Optimization for boundary layer flows using DNS

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Transition Mechanisms, Prediction and Control

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Outline

- Motivation
- Optimal Initial Conditions
- Optimal Forcing
- Results
- Conclusions and outlook

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Motivation

- Examine stability of flows in complex geometries
  - Spatially growing boundary layer
  - Jet in cross flow
  - Parabolic Leading edge
- Prohibitively large eigenvalue problems
- Matrix-free optimisation methods can help with
  - Asymptotic stability (modal)
  - Short time stability (non-modal)
  - Designing controllers

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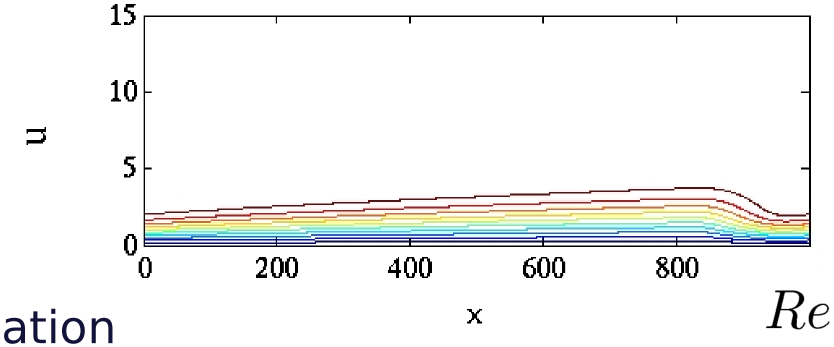
Problem Formulation

- Linearized Navier-Stokes

$$\partial_t \mathbf{u} = -\mathbf{U}\nabla\mathbf{u} - \mathbf{u}\nabla\mathbf{U} - \nabla\pi + Re^{-1}\nabla^2\mathbf{u}$$

- 2D Base flow:
  - Blasius flow
  - Fringe region



$$Re_{\delta_0^*} = 1000$$

- State-space formulation
  - Define pressure through Poisson

$$\Delta\pi = \nabla\cdot(-(\mathbf{U}\cdot\nabla)\mathbf{u} - (\nabla\mathbf{U})\mathbf{u}) \Rightarrow \pi = \mathcal{K}\mathbf{u}$$

$$\partial_t \mathbf{u} = -\mathbf{U}\nabla\mathbf{u} - \mathbf{u}\nabla\mathbf{U} - \nabla\mathcal{K}\mathbf{u} + Re^{-1}\nabla^2\mathbf{u} \Rightarrow \partial_t\mathbf{u} = \mathcal{A}\mathbf{u}$$

- Choose a norm $(\mathbf{a},\mathbf{b}) = \frac{1}{2}\int_0^T\int_\Omega \mathbf{a}^T\mathbf{b}\,d\Omega dt$

- Define adjoint $(\mathbf{p},\mathcal{T}\mathbf{u}) = (\mathcal{T}^\dagger\mathbf{p},\mathbf{u}) + \text{B.T.}$

Optimal Disturbances using Lagrange approach

Looking for the initial condition that optimizes the energy of the final condition.

- Governing equations and objective function

$$\partial_t \mathbf{u} = \mathcal{A}\mathbf{u} \qquad \mathcal{J} = (\mathbf{u}(T), \mathbf{u}(T))$$

- Lagrange functional

$$\mathcal{L} = (\mathbf{u}(T), \mathbf{u}(T)) - \int_0^T (\mathbf{p}, (\partial_t - \mathcal{A})\mathbf{u})dt - \lambda((\mathbf{p}(T), \mathbf{u}(T)) - (\mathbf{p}(0), \mathbf{u}(0))$$

- Lagrange multipliers: $\mathbf{p}$ and λ

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- Variations of the Lagrange function

$$\frac{\delta \mathcal{L}}{\delta \mathbf{p}} = (-\partial_t + \mathcal{A})\mathbf{u} = 0$$ ➡ DNS of NS

$$\frac{\delta \mathcal{L}}{\delta \mathbf{u}} = (-\partial_t - \mathcal{A}^\dagger)\mathbf{p} = 0$$ ➡ DNS of Adjoint NS

$$\frac{\delta \mathcal{L}}{\delta \lambda} = (\mathbf{p}(T), \mathbf{u}(T)) - (\mathbf{p}(0), \mathbf{u}(0)) = 0$$ ➡ IC for the Adj DNS

