Global stability of a jet in crossflow

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Jet in cross-flow

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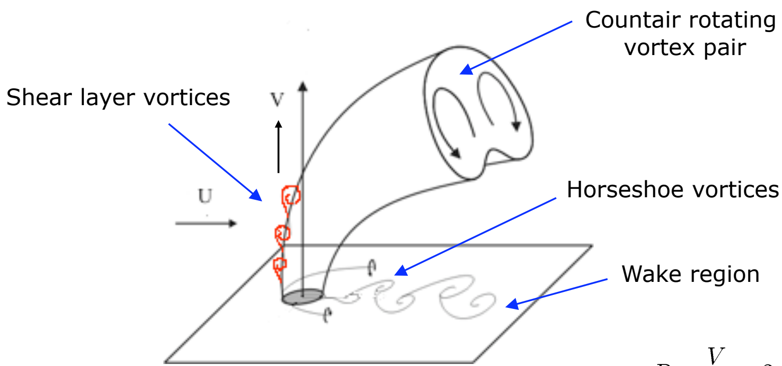

$$R = \frac{V}{U} = 3$$

$$\mathrm{Re} = \frac{U\delta_0}{\nu} = 165$$

- Is the flow linearly globally stable?
- What type of instability is it?

Direct numerical simulations

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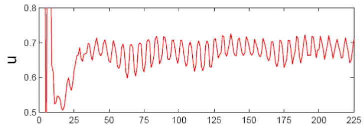
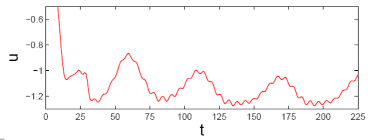
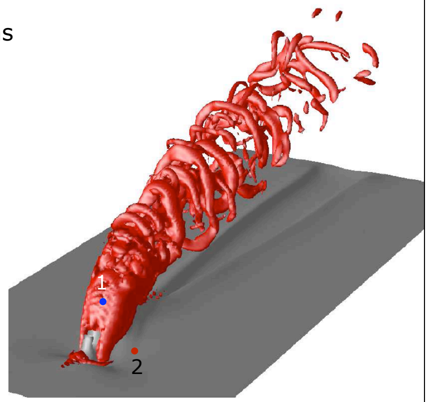
- DNS: Fully spectral and parallelized
- Self-sustained global oscillations
- Probe 1– shear layer



- Probe 2 – separation region





λ_2 Vortex identification criterion

Basic state

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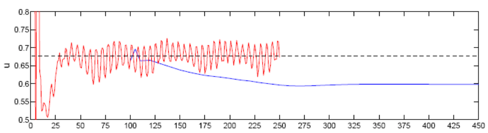
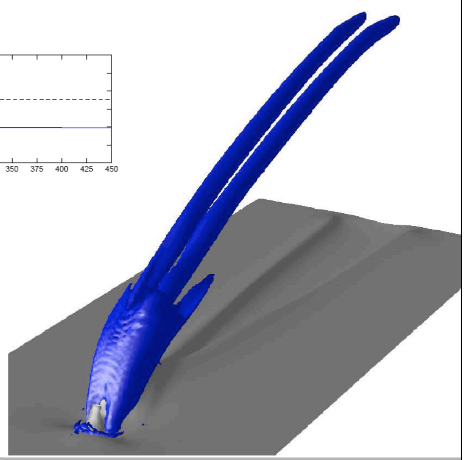
- Steady state computed using the SFD method
  (Åkervik *et.al.)*



Unsteady
Time-averaged
Steady-state



Linearized Navier-Stokes for jet in crossflow

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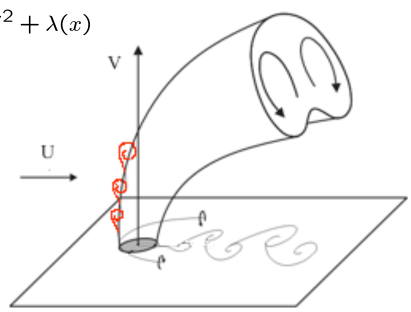
Continuous formulation

$$\begin{aligned} \frac{\partial u}{\partial t} &= \mathcal{A}u - \nabla p \\ 0 &= \nabla \cdot u \\ u &= u_0 \qquad \text{at} \quad t = 0 \end{aligned}$$

$$\mathcal{A} = -(U \cdot \nabla) - (\nabla U^T)^T + \frac{1}{Re}\nabla^2 + \lambda(x)$$

$$Re = \frac{U_\infty \delta_0^*}{\nu} = 165$$



Discrete formulation

$$\begin{aligned} \frac{du}{dt} &= Au \\ u &= u_0 \quad \text{at} \quad t = 0 \end{aligned}$$

