Optimal perturbations and very large scale structures in zero pressure gradient turbulent boundary layers

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OPEN ISSUES IN TRANSITION AND FLOW CONTROL

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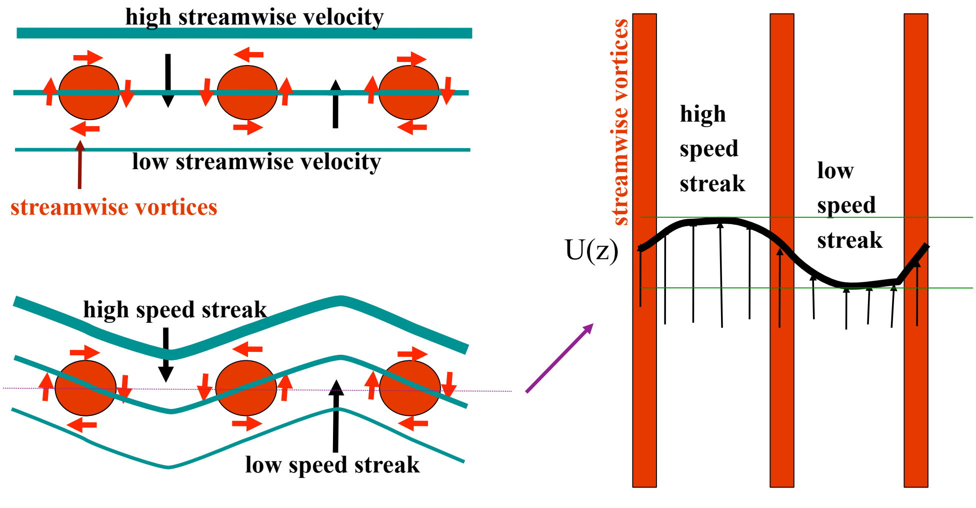
Streaks formation: the lift-up effect

Streamwise vortices in a shear flow => streamwise streaks

Taylor 1939, Moffatt 1967,..., Landahl 1980, Ellingsen & Palm 1985...



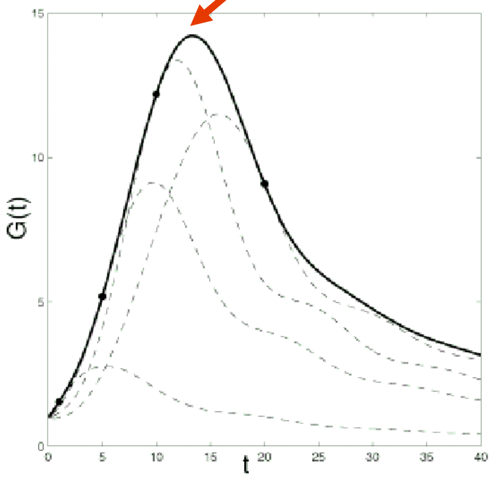
Strongly related to non normality of the linearised operator

reviews in Trefethen et al 1993, Schmid & Hennigson 2001

Optimal transient growth (fixed Re, α, β)

$$G(t) = \max_{\hat{q}_0 \neq 0} \frac{\|\hat{q}(t)\|^2}{\|\hat{q}_0\|^2}$$

(G_{max}, t_{max})



from Schmid & Henningson 2001

Optimal wavelengths for laminar plane shear flows :

$\lambda_x \sim \infty, \lambda_z = 3h$

$G_{max} \sim Re^2$

Gustavsson 1991,...

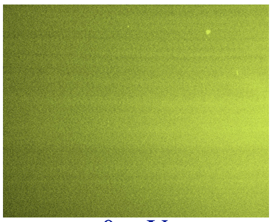
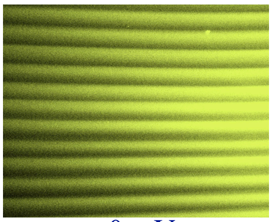
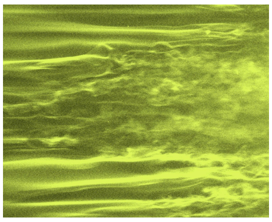
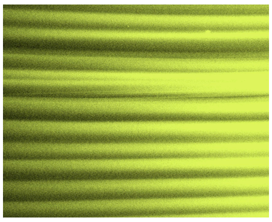
Optimal streamwise vortices & streaks

Optimal growth, vortices & streaks computed for virtually all canonical laminar shear flows (Couette, Poiseuille, pipe Poiseuille, Blasius BL,...)