Optimal Forcing

Looking for the time periodic volume forcing *f* that optimizes the time integral of the kinetic energy of the response at the asymptotic limit

- Governing equations and objective function

$$\partial_t \mathbf{u} = \mathcal{A}\mathbf{u} + \mathbf{f}\exp(i\omega t) \qquad \mathcal{J} = \int_{T-\frac{2\pi}{\omega}}^{T} (\mathbf{u}(t), \mathbf{u}(t))dt$$

Important! Time of integration T must be large -> All the transient behavior has died out.

- Lagrange function

$$\mathcal{L} = \int_{T-\frac{2\pi}{\omega}}^{T} (\mathbf{u}(t), \mathbf{u}(t))dt - \int_0^T (\mathbf{p}, (\partial_t - \mathcal{A})\mathbf{u} + \mathbf{f}\exp(i\omega t))dt - \lambda((\mathbf{f}, \mathbf{f}) - 1)$$

- Variations of the Lagrange function

$$\frac{\delta \mathcal{L}}{\delta \mathbf{p}} = (-\partial_t + \mathcal{A})\mathbf{u} + \mathbf{f}\exp(i\omega t) = 0$$ → DNS of NS

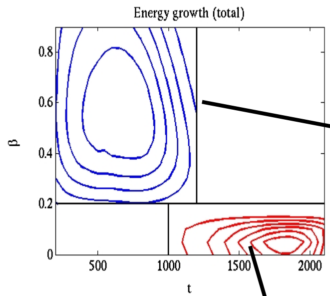
$$\frac{\delta \mathcal{L}}{\delta \mathbf{u}} = (-\partial_t - \mathcal{A}^\dagger)\mathbf{p} + \mathbf{u}H = 0$$ → Adj DNS of NS forced by forward solution

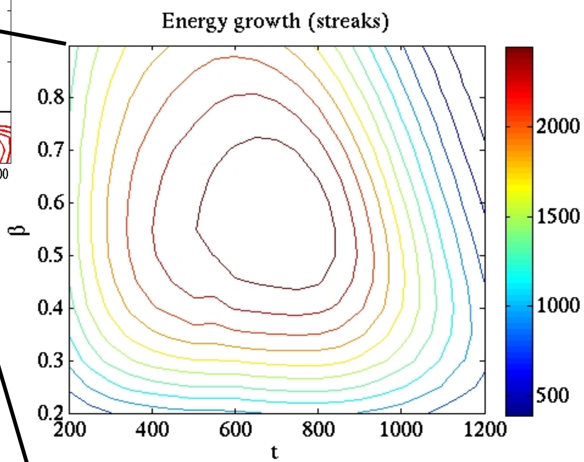
$$\frac{\delta \mathcal{L}}{\delta \mathbf{f}} = \int_0^T (\mathbf{p}\exp(-i\omega t))dt + \gamma \mathbf{f} = 0$$ → Optimality condition

- Equivalent eigenvalue problem to be solved

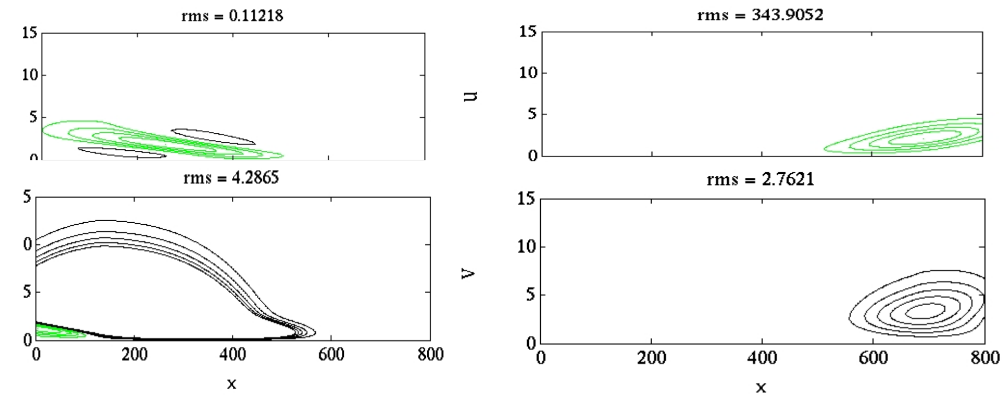
$$(i\omega\mathcal{I} - \mathcal{A}^\dagger)^{-1}(i\omega\mathcal{I} - \mathcal{A})^{-1}\mathbf{f} = \lambda \mathbf{f}$$