Dimension of discretized system

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	Base Flow	Inhomogeneous direction(s)	Dimension of $\boldsymbol{u}(t)$	Storage of A
Ginzburg-Landau	$\boldsymbol{U}(x)$	1D	10^2	1 MB
Blasius	$\boldsymbol{U}(x,y)$	2D	10^5	25 GB
Jet in crossflow	$\boldsymbol{U}(x,y,z)$	3D	10^7	500 TB

- Matrix A very large for complex spatially developing flows
- Consider eigenvalues of the matrix exponential, related to eigenvalues of A

$$\lambda_j = e^{\omega_j t}$$

- Use Navier-Stokes solver (DNS) to approximate the action of matrix exponential or evolution operator

$$u(t) = e^{At}u_0 = T(t)u_0$$

Krylov subspace with Arnoldi algorithm

- Krylov subspace created using NS-timestepper
- Orthogonal basis created with Gram-Schmidt
- Approximate eigenvalues from Hessenberg matrix H

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Krylov subspace: $\{v_1, e^{At}v_1, \cdots, (e^{At})^{m-1}v_1\}$

orthogonal basis: $V = \{v_1, v_2, \cdots, v_m\}$

$$\Rightarrow \quad e^{At} \approx VHV^T \qquad H : m \times m$$

eigenvalues: $H = E\tilde{\Lambda}E^{-1} \quad \Rightarrow \quad e^{At} \approx VE\tilde{\Lambda}E^{-1}V^T$

Verification: Global spectrum for Blasius flow

- Least stable eigenmodes equivalent using time-stepper and matrix solver
- Least stable branch is a global representation of Tollmien-Schlichting (TS) modes

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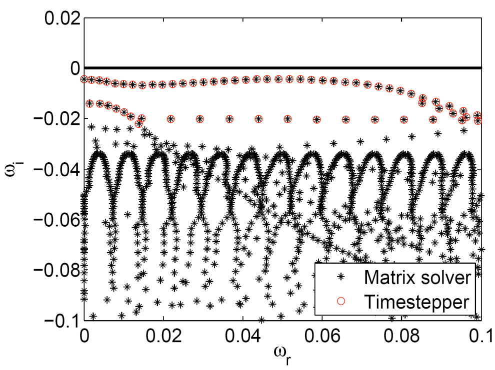

Åkervik et al *EJMB* (2008)
Bagheri et al *AIAA* (2008)

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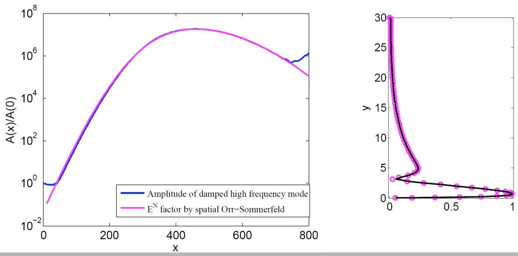
Verification: Global TS-waves

- Streamwise velocity of least damped TS-mode

- Envelope of global TS-mode identical to local spatial growth
- Shape functions of local and global modes identical



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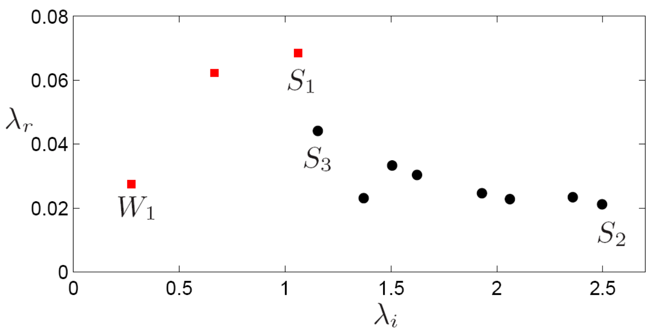
Global spectrum of jet in crossflow



Eleven first global eigenmodes
Fully three-dimensional
Highly unstable
Shear layer and wall modes
Symmetric (●) and anti-symmetric (■) modes

Global eigenmode S1

- Most unstable shear layer mode (anti-symmetric)
- Growth rate: 0.069
- Strouhal number: 0.17
- Strouhal number DNS: 0.16