They are relevant to:
subcritical transition to turbulence,
bypass transition induced by free-stream turbulence,
self-sustained turbulent processes

They are also useful to manipulate shear flows with low or no energy (passive control):
Lift-up effect used to amplify the control input energy

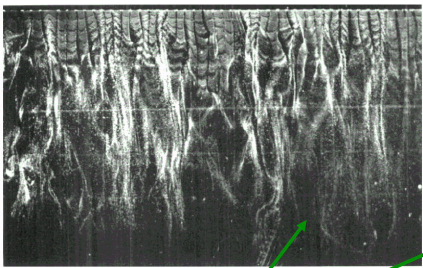
Boundary layer transition delay with streaks

	No streaks forced	'Controller streaks' on
No forced perturbations	 0 mV	 0 mV
With forced perturbations	 205 mV	 450 mV

Fransson, Talamelli, Brandt & Cossu, *Phys. Rev. Lett.* 2006

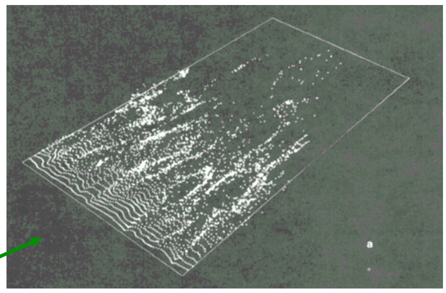
Streaky structures in turbulent flows

Near wall streaks in a turbulent boundary layer (experimental)



Kline et al. JFM 1967

Near wall streaks in a turbulent channel flow (numerical)

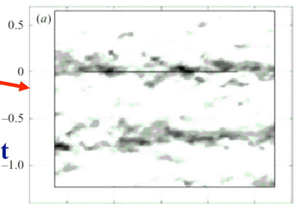


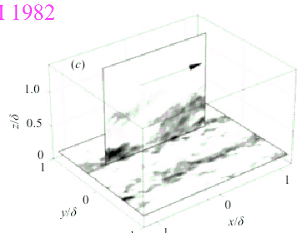
Moin & Kim JFM 1982

mean streaks spacing $\lambda^+ = 100$

streaks spacing at least 2δ

Large scale streaks in a turbulent boundary layer (experimental)





Hutchins & Marusic JFM 2007

Turbulent channel: Butler & Farrell approach (1993)

$$\frac{\partial \mathbf{u}}{\partial t} + U\frac{\partial \mathbf{u}}{\partial x} + (v\,U',0,0) = -\nabla p + \nu\nabla^2\mathbf{u}$$

molecular viscosity

Reynolds-Tiedermann (1967)
turbulent mean velocity profile

unconstrained optimal λ_z = 3h

λ^+ = 100 obtained constraining the optimization time

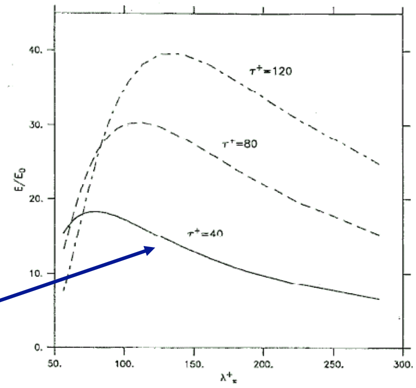


FIG. 3. Optimal energy growth versus wavelength for optimization times representative of the eddy turnover time near the wall.