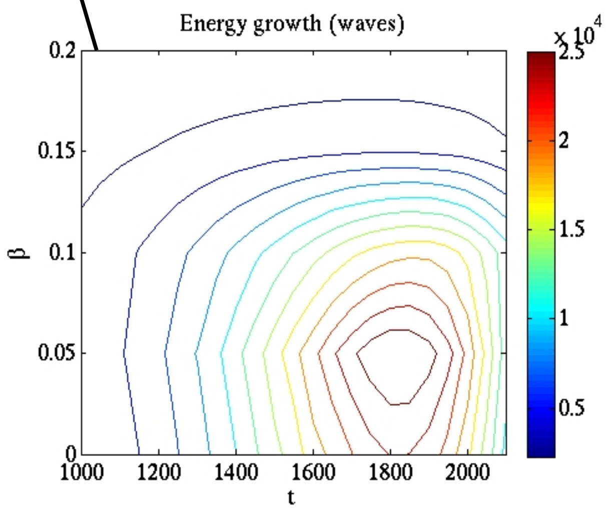
Results for initial conditions



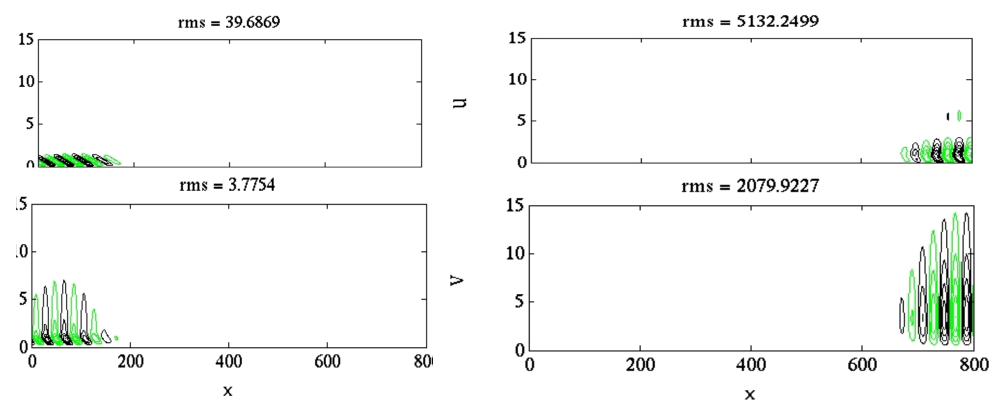


Disturbance Response





Disturbance Response



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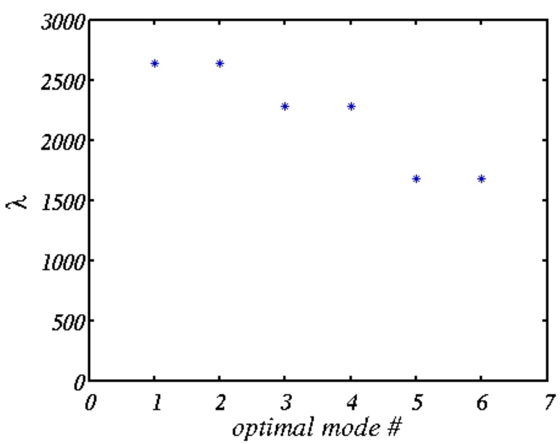
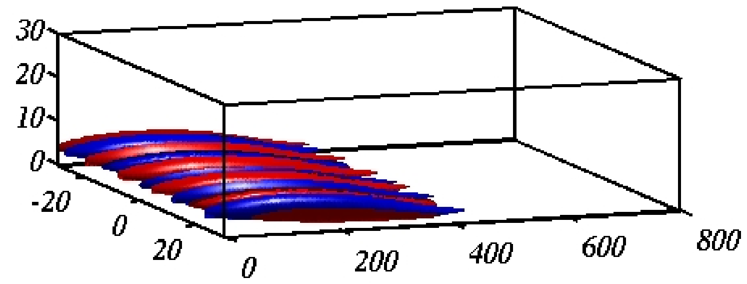
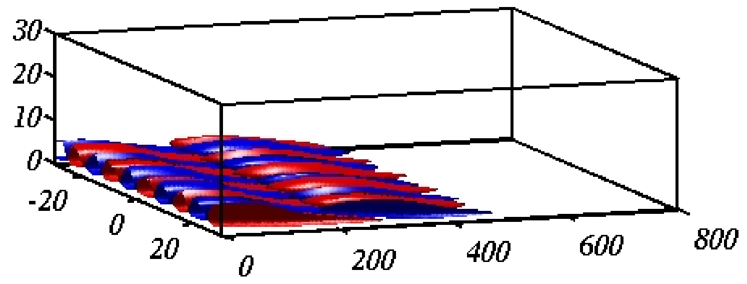
Results for initial conditions (Arnoldi)

