

**ERCOFTAC**

European Research Community  
On Flow, Turbulence And Combustion

**10th ERCOFTAC SIG 33 Workshop**

**Progress in Transition Modeling and Control**

**Sandhamn, Sweden**

**May 29-31, 2013**







NORDITA

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**Organisers:**

**Ardeshir Hanifi (FOI/KTH)**

**Dan Henningson (KTH)**



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Wednesday	<b>Lunch</b>	<b>12:30</b>	
	<b>Session 1</b>	<b>Chair: U. Rist</b>	
	14:00-14:45	Denis Sipp	<b>Open-loop and closed-loop control of open cavity flows : manipulating the limit-cycle with linearized methods</b>
	14:45-15:05	Shervin Bagheri	Effects of small noise on the DMD/Koopman spectrum <i>S. Bagheri</i>
	15:05-15:25	Stefania Cherubini	Non-linear optimal control in the boundary layer <i>S. Cherubini, J.-Ch. Robinet &amp; P. De Palma</i>
	<b>Coffee break</b>		
	<b>Session 2</b>	<b>Chair: D. Sipp</b>	
	15:50-16:10	Georgios Rigas	Model-based framework for feedback control in the wake of axisymmetric bluff bodies <i>G. Rigas, A. S. Morgans &amp; J. F. Morrison</i>
	16:10-16:30	Reza Dadfar	Control of Boundary Layer Instabilities on an Airfoil using an Output Feedback Controller with Plasma Actuator <i>R. Dadfar, A. Hanifi &amp; D. S. Henningson</i>
	16:30-16:50	Sohrab Sattarzadeh	Boundary layer transition delay using passive flow control: AFRODITE <i>S. S. Sattarzadeh, S. Shahinfar &amp; J. H. M. Fransson</i>
16:50-17:10	Gabriele Rocco	Analysis of the underlying physics of the vortex shedding suppression in a flow past a cylinder via spanwise forcing <i>G. Rocco &amp; S. J. Sherwin</i>	
17:10-17:30	Vassilis Theofilis	Linear spatial global instability analysis of hypersonic flow over an elliptic cone <i>P. Parades &amp; V. Theofilis</i>	
17:30-17:50	Jens Sørensen	Multi Vortex Breakdown as a Manifestation of Helical Instability <i>J. Sørensen &amp; V. Okulov</i>	
<b>Dinner</b>	<b>19:45</b>		

Thursday	<b>Session 3</b>	<b>Chair: S. Sherwin</b>	
	09:00-09:45	Stephane Le Dizes	<b>Radiative instability in shear flows</b>
	09:45-10:05	Sergei Chernyshenko	Analysis of global stability of fluid flows using sum of squares of polynomials <i>S. Chernyshenko &amp; P. Goulart</i>
	10:05-10:25	Shuisheng He	Transition in unsteady channel flow <i>S. He &amp; M. Seddighi</i>
	<b>Coffee break</b>		
	<b>Session 4</b>	<b>Chair: S. Chernyshenko</b>	
	10:40-11:00	Jean-Christophe Robinet	Cylindrical roughness-induced transition on a flat plate <i>J.-Ch. Loiseau, J.-Ch. Robinet &amp; E. Leriche</i>
	11:00-11:20	Ulrich Rist	Experimental Investigation of 3D Instability Modes in the Wake of a Circular Hump in a Boundary Layer <i>U. Rist &amp; Y. Shin</i>
	11:20-11:40	Bastien Jordi	Encapsulated Selective Frequency Damping method <i>B. Jordi, C. Cotter &amp; S. Sherwin</i>
	11:40-12:00	Andrea Mazzino	Elastic-like instabilities in non-Newtonian parallel flows <i>S. Boi, A. Mazzino &amp; J. O. Pralits</i>
<b>Lunch</b>			
<b>Session 5</b>	<b>Chair: R. Kerswell</b>		
13:30-14:15	Predrag Cvitanovic	<b>You Think You Know How To Cycle? That's Rich</b>	
14:15-14:35	Taras Khapko	From periodic to chaotic self-sustaining process in boundary-layer flows <i>T. Khapko, T. Kreilos, P. Schlatter, Y. Duguet, B. Eckhardt &amp; D. S. Henningson</i>	
14:35-14:55	Tobias Kreilos	Pockets of Turbulence in Plane Couette Flow <i>T. Kreilos, B. Eckhardt &amp; T. M. Schneider</i>	
14:55-15:15	Philipp Schlatter	Oblique laminar-turbulent interfaces in transitional shear flows <i>Y. Duguet &amp; P. Schlatter</i>	
<b>Coffee break</b>			

<b>Session 6</b>		<b>Chair: V. Theofilis</b>
15:40-16:00	Alexandre Suryadi	Unstable structures in rotating plane Couette flow - quantitative measurements of flow velocities <i>A. Suryadi &amp; P. H. Alfredsson</i>
16:00-16:20	Shintaro Imayama	Laminar-turbulent transition of rotating-disk flow - an experimental and numerical approach <i>S. Imayama, E. Appelquist, R. J. Lingwood, P. Schlatter &amp; P. H. Alfredsson</i>
16:20-16:40	Robert Downs	Experimental Investigation of Swept-Wing Boundary Layer Receptivity to Modest Freestream Turbulence <i>R. S. Downs &amp; E. B. White</i>
16:40-17:00	S. Mohammad Hosseini	Optimal initial perturbations in streamwise corner-flow <i>O. Schmidt, S. M. Hosseini, U. Rist, A. Hanifi &amp; D. S. Henningson</i>
17:00-17:20	Oliver Schmidt	A discrete mode representation of the spatial transient growth mechanism found in streamwise corner-flow <i>O. T. Schmidt &amp; U. Rist</i>
17:20-17:40	Jean-Michel Lucas	Optimal disturbances on curved surfaces for two-dimensional compressible boundary layers <i>J.-M. Lucas, O. Vermeersch &amp; D. Arnal</i>
<b>Dinner</b>		<b>20:00</b>