Butler & Farrell *Phys. Fluids* 1993

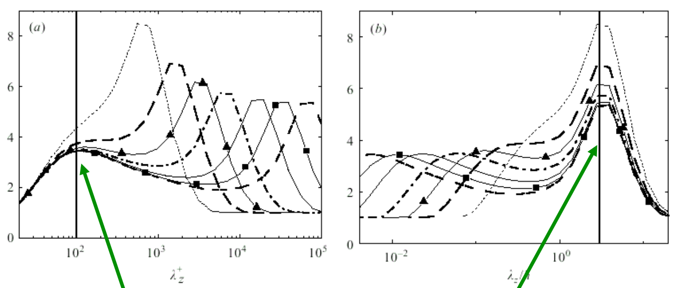
del Alamo & Jiménez approach (2006)

$$\frac{\partial \mathbf{u}}{\partial t} + U\frac{\partial \mathbf{u}}{\partial x} + (v\,U',0,0) = -\nabla p + \nabla\cdot\left[\nu_T(y)\left(\nabla\mathbf{u} + \nabla\mathbf{u}^T\right)\right]$$

Reynolds-Tiedermann profile

ν_T includes **turbulent eddy viscosity**

208 *J. C. del Álamo and J. Jiménez*



Secondary Gmax peak scaling in inner variables with optimal at λ^+ = 100

del Alamo & Jiménez *JFM* 2006

unconstrained optimals!

Primary Gmax peak scaling in outer variables with optimal at λ_z = 3 h

Turbulent boundary layers are relevant for industrial, meteorological and many other applications

Can we extend these channel flow results to the turbulent boundary layer?

Double peak structure?

Which is the relevant external scale?

What is the optimal spacing?

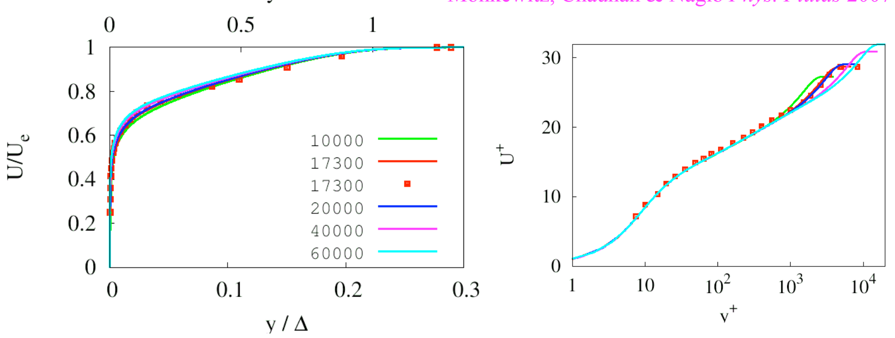
How does the external peak scale with Re?

…

Asymptotic self-consistent mean velocity profile

$$U = u_\tau \left[U_i^+(y^+) - U_{log}^+(y^+) + {U_e}^+(Re_{\delta_*}) - U_w^+(\eta) \right]$$

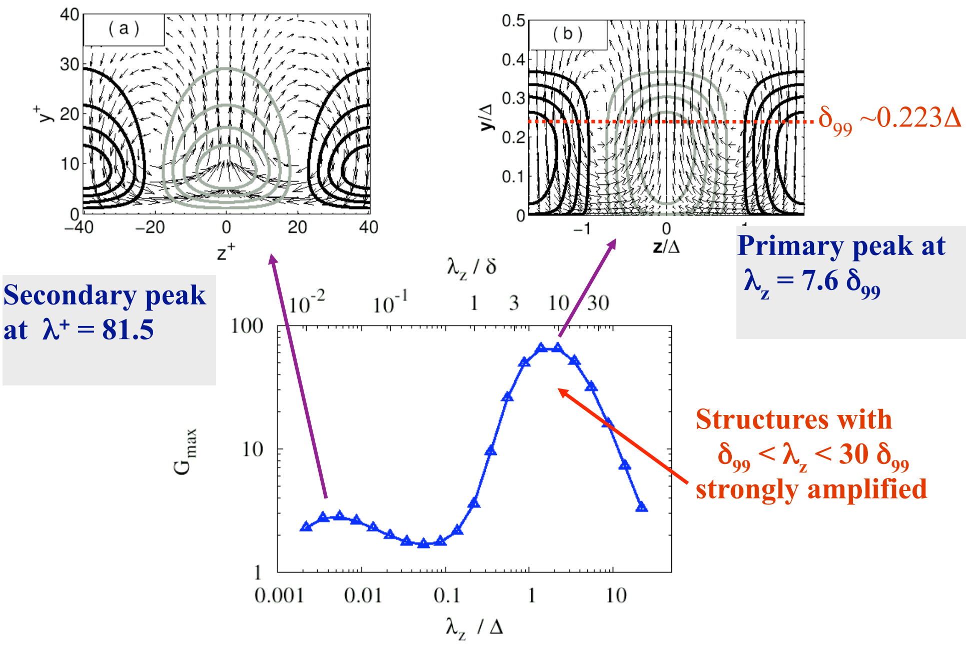
Monkewitz, Chauhan & Nagib *Phys. Fluids* 2007