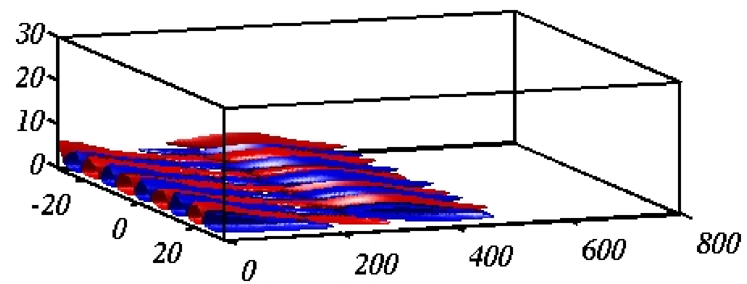
Eigenvalues gives energy growth



- Eigenvalues come in pairs with corresponding mode shifted in z







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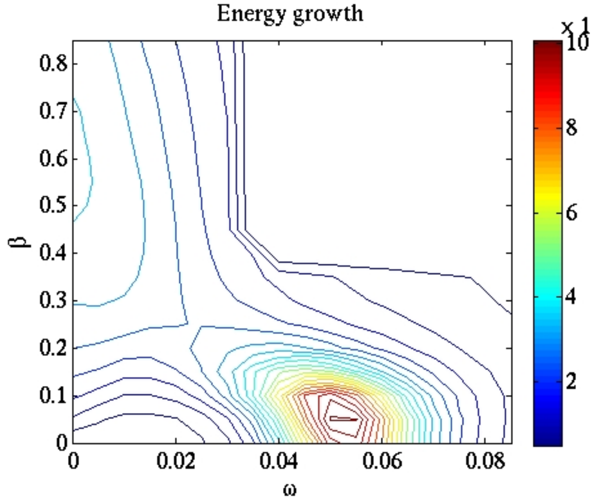
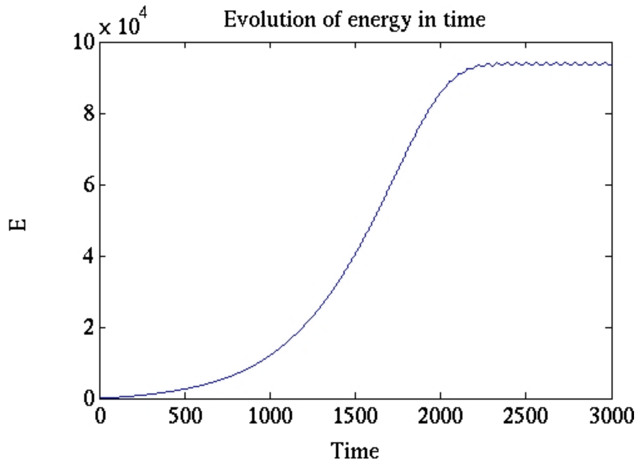
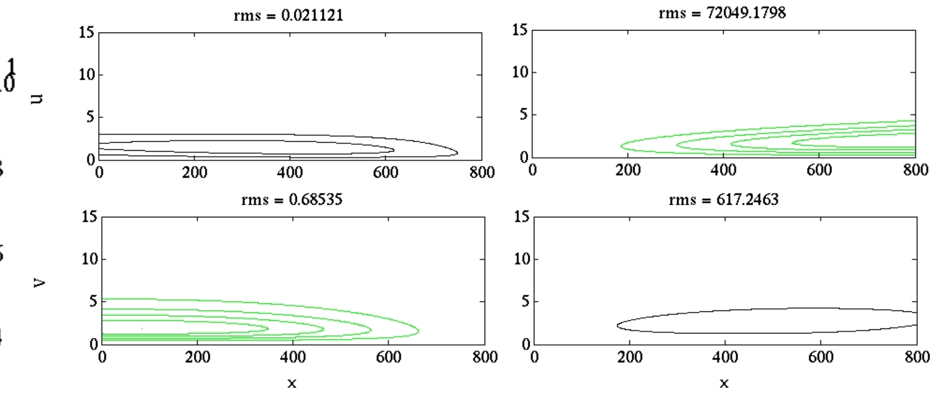
Results for forcing