<b>Session 7</b>		<b>Chair: P. Cvitanovic</b>
09:00-09:45	Rich Kerswell	<b>Nonlinear Transient Growth: A New Tool for Analysing Nonlinear Stability</b> <i>R. Kerswell, C. Caulfield, C. Pringle, S. Rabin &amp; A Willis</i>
09:45-10:05	Masato Nagata	New exact coherent states in plane Poiseuille flow <i>M. Nagata &amp; K. Deguchi</i>
10:05-10:25	Genta Kawahara	The onset of transient turbulence in minimal plane Couette flow <i>G. Kawahara, J. R. T. Lustro &amp; L. van Veen</i>
<b>Coffee break</b>		
<b>Session 8</b>		<b>Chair: M. Nagata</b>
10:45-11:05	Yohann Duguet	Minimal transition thresholds in plane Couette flow <i>Y. Duguet, A. Monokrousos, L. Brandt &amp; D. S. Henningson</i>
11:20-11:40	Matthew Chantry	Decaying past the edge: studying decay in plane Couette flow. <i>M. J. Chantry and T. Schneider</i>
11:40-12:00		
<b>Lunch</b>		

Friday

Open-loop and closed-loop control of open cavity flows :  
manipulating the limit-cycle with linearized methods

Denis Sipp  
ONERA/DAFE

In this talk we will consider a prototypical oscillator flow, an open cavity flow in the incompressible regime for Reynolds numbers in the range [4000-7500]. For a supercritical Reynolds number, there are several unstable global modes, which are all linked to the Rossiter mechanism. The flow converges on a limit-cycle, which may be detrimental to the applications (vibrations, noise, etc). We wish either to suppress the unsteadiness (stabilize the global modes) or shift the frequency away from its natural value (for example to avoid resonance with a structural mode). We will analyse two different ways to achieve these goals, open-loop and closed-loop control. We will give an overview of these two control strategies. In particular, we will highlight how linearized methods are able to tackle strongly non-linear dynamics, such as those observed when control is switched on from the limit-cycle.

In the case of open-loop control (Sipp JFM 2013), we consider a time-periodic momentum forcing characterized by a frequency  $\omega_f$  and an amplitude  $E'$ . Thanks to a weakly-non linear analysis valid in the vicinity of the bifurcation threshold, we will build amplitude equations governing the dynamics of the unstable global mode. Depending on the forcing frequency  $\omega_f$ , we will show that the forcing term may enter the amplitude equations in different forms. If  $\omega_f$  is close to the natural frequency of the flow, a locking phenomenon may occur. The forcing amplitude  $E'$  that achieves locking is shown to be very small so that forcing in the vicinity of the natural frequency of the flow is a very efficient means to slightly modify the frequency of an oscillator flow. If  $\omega_f$  is not close to the natural frequency of the flow, then the unstable global mode is either stabilized or destabilized. We will show that most of the forcing frequencies achieve stabilization of the global mode. From a physical point of view, this partly stems from the fact that the forcing term generates a mean-flow which has a stabilizing effect on the global mode. Yet, the forcing amplitudes to achieve stabilization are much higher than those which achieve locking of the flow. We will show that direct numerical simulations in which the control has been switched on from the limit cycle accurately follow the predictions of the amplitude equations: both the locking and the stabilizing phenomena could be observed at the predicted forcing amplitudes.

In the case of closed-loop control, we consider an actuator located upstream and a sensor located downstream. We will first quickly recall 1/ how a reduced-order-model based on the unstable global modes and few balanced modes may be built so that it accurately captures the dynamics from the actuator to the sensor 2/ how a compensator combining a controller and an estimator may be obtained within the Linear-Quadratic-Gaussian control framework. We will show thanks to compensated direct numerical simulations that such a strategy is able to maintain a system in the vicinity of the base-flow. Yet, we will show that this strategy is not able to drive the system from the limit-cycle toward the base-flow. We will show that this may be understood in the light of a robustness analysis. Several techniques to increase robustness of the compensators will be analysed. In particular, we will highlight the importance of some parameters (Reynolds number, location of sensor, delay between the actuator and the sensor, number of sensors) for the robustness of the compensator. Note that closed-loop control is a joint work with Peter Schmid (Ladhyx).

# Effects of small noise on the DMD/Koopman spectrum

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Linné Flow Centre,  
KTH Mechanics, 10044 Stockholm, Sweden

Koopman modes and Dynamic Mode Decomposition (DMD) have quickly become popular tools for extracting coherent structures associated with different frequencies from both (nonlinear) numerical and experimental flows. As an example of a DMD spectrum, we show in figure 1 the typical parabolic branches that the eigenvalues form. This parabolic shape is observed in nearly all DMD spectra, and it is often the case that the tails of the parabolas are sensitive to the quality of the data set. In this talk, we provide a theoretical explanation for this parabolic form of the spectrum, and show that it arises due to the presence of noise. We show analytically that the presence of noise induces a damping on the eigenvalues, which increases quadratically with the frequency, and linearly with the non-normality of the linearized (Floquet) system. Thus the location of Koopman eigenvalues in the complex plane varies depending on the amount of noise in the environment, and one cannot expect any variant of the Dynamic Mode Decomposition algorithm to be fully robust to noise.