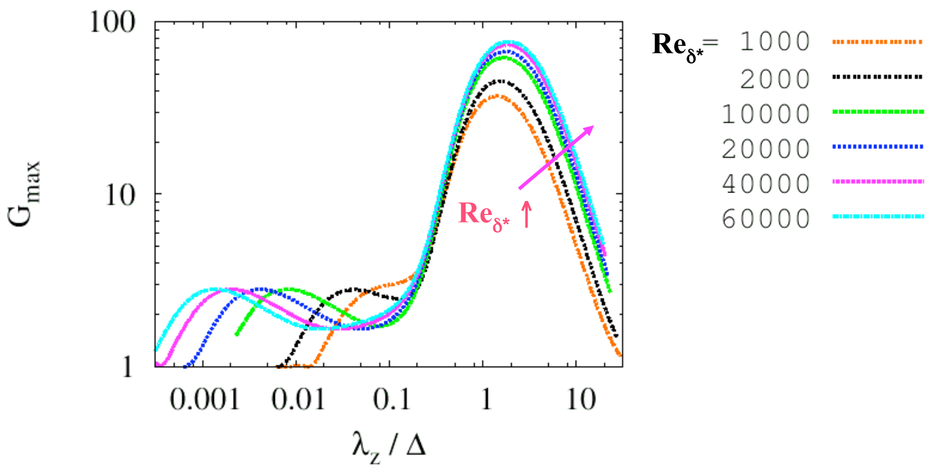
Velocity profiles at different $Re_{\delta*}$ in outer & inner units.
Comparison with data of de Graaf & Eaton (JFM 2000)

$$U_{log}^+(y^+) = \frac{1}{\kappa}\ln(y^+) + B \quad {U_e}^+(Re_{\delta_*}) = \frac{1}{\kappa}\ln(Re_{\delta_*}) + C \quad \kappa = 0.384,\ B = 4.17,\ C = 3.30$$

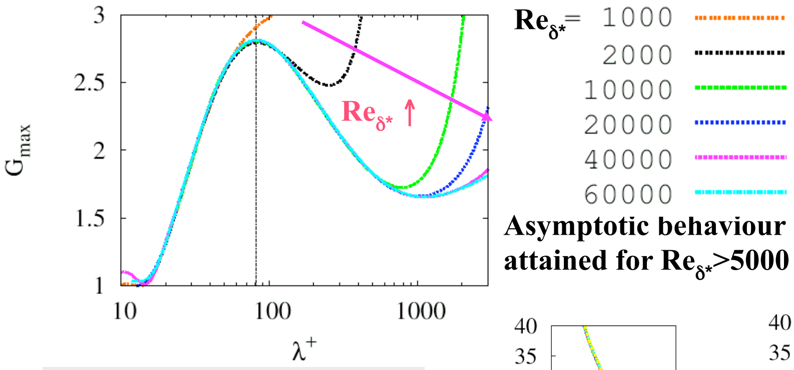
Optimal α=0 growth & perturbations at Re_{δ^*}=17300



Gmax(α=0,β) at selected Reδ*



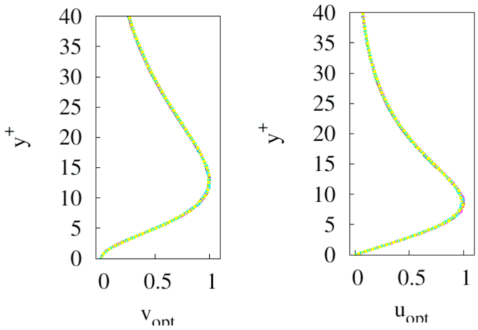
Zoom on the secondary peak in inner units



