = \sqrt{U_e \nu x} f(\xi, \eta)$$

Separated states recovery:

- Reyhner and Flügge-Lotz approximation
- Displacement thickness imposed





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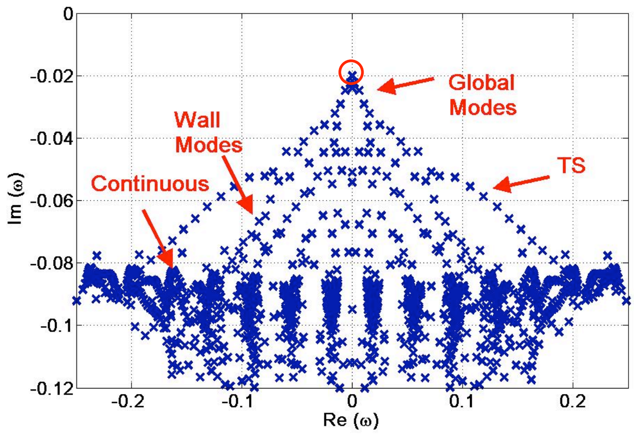
Global Modes in the LSB

Case: Re_{δ^*}=450 at inflow, Re_{δ^*}=700 at outflow, **Separation Bubble**
3D Analysis: $0 < \beta < 1$
Boundary conditions: Dirichlet at inflow & Extrapolation at outflow



β = 1

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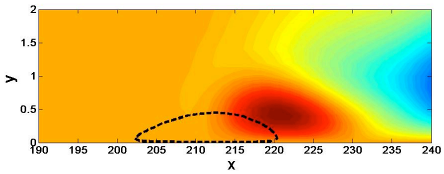
Global Modes in the LSB (II)

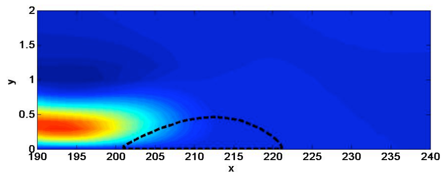
The steady 3D global mode – Amplitude functions ($\beta = 1$)

Case: $Re_{\delta*}$=450 at inflow, $Re_{\delta*}$=700 at outflow, **Separation Bubble**
3D Analysis: $0 < \beta < 1$
Boundary conditions: **Dirichlet** at inflow & Extrapolation at outflow

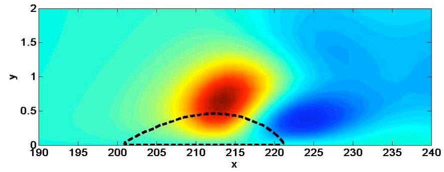
u - Direct



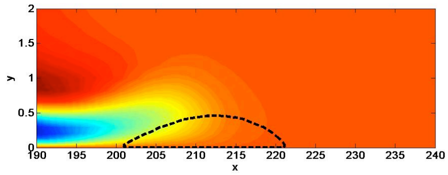
u - Adjoint



w - Direct



w - Adjoint



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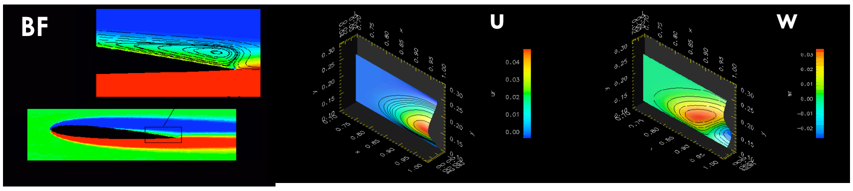
Theofilis, Hein and Dallmann 2000

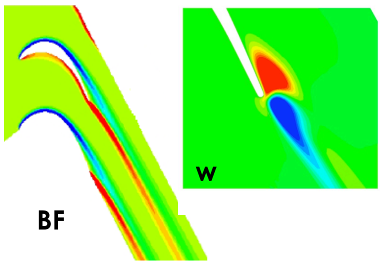
Steady Global Mode

Present in different geometries

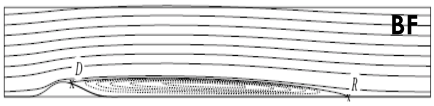
- NACA 0012 (Theofilis, Barkley and Sherwin, 2002)



- LPT blade (Abdeseemed, Sherwin and Theofilis 2004)



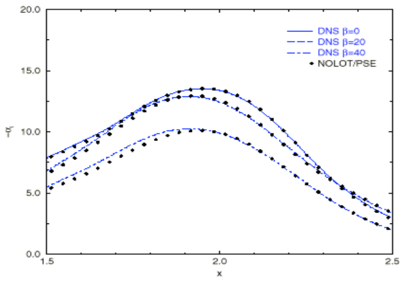
- Bump (Gallaire, Marquillie and Ehrenstein 2007)



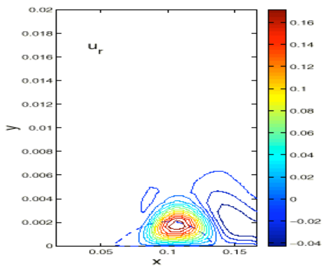
The dominant mechanism?