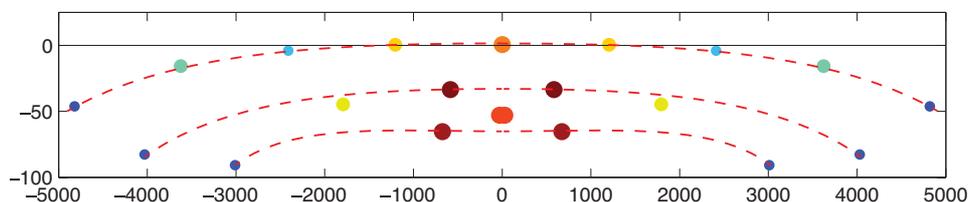


Figure 1: The DMD/Koopman spectrum obtained from processing Schlieren snapshots of a turbulent helium jet (from *Schmid, Li, Juniper and Pust, Theoret. Comput. Fluid Dyn., vol 25, 2011*), The typical parabola one would expect for a noisy limit cycle are shown with dashed lines.

# Non-linear optimal control in the boundary layer

S. Cherubini<sup>1</sup>, J. C. Robinet<sup>1</sup>, P. De Palma<sup>2</sup>

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<sup>2</sup> DIMMM, Politecnico di Bari, Italy

When excited by finite-amplitude ambient perturbations, shear flows can undergo sub-critical transition. To unravel the mechanisms triggering such a finite-amplitude route to transition, some authors<sup>1,2</sup> have computed *non-linear optimal perturbations*, i.e., finite-amplitude perturbations which induce the largest energy growth in the flow, due to both linear and non-linear effects. In this work, we aim at controlling the growth of such optimal perturbations in a fully non-linear framework, by using a blowing and suction strip placed on the flat plate where a boundary-layer flow develops. The inlet of the domain is placed at  $x = 150$  displacement thickness units, where the Reynolds number,  $Re_{in} = 600$ , is supercritical, whereas the blowing and suction strip is placed at  $215 < x < 280$ . We use a Lagrange multiplier method to minimize an objective function based on both the energy of the perturbations at a target time, and a cost function depending on the control law. Thus, the minimization procedure is based on the iterative computation of the direct and adjoint three-dimensional Navier-Stokes equations, together with a steepest descent algorithm, in order to reduce the gradient of the functional to zero machine. For large enough target times ( $T > 75$ ), the optimal control law has been found effective to induce relaminarization, as shown in the left frame of Figure 1, providing the evolution in time of the maximum value of the wall-normal velocity. In particular, the optimal blowing/suction law appears to affect the initial vortices, weakening their amplitude as well as their inclination with respect to the streamwise direction (compare the top and the bottom right frames of Figure 1, for the controlled and the uncontrolled case, respectively). As a consequence, the streamwise velocity does not grow enough to induce the non-linear coupling effects which lead the non-linear optimal perturbation to turbulence<sup>3</sup>. The optimal control law has been computed also in a linearized case, and the results have been compared with the non-linear ones in order to establish the influence of non-linearity for the determination of an effective control law. Moreover, different initial perturbations, leading the flow to turbulence, have been considered in order to validate the control approach. Finally, the influence of the independent parameters of the minimization has been investigated.

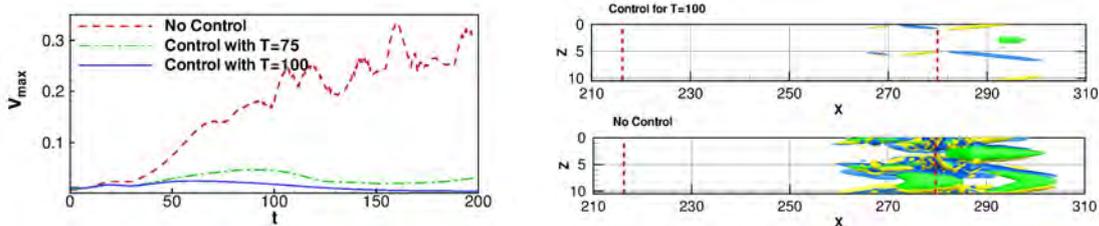


Figure 1: Time evolution of the maximum of the wall-normal velocity component for the uncontrolled case, and two controlled cases with  $T = 75$  and  $T = 100$  (a). Snapshots of the DNS in the controlled (top) and uncontrolled (bottom case) for  $t = 100$ : streamwise velocity (green), and streamwise vorticity (yellow and blue). The red dashed lines indicates the location of the control strip.

<sup>1</sup>Cherubini, S., De Palma, P., Robinet, J.-Ch. & Bottaro, A. *Phys. Rev. E* **82**, 066302, 2010.

<sup>2</sup>Pringle, C. C. T. and Kerswell, R.R. *Phys. Rev. Lett.* **105**, 154502, 2010.

<sup>3</sup>Cherubini, S., De Palma, P., Robinet, J.-Ch. & Bottaro, A. *J. Fluid Mech.* **689**, 221–253, 2011.

# Model-based framework for feedback control in the wake of axisymmetric bluff bodies

G. Rigas\*, A. S. Morgans and J. F. Morrison

*Department of Aeronautics, Imperial College London, London, SW7 2AZ, UK*

Recent studies<sup>1</sup> of the flow past axisymmetric bluff bodies have shown that the laminar wake loses its stability due to a sequence of bifurcations with increasing Reynolds number. Simple nonlinear models<sup>2</sup> allow most of the wake dynamics to be properly captured. These models have been validated numerically at low Reynolds numbers close to the critical bifurcation points.

In the present theoretical and experimental study, we extend these theories to high Reynolds numbers ( $2 \times 10^5$ ) and we include the effect of actuation in the models. The derived Stuart-Landau-type models, which are based on a weakly nonlinear analysis of the Navier-Stokes equations, describe the nonlinear interaction and the amplitude evolution of the modes under different types of forcing. The truncated number of modes is validated from experimental measurements.