Asymptotic behaviour attained for Re_{δ^*}>5000

Secondary peak at $\lambda^+ = 81.5$

optimal perturbations self similar in inner units when $Re_{\delta^*} > 5000$

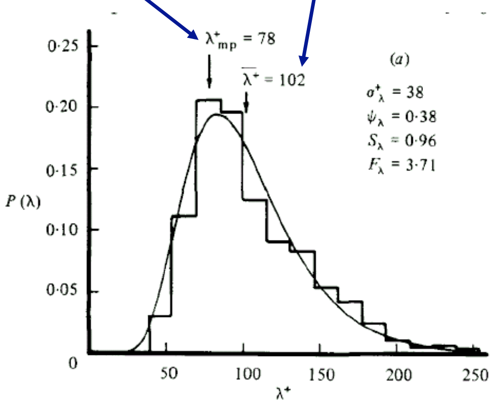


Why λ^+=81.5 and not the commonly accepted value of 100?

most probable value

mean value

Probability of observing streaks with a given λ^+ at y^+=5 a Re_θ=1490



Smith & Metzler *JFM* 1983

Scaling of the outer optimal

β_{opt} does not scale on Δ

Laminar non self-similar boundary layers:
relevant length is θ (e.g. Corbett & Bottaro 2000)

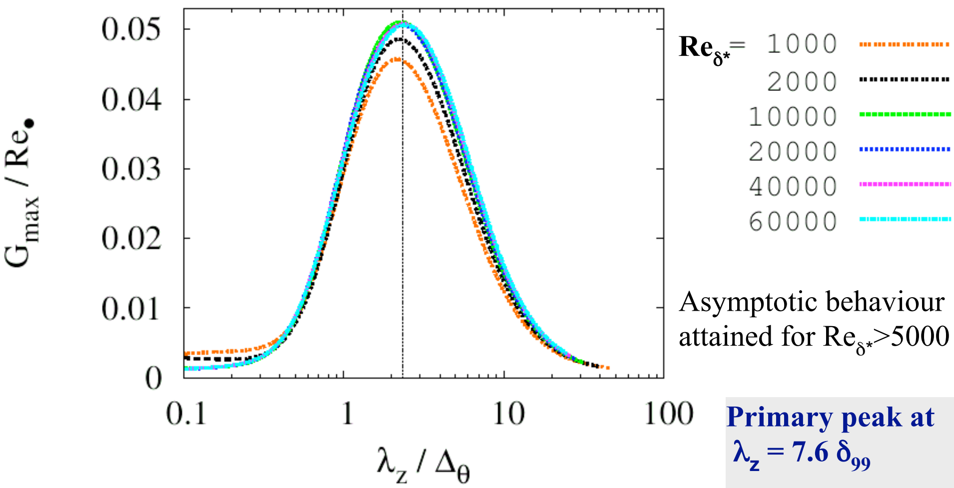
modified Rotta-Clauser length: $\Delta_\theta = \theta U_e / u_\tau$

G_{max} does not scale on Re_{δ^*}

'effective turbulent' Reynolds number

$$Re_\bullet = \frac{U_e \Delta_\theta}{\nu_{T\,max}}.$$

Scaling of the primary peak

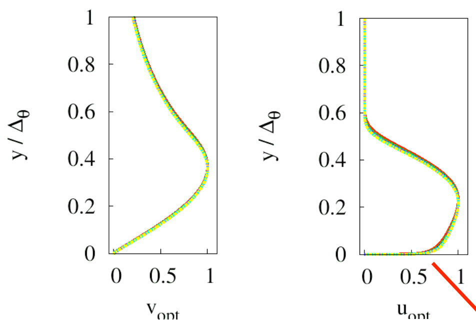


Primary peak at $\lambda_z = 7.6\ \delta_{99}$

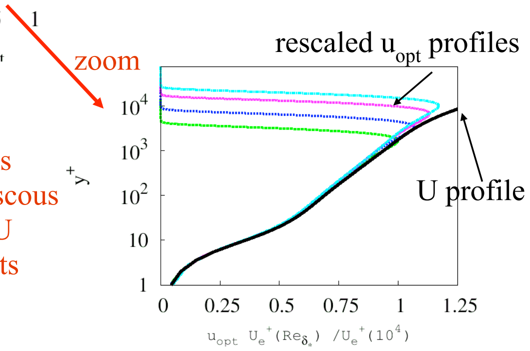
$$G_{max}^{(out)} = (0.0508 \pm 0.0004) Re_\bullet$$