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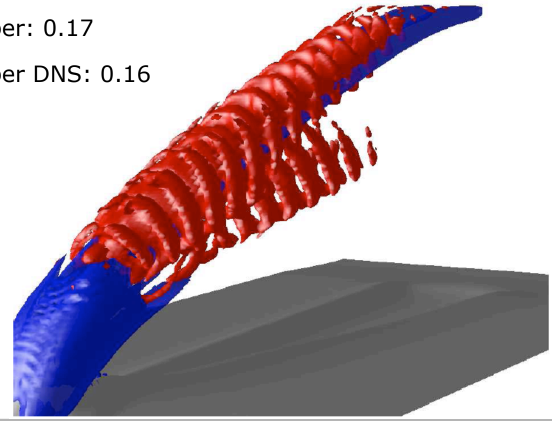

Global eigenmode W1

- Low-frequency wall-mode (anti-symmetric)
- Growth rate: 0.027
- Strouhal number: 0.04
- Strouhal number DNS: 0.016

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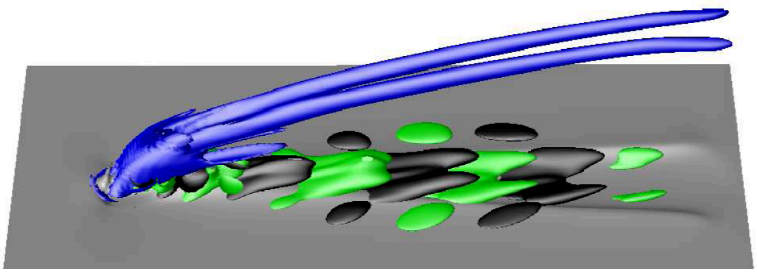
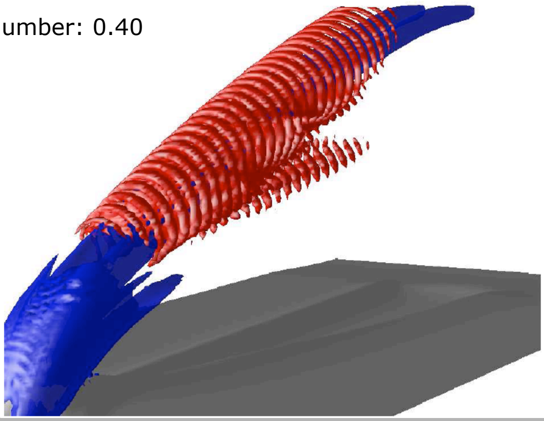


Global eigenmode S2

- High-frequency shear layer mode (symmetric)
- Growth rate: 0.021
- Strouhal number: 0.40



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Global eigenmode S3

- Most unstable symmetric shear layer mode, found by imposing symmetric impulse response in LNS
- Growth rate: 0.044
- Strouhal number: 0.18

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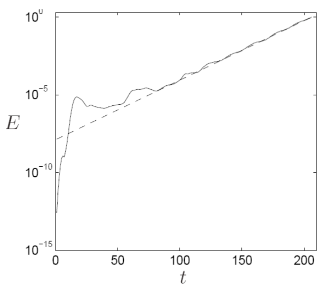

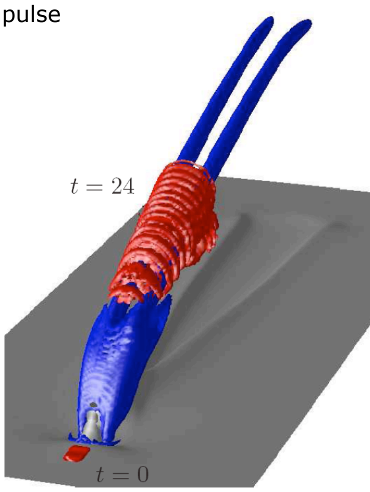

Conclusions

- Complex stability problems solved using Krylov-Arnoldi methods based on Navier-Stokes timestepper
- Globally unstable symmetric, anti-symmetric, wall and shear layer modes found
- Self-sustained synchronized oscillations at R=3:
  - Linear 3D global stability analysis
  - Observed in Direct Numerical Simulation
- Future work:
  - Bifurcation analysis: find critical velocity ratio
  - Sensitivity to forcing (adjoint global modes)
  - Optimal disturbances

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Seventh IUTAM Symposium on Laminar-Turbulent Transition

June 23-26, 2009
KTH Royal Institute of Technology
Stockholm, Sweden
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http://www.flow.kth.se/iutam09