The axisymmetric bluff body we used to perform the experiments is a bullet-shaped body of revolution, as shown in figure 1. Actuation and sensing locations are taken on the base of the body. The actuator is a Zero-Net-Mass-Flux device driven by 4 speakers and interacts with the flow through an annular orifice at the circumference of the base. Sensing is achieved by measuring the mean and fluctuating pressure on the base.

A wide range of forcing amplitudes and frequencies are tested and the response of the vortex shedding (global) mode is measured. The wake is forced either axisymmetrically ( $m = 0$ ) or helically ( $m = \pm 1$ ), and for both cases the vortex shedding mode “locks-in” when forcing close to the shedding frequency or its rational multiples, revealing nonlinear oscillator behavior in accordance with the model predictions. An identification procedure uses forced transients to identify the unknown parameters of the model parameters from experimental data. Very good agreement between model simulations and experiments is obtained, showing for the first time that Landau-type models are capable of capturing the dynamic behavior of a three-dimensional turbulent wake.

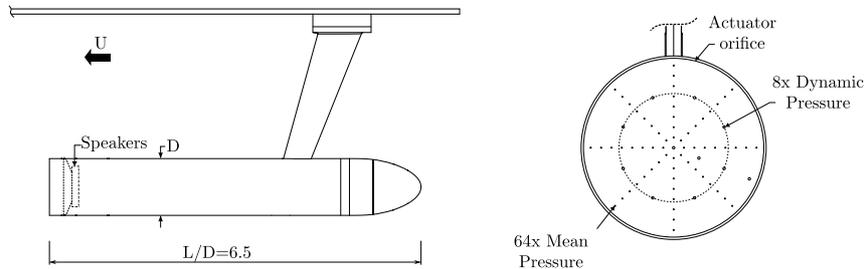


Figure 1: Schematic representation of the axisymmetric bluff body. (a) side view. (b) rear view.

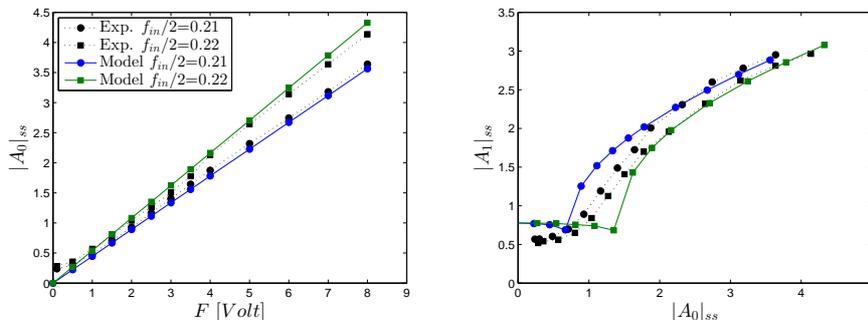


Figure 2: Parametric subharmonic resonance of the global mode  $A_1$  forcing the axisymmetric mode  $A_0$ : comparison of the steady-state amplitude between experiment and model at two different forcing frequencies.

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<sup>1</sup>D. Fabre, F. Auguste, and J. Magnaudet, *Phys. Fluids* 20 (2008).

<sup>2</sup>P. Meliga, J.-M. Chomaz, and D. Sipp, *J. Fluid Mech.* 633 (2009).

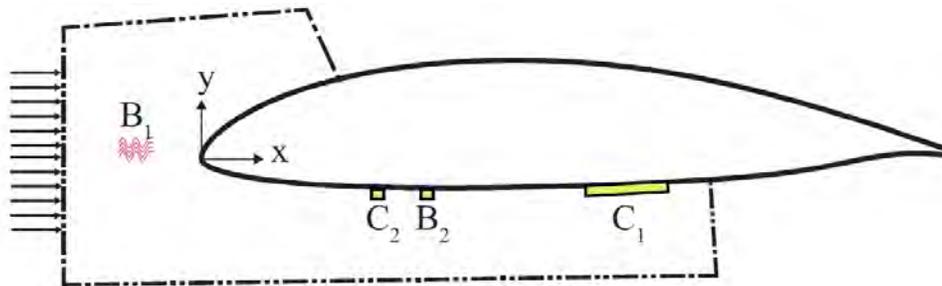
# Output Feedback Control For Transition Delay Over Airfoil Using Plasma Actuators

Reza Dadfar<sup>1</sup>, Ardeshir Hanifi<sup>1,2</sup>, and Dan S. Henningson<sup>1</sup>

<sup>1</sup>Linné FLOW Centre, Mechanics, KTH

<sup>2</sup>Swedish Defence Research Agency, FOI

In this study, we investigate active control of a flow developing over an airfoil using direct numerical simulations (DNS) including plasma actuators. The initial perturbation is located upstream of the leading edge; the impulse response of the system is characterized by free-stream perturbations quickly advected downstream, penetrating inside the boundary layer through the receptivity mechanism and triggering the unstable TS waves. A localized sensor and actuator are located close to the wall, used to damp the amplitude of the propagating TS wave. The controller is designed on a low dimensional model of the linearized Navier-Stokes equations where the model is identified by using the Eigensystem Realization Algorithm (ERA). Once the reduced order model of the system is obtained, the controller can be easily built by using the standard tools of control theory. A Linear Quadratic Gaussian (LQG) controller is introduced for computing the control law. As the objective of the controller, output projection is used, by introducing a basis of proper orthogonal decomposition (POD) modes. The modes are selected such that only the disturbances characterized by certain frequencies are included in the basis; this choice allows discriminating the TS waves from other types of disturbances penetrating inside the boundary layer.