Two different classes of eigenmodes:

Waves amplified by the shear layer



Steady "global" mode



Theofilis, Hein & Dallmann 2000

- Spatial amplification rates of the global mode are **substantially smaller** than those of the shear layer

- The global mode can be **temporally unstable**

TS & bubble interaction

Case: $Re_{\delta*}$=450 at inflow, $Re_{\delta*}$=700 at outflow, **Separation Bubble**
3D Analysis: $0 < \beta < 0.30$
Boundary conditions:

Inflow:
- **Dispersion relation** from local spatial analysis: $D(\alpha, \omega, \beta, Re) = 0$
- **Gaster-type** transformation

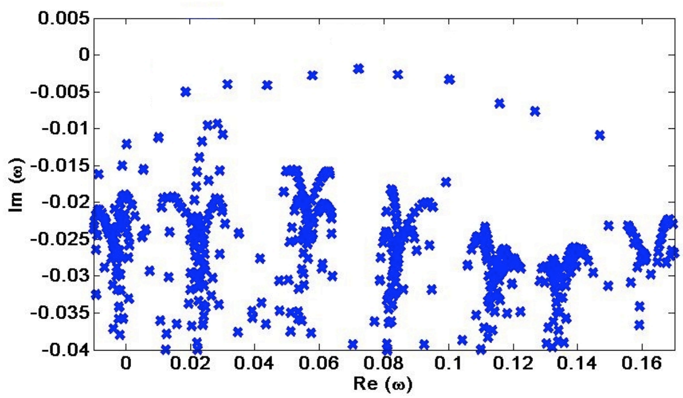
$$\frac{\partial \hat{\mathbf{q}}}{\partial x} = \mathrm{i}\left(\alpha_{r,0} + \frac{\partial \alpha_r}{\partial \omega_r}(\omega_0)\cdot(\omega - \omega_0)\right)\hat{\mathbf{q}}$$

Same BC as 2D Ehrenstein & Gallaire's

Outflow: Linear extrapolation

TS & bubble interaction

Case: $Re_{\delta*}$=450 at inflow, $Re_{\delta*}$=700 at outflow, **Separation Bubble**
3D Analysis: $0 < \beta < 0.30$
Boundary conditions: Dispersion relation inflow, extrapolation outflow

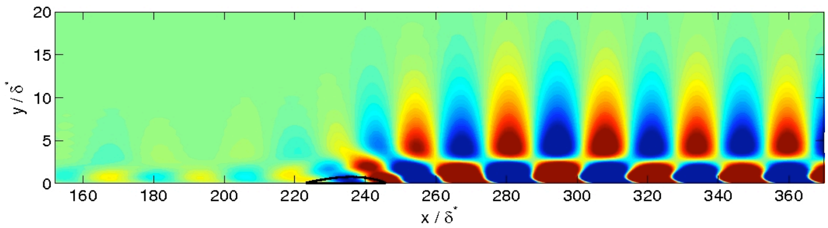


β = 0.15

TS & bubble interaction

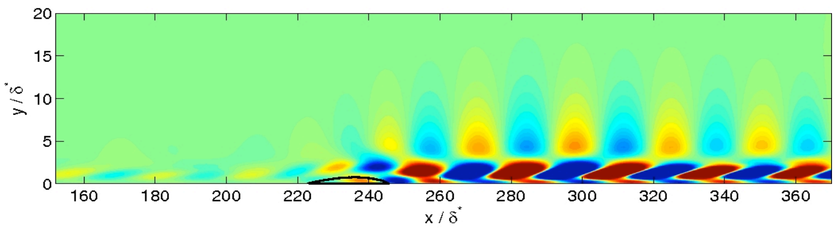
Amplitude functions for plane (2D) TS waves

$\beta = 0, \quad \omega = 0.0963 - i\, 0.0023$



Amplitude functions for oblique (3D) TS waves

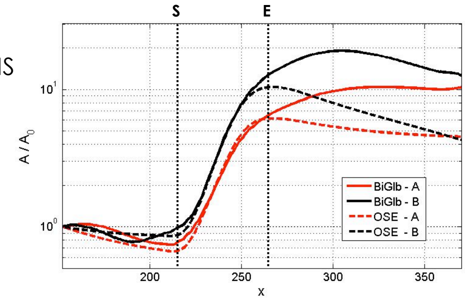
$\beta = 0.15, \ \omega = 0.1003 - i\, 0.0077, \ \Psi = 31°$

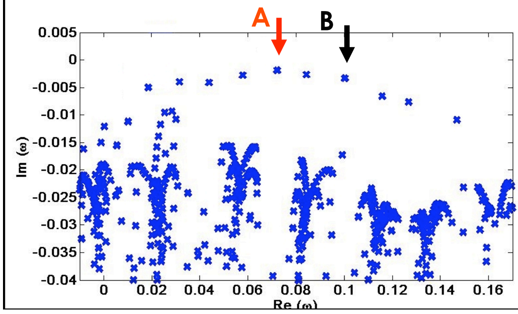


TS & bubble interaction

Amplification of Tollmien-Schlichting eigenmodes

- Max spatial amplification ~ **10 – 20** orders of magnitude smaller than in DNS
- Max amplified **spatially** is not the least damped **temporally**
- Study of **wave-packet evolution** required to reproduce physics





$\beta = 0.15$

A : $\omega = 0.07201 - i\, 0.0019$

B : $\omega = 0.1003 - i\, 0.0033$

Summary

- **Parallel flow asumptions: OSE eigenvalues** recovered
- **Cuasi-parallel flow:** Same **structure** of the spectrum
- **Non-parallel flow:** New family of **non wave-like** modes
- **Mechanisms competing:**
  - Global mode (**one eigenmode alone**) self-excited
  - Incoming wave-packets (**several eigenmodes**) amplified by the shear layer