Gmax scales on Re and not Re^2 like in the laminar case

Similarity of the optimal perturbations



optimal perturbations self similar in Δ_θ units when $Re_{\delta^*} > 5000$



The outer optimal streaks are self-similar in the viscous layer & proportional to U when scaled in inner units and on Ue

Open issues

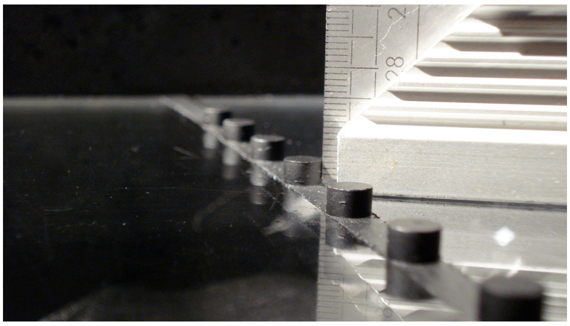
Can *natural* optimal very large scale structures be observed in turbulent boundary layers? Need for new experiments with very large spanwise acquisition windows.

Can optimal very large scale structures be *artifically forced* in channel flows and boundary layers?
What is their effect? Relation with VG?

Nonlinear interaction of large and small scale optimals?

Preliminary results for Couette flow: outer optimal seems to match experimental & DNS evidence of large scale structures

Preliminary experimental results



Roughness elements

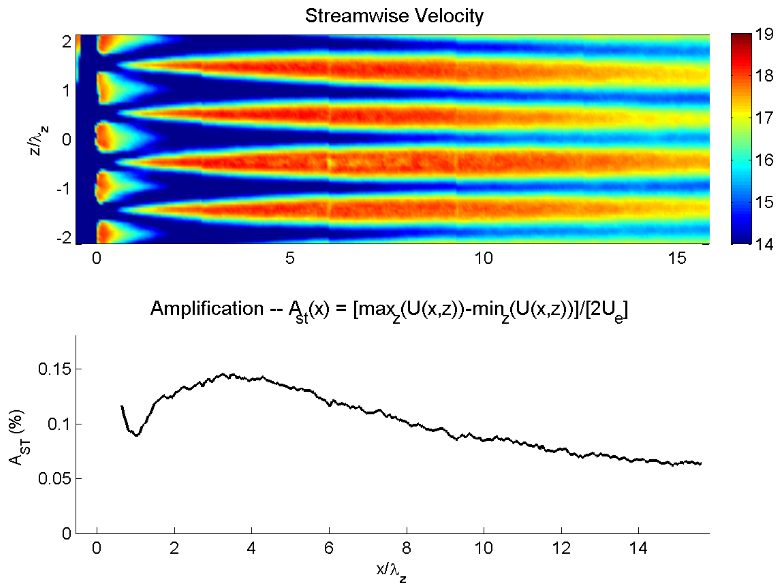
height: k=4mm
diameter: D=6mm
spacing: z=24mm

Turbulent boundary layer : U_e=20m/s

x=0: $\delta \sim$ 4mm, $\lambda_z/\delta \sim 6$, $Re_{\delta*} \sim 1100$
x=150mm: $\delta \sim$ 8mm, $\lambda_z/\delta \sim 3$, $Re_{\delta*} \sim 1800$

Windtunnel of the Peugeot-Citroën Aerodynamics Dept. in Velizy

Turbulent streaks mean amplitude



PIV data: horizontal plane y=3/4k (roughness height) / meaned on 600 samples

Thank you for your attention!

p(re)prints on:

http://www.ladhyx.polytechnique.fr/people/carlo/