*Schematics of computational domain including actuators and sensors.*

Here, we model single dielectric barrier discharge (SDBD) plasma actuators by a volume forcing measured in experiments. Limitations governing these actuators are accounted during the design of our controller. Indeed, the plasma actuators can force the flow only along one direction, approximately oriented with the streamwise coordinate, while the sign of the signal can dictate the direction of the forcing. In general such a constraint requires nonlinear control design; to address this limitation, we investigate two possible alternatives in order to extend the use of the LQG controllers. A first possibility relies on the introduction of a constant forcing (analogous to modifying the baseflow) and on the top of which the optimal control signal is actually applied. The resultant forcing is applied in only one direction, meanwhile preserving the ability of canceling wavy perturbations. A second alternative is represented by the design of a controller based on two adjacent actuators, each of them characterized by a specific direction. Both the procedures result in a successful attenuation of the disturbance amplitudes, with efficiency comparable to the standard LQG method.

## Boundary layer transition delay using passive flow control: AFRODITE

S. S. Sattarzadeh, S. Shahinfar and J. H. M. Fransson

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Classical vortex generators, known for their efficiency in delaying or even inhibiting boundary layer separation, are here shown to be coveted devices for transition to turbulence delay. The present devices are miniature with respect to classical vortex generators but are tremendously powerful in modulating the laminar boundary layer in the direction orthogonal to the base flow and parallel to the surface. The modulation generates an additional term in the perturbation energy equation, which counteracts the wall-normal production term, and hence stabilizes the flow. Our experimental results show that these devices are really effective in delaying transition but we also reveal their Achilles' heel<sup>1</sup>.

The physical mechanism of the stabilizing effect is known<sup>2</sup> and has previously been shown to be strong enough to delay transition to turbulence in wind tunnel experiments<sup>3</sup>, where the base flow was modulated by means of cylindrical roughness elements. Later this result has been confirmed numerically in ref.<sup>4</sup>. The experimental design (as well as the numerical simulation) was, however, fairly laboured, since the artificial disturbance was introduced downstream of the cylindrical roughness array avoiding any potential non-linear interaction of the incoming disturbance with the roughness array. In a recent study the flow configuration and experimental setup has challenged the passive flow control method by generating controlled disturbances upstream of the boundary layer modulators and showed promising results in being capable of delaying transition to turbulence<sup>1</sup>.

In this experimental investigation we show that miniature classical vortex generators really are suitable devices in accomplishing transition delay and plausible to work in real flow applications. MVGs are clearly superior to circular roughness elements, since the flow is allowed to pass right through them, possibly reducing the absolute instability region behind the devices and allowing for twice as high amplitude streaks to be generated, but still with some margin to the threshold amplitude beyond which the streaky base flow becomes unstable. This makes the streaky base flow much more robust for external perturbations, a prerequisite for real flow applications. Furthermore, in the present setup the TS waves are being generated upstream of the MVG array, leaving the full and nasty receptivity process of the incoming wave by the MVG array, which really challenges the present passive flow control method. Despite this, transition delay is convincingly accomplished.

This work is financially supported by the European Research Council and is performed within the **AFRODITE** research programme, which stands for Advanced Fluid Research On Drag reduction In Turbulence Experiments.

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<sup>1</sup>Shahinfar, S., Sattarzadeh, S.S., Fransson, J.H.M. & Talamelli, A. (2012). *Phys. Rev. Lett.* **109**, 074501.

<sup>2</sup>Cossu, C. & Brandt, L. (2004). *Eur. J. Mech./B Fluids* **23**, 815–833.

<sup>3</sup>Fransson, J.H.M., Talamelli, A., Brandt, L. & Cossu, C. (2006). *Phys. Rev. Lett.* **96**, 064501.

<sup>4</sup>Schlatter, P., Deusebio, E., de Lange, R. & Brandt, L. (2010). *Int. J. Flow Contr.* **2**, 259–288.

# Analysis of the underlying physics of the vortex shedding suppression in a flow past a cylinder via spanwise forcing

Gabriele Rocco<sup>1</sup>, Spencer J. Sherwin<sup>1</sup>

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Controlling efficiently the vortex dynamics of wakes of bluff bodies is a fundamental problem in many applications. Earlier direct numerical simulations [1] of three-dimensional bluff bodies demonstrated that the introduction of a spanwise waviness at both the leading and trailing surfaces suppresses the vortex shedding and reduces the amplitude of the fluctuating aerodynamic forces. Under this motivation, direct numerical simulations and stability analysis of the flow past a three-dimensional cylinder in the supercritical regime ( $Re = 60$ ) were performed. Starting from a fully developed shedding, a sufficiently high spanwise forcing is introduced on the surface of the cylinder, in the regions where separation effects occur, resulting in the stabilisation of the near wake in a time-independent state, similarly to the effect of a sinusoidal stagnation surface. Numerical experiments were conducted to detect the critical values of the amplitude of the forcing able to suppress the vortex street and the respective physical structure of the wake was identified with three different regimes, namely an under-suppressed, suppressed and over-suppressed regime. The wake topology of the over-suppressed regime was investigated. At the current Reynolds number the flow does not show any significant variations with the increase of the forcing over the critical value and the dynamics is characterised by a steady near-base region where the von Kármán vortex street is completely suppressed. Stability analysis of the linearised Navier-Stokes equations was then performed on the three-dimensional flow to investigate the role of spanwise modulation on the classical absolute instability associated with the von Kármán street. Figure (1) shows the iso-contours of the three-dimensional eigenmode and the decay rate of the instabilities was found to be roughly the same of a flow at  $Re \simeq 30$ . The introduction of a symmetric axial condition in the basic state allowed us to repeat the stability analysis to study the refinement of the onset of the instabilities and it showed, even in stable flows, residual traces of the periodic shedding in the far wake and a shift of the maximum value towards the cylinder.