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Optimal disturbance growth

- Optimal growth from eigenvalues of $T^*(t)T(t)$ $\quad T(t) = e^{At}$

$$G(t) = \max_{\|u_0\|=1} (u(t), u(t)) = \max_{\|u_0\|=1} (u_0, T^*(t)T(t)u_0)$$

$$T^*(t)T(t)u_0 = \lambda_E u_0$$

- Krylov sequence built by forward-adjoint iterations

$$T^*(t)T(t) \approx VHV^T = VE\Lambda_E E^{-1}V^T$$

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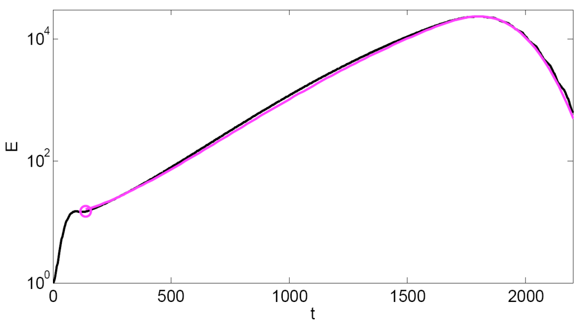
Evolution of optimal disturbance in Blasius flow

- Full adjoint iterations (black)
  sum of TS-branch modes only (magenta)
- Transient since disturbance propagates out of domain



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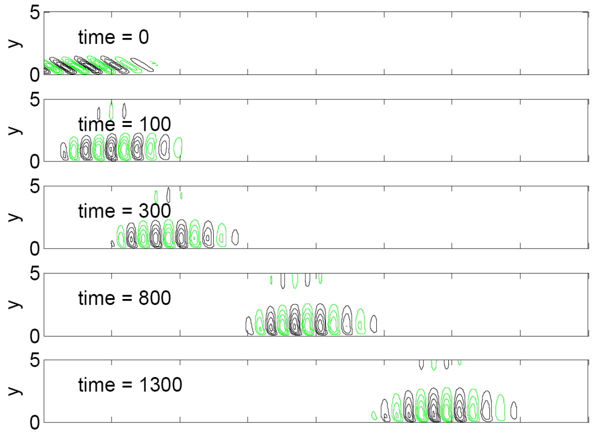
Snapshots of optimal disturbance evolution

- Initial disturbance leans against the shear raised up by Orr-mechanism into propagating TS-wavepacket



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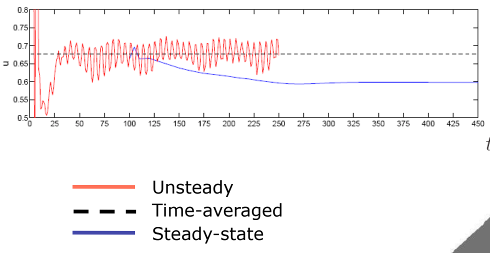
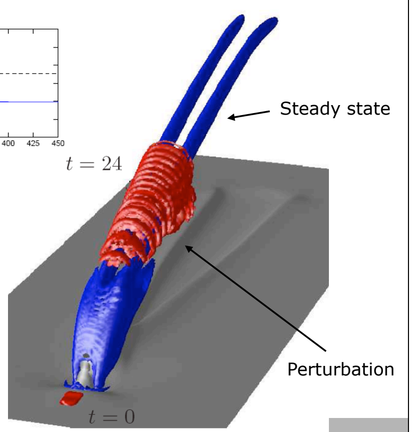
Basic state and impulse response

- Steady state computed using the SFD method
(Åkervik *et.al.)*





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Background

- Global modes and transient growth
  Ginzburg-Landau: Cossu & Chomaz (1997); Chomaz (2005)
  Waterfall problem: Schmid & Henningson (2002)
  Blasius boundary layer, Ehrenstein & Gallaire (2005); Åkervik et al. (2008)
  Recirculation bubble: Åkervik *et al.* (2007); Marquet et al. (2008)

- Matrix-free methods for stability properties
  Krylov-Arnoldi method: Edwards *et al.* (1994)
  Stability backward facing step: Barkley *et al.* (2002)
  Optimal growth for backward step and pulsatile flow: Barkley *et al.* (2008)

- Model reduction and feedback control of fluid systems
  Balanced truncation: Rowley (2005)
  Global modes for shallow cavity: Åkervik *et al.* (2007)
  Ginzburg-Landau: Bagheri *et al.* (2008)

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