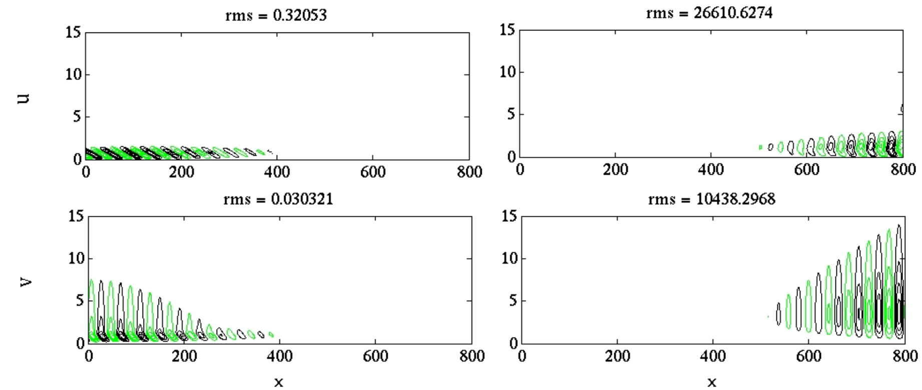




Forcing Response



Forcing Response



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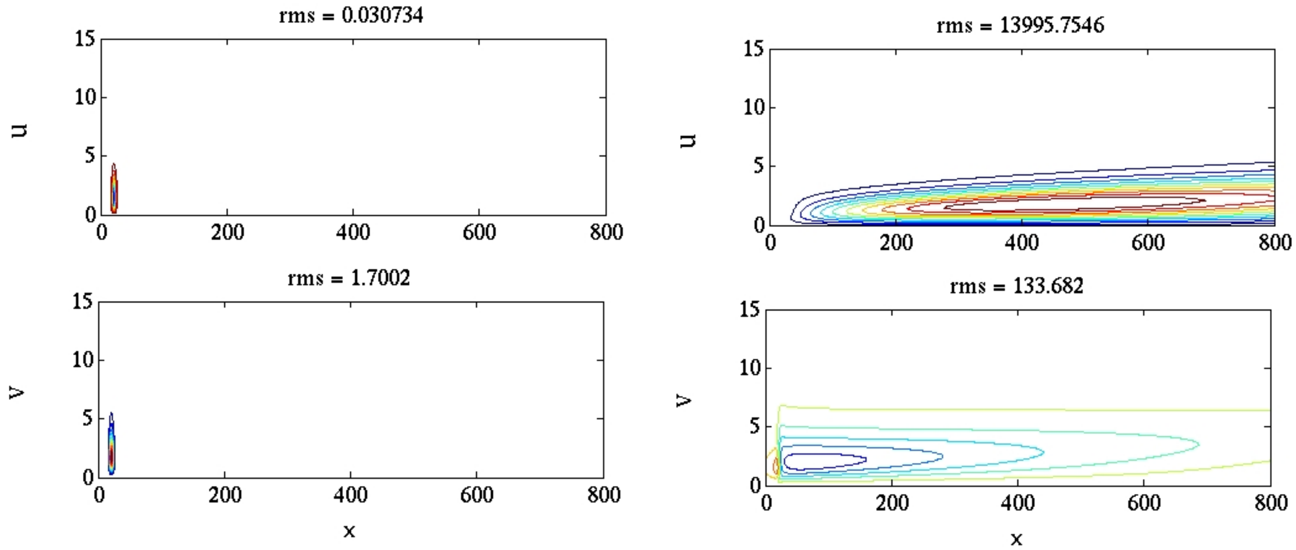
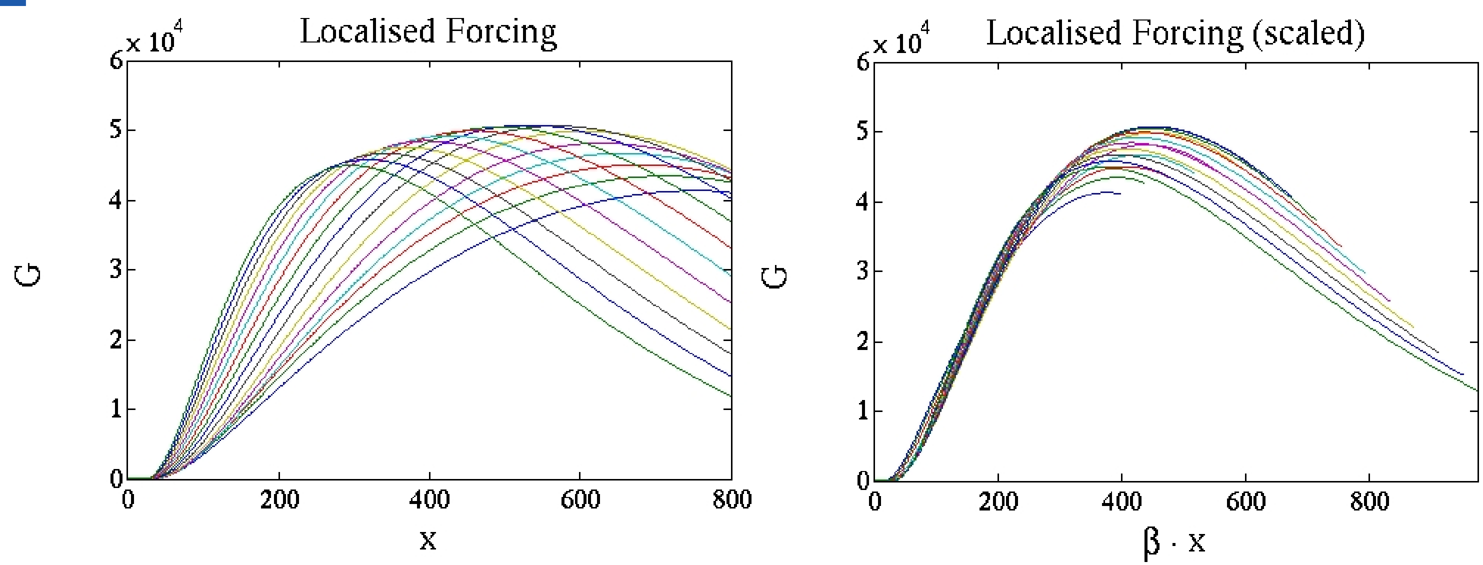
Typically integration time T=5000

Results for localized forcing

Forcing | Response





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Conclusions

- TS-mechanism gives more growth than lift-up since computational box is long and initial position is far down stream
- Smaller difference between maximum growth in TS-mechanisms and lift-up for optimal forcing
- Results are similar to previous studies with the boundary layer equations

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