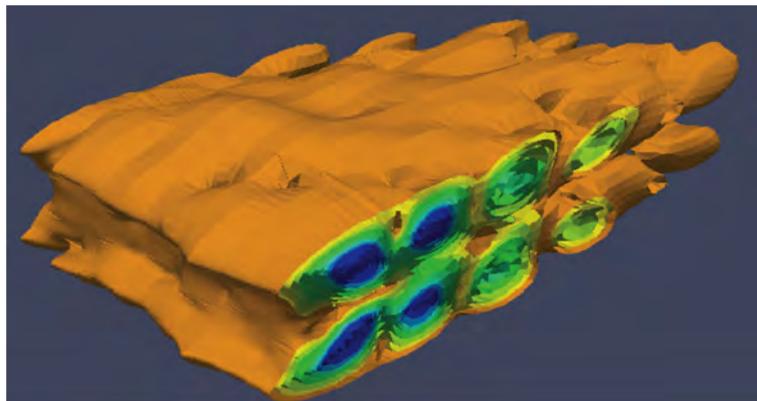


Figure 1: 3D eigenmode of the over-suppressed regime

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- [1] Darekar R., M., Sherwin S. J. , Flow past a square-section cylinder with wavy stagnation face, *J. Fluid Mech* **426**, 263-295, 2001.

# LINEAR SPATIAL GLOBAL INSTABILITY ANALYSIS OF HYPERSONIC FLOW OVER AN ELLIPTIC CONE

P. PAREDES, V. THEOFILIS

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Linear instability of the three-dimensional laminar boundary-layer developing on a rounded-tip 2:1 elliptic cone, serving as model of the HIFiRE-5 flight test geometry [1], is analyzed at hypersonic speeds and a flight altitude of  $\sim 30km$  in an effort to understand laminar-turbulent transition mechanisms and predict heat loads on next-generation flight vehicles.

Spatial BiGlobal linear theory is employed to analyze flow at an intermediate axial section at which  $Re_{\delta^*} = 11354$ , formed with the local freestream values and boundary-layer thickness,  $\delta^*$ , at the minor-axis center-line. Figure 1(a) shows the Mach number distribution,  $Ma_x$ , at this section. Eigenspectrum results reveal three different families of spatially unstable global modes, the attachment-line (AL, Figure 1(b)) and cross-flow (CF, Figure 1(c)) modes known from earlier analyses [2, 3], and a new mode discovered in the present work, termed center-line mode (CL, Figure 1(d)). At the conditions analyzed, the AL modes are found to be the most unstable linear instabilities, a result which is in-line with the predictions of [4], who analyzed the same base flow using (classic) PSE. At the specific flow altitude and axial section of the cone, the center-line mode is the least unstable of the three; however, a different scenario may be operative at lower Reynolds number, at which early transition near the top centerline has been observed experimentally. Characterization of the linear eigenmodes at other locations and flow conditions is currently underway using the available base flows[4].

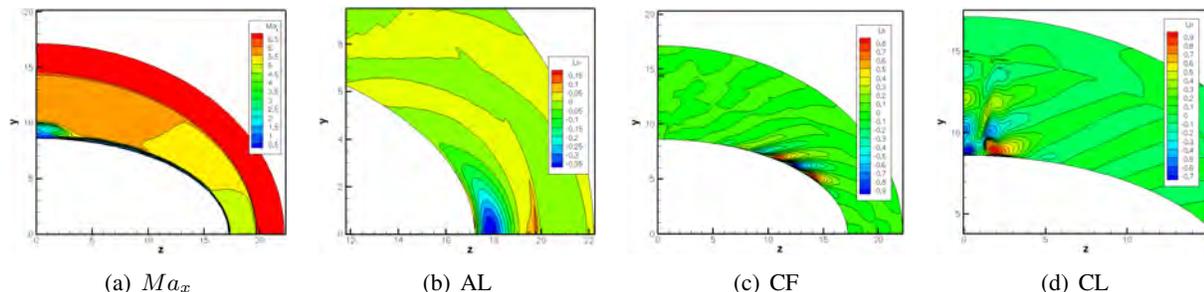


Figure 1: Base flow Mach number in (a) and the real parts of the streamwise velocity of the attachment-line (b), cross-flow (c) and center-line (d) eigenmodes obtained at  $F^* = 26$  kHz.

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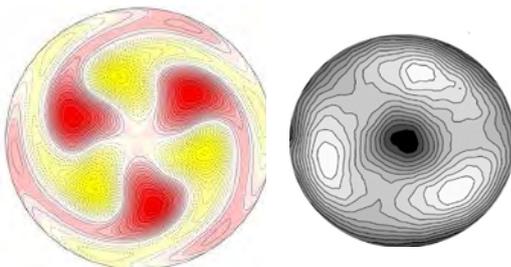
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# Multi Vortex Breakdown as a Manifestation of Helical Instability

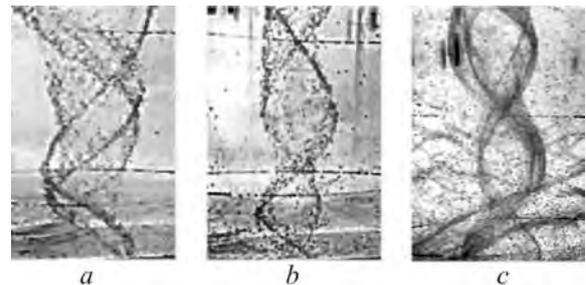
**Jens N. Sørensen & Valery L. Okulov**

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Vortex breakdown is a phenomenon inherent to many practical problems in aerodynamics, geophysics, and engineering, which may appear e.g. in tip vortices over wings and behind propellers, in atmospheric tornadoes and cyclones, or in flame-holder vortices in combustion devices. The breakdown of the vortices is associated with an abrupt deceleration of the axial velocity on the vortex axis which sometimes develops to a recirculation zone. Although now about ten forms of vortex breakdown have been identified [1-3], the gist of the phenomenon is, however, not yet fully understood. There have been many attempts to explain vortex breakdown, but most theories only deal with a part of the phenomenon [1]. In the present work we seek to explain vortex breakdown as a change between different flow states with different vorticity distributions using the concept of helical symmetry, implicitly understanding that the initial vortex state is subject to instability. In earlier investigations of swirling flows in a rotating cavity we have made parametrical studies of vortex breakdown [4] and, by using stability analysis, detected new vortex breakdown states [3]. The numerically determined stability boundaries were found to be in very good agreement with experimental observations [4]. These states describe three different non-axisymmetric forms of the central vortex structures, which were transformed from an initial axisymmetric state. The appearance of the new stable forms has recently been confirmed by 3-D simulations [5]. The iso-vorticity plots in Fig. 1 show the good comparison between experimental and numerical results. The visualizations in Fig. 2 show the three-dimensional multi-plet structure of the vortices. In the presentation, using the concept of helical symmetry, we combine the approaches of conjugate state theory and stability analysis to explain vortex breakdown as a transition between two stable vortex states. Analyzing vortex breakdown of columnar vortices as well as multi vortex breakdown shows that that a combined approach may explain ‘classical’ configurations of single vortex breakdown as well as multi vortex breakdown.



**Fig 1.** Example of two types of disturbed vorticity: Left: Multipole or rot. wave [5]; Right: Multiplet [3]



**Fig 2.** Bubble visualization of multi vortex breakdown in cavity flow: (a) triplet, (b) doublet, (c) quadruplet.

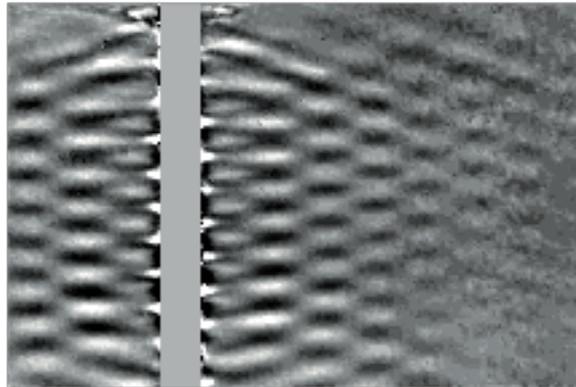
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## **Radiative instability in shear flows**

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In a compressible or stratified medium, or any medium that can support waves, open shear flows can become unstable by spontaneously emitting waves. The mechanism of this radiative instability will be described in details. We shall show that this instability is active in boundary layer flows, jets and vortices. Numerical results will be provided in each case. Experimental evidence will also be shown.



Experimental evidence of the radiative instability of the flow around a rotating cylinder in a stratified fluid. (From Riedinger, Le Dizes, Meunier, 2011)

## Analysis of global stability of fluid flows using sum of squares of polynomials

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The progress on the new method of the analysis of global stability of fluid flows is reported.

A global stability of a particular steady flow can be proven by constructing a Lyapunov functional  $V[\mathbf{u}]$  of the velocity perturbation  $\mathbf{u}$ , that is a positive-definite functional of  $\mathbf{u}$  that decays monotonously on any solution of the Navier-Stokes equations. In the well-known energy stability approach the Lyapunov functional is chosen to be the perturbation energy  $E = \|\mathbf{u}\|^2/2$ , where the norm is defined as an integral of  $u^2$  over the flow domain. In this case the problem of finding the range of the Reynolds numbers in which  $E$  is a Lyapunov functional reduces to a linear eigenvalue problem, which is tractable. However, the estimates of the global stability range, obtained using the energy stability approach, frequently happen to be very conservative. To improve on this, [1] proposed to use a functional of the form  $V[\mathbf{u}] = V(\mathbf{a}, q^2)$ , where  $\mathbf{a} = (a_1, \dots, a_n)$  are the first  $n$  coefficients of the Galerkin expansion of  $\mathbf{u}$  with respect to a basis  $\{\mathbf{e}_i\}$ ,  $q^2 = \|\mathbf{u}_s\|^2/2$ , and  $\mathbf{u}_s = \mathbf{u} - \sum_{i=1}^n a_i \mathbf{e}_i$ . Then, selecting  $\{\mathbf{e}_i\}$  to be the eigenfunctions of the energy stability problem, the condition of monotone decay of  $V$  can be reduced to the form

$$\frac{\partial V}{\partial \mathbf{a}} \cdot \mathbf{f}(\mathbf{a}) + \frac{\partial V}{\partial (q^2)} \Gamma[\mathbf{u}_s] + \left( \frac{\partial V}{\partial \mathbf{a}} - \frac{\partial V}{\partial (q^2)} \mathbf{a}^\top \right) \Theta[\mathbf{u}_s] < 0 \quad (1)$$

where the expressions for the vector polynomial  $\mathbf{f}$ , the functional  $\Gamma[\mathbf{u}_s]$  and the vector functional  $\Theta[\mathbf{u}_s]$  are immediately obtainable from the Navier-Stokes equations. It was demonstrated [1] that  $\Gamma[\mathbf{u}_s]$  and  $|\Theta[\mathbf{u}_s]|^2$  can be bounded by polynomial functions of  $\mathbf{a}$  and  $q^2$ . For a polynomial  $V(\mathbf{a}, q^2)$  condition (1) can be reduced to a condition of positive-definiteness of a certain other polynomial using the method proposed in [1]. Then the problem of finding  $V$  can be efficiently solved using the recent results in the emerging field of sum-of-squares (SOS) optimization over polynomials [2, 3]. It was shown that this approach is guaranteed to produce results at least as good as the results of the energy stability theory.

The further progress to be reported as compared to [1] is the way of finding the bounds for  $\Gamma[\mathbf{u}_s]$  and  $|\Theta[\mathbf{u}_s]|^2$  analytically, and an application of this approach to the rotating Couette flow.

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# Transition in unsteady channel flow

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## ABSTRACT

We have recently conducted a direct numerical simulation (DNS) study of a transient channel flow following a sudden increase of flow rate of an initially turbulent flow to investigate the response of turbulence [1]. In this work, we have shown that a low-Reynolds-number turbulent flow can undergo a process of transition that resembles the laminar-turbulent transition. In response to the rapid increase of flow rate, the flow does not progressively evolve from the initial turbulent structure to a new one, but undergoes a process involving three distinct phases (pre-transition, transition and fully turbulence) that are equivalent to the three regions of the boundary layer bypass transition, namely, the buffeted laminar flow, the intermittent flow and the fully turbulent flow regions. This transient channel flow represents an alternative bypass transition scenario to the free-stream turbulence (FST) induced transition, whereby the initial flow serving as the disturbances is a low-Reynolds-number turbulent wall shear flow with pre-existing streaky structures. A thin boundary layer of high strain rate is formed adjacent to the wall following the rapid increase of flow rate, which grows into the core of the flow with time providing the main reasons for further changes of the flow. The pre-existing turbulent structures act as background perturbations to this boundary layer, much like the role the free stream turbulence plays in a bypass transition. These turbulent structures are modulated by the time-developing boundary layer and stretched to produce elongated streaks of high and low streamwise velocities, which remain stable in the pre-transitional period. At this stage, the axial fluctuating velocity increases steadily but the other two components remain effectively unchanged. In the transitional phase, localised turbulent spots are being generated which are distributed randomly in space. Such turbulent spots grow longitudinally as well as in the spanwise direction, merging with each other and eventually occupying the entire wall surfaces when the transition completes and the flow becomes fully turbulent.

In the study reported in [1], only one case was considered, whereby the initial and the final Reynolds numbers ( $Re_b = U_b \delta / \nu$ , where  $U_b$  is the bulk velocity of the flow and  $\delta$  the half channel height) were 2800 and 7400 respectively. The ramp period was very short ( $t^* = 0.22$ ) and the flow variation can be viewed as a step change ( $t^* = t / (\delta / U_{b1})$ , where  $U_{b1}$  is the bulk velocity of the final flow at  $Re_b = 7400$ ). In the proposed presentation, we will report further studies on the effect of the initial and final Reynolds numbers on the characteristics of the response of turbulence. We also consider ‘slowly’ increased acceleration whereby the ramp period is varied from  $\Delta t^* = 0.24$  to 96.

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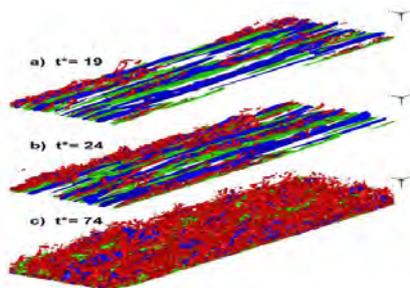


Fig. 1 Flow structures (Green:  $u' = +0.12$ ; Blue:  $u' = -0.12$ ;  $\lambda_2 = -0.5$ )

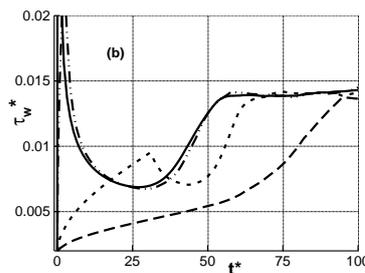


Fig. 2 Effect of acceleration rate

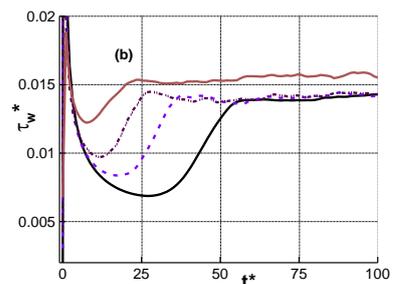


Fig. 3 Effect of initial Reynolds number

# Cylindrical roughness-induced transition on a flat plate

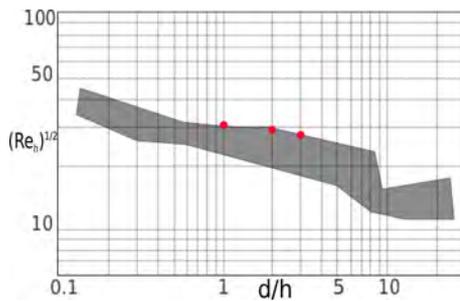
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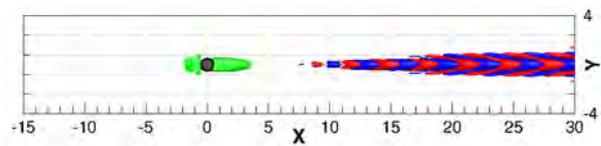
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Surface roughness can have a profound effect on boundary layer transition. The mechanisms associated with single roughness elements are only partially understood while those responsible for transition with distributed roughness are not yet known. This has led to a large body of empirical information in the literature that is not fully consistent. Many studies have been conducted during half a century to understand the mechanisms of transition. Unfortunately, beyond a critical Reynolds number dependant on the diameter  $d$  and height  $h$  of the rugosities, the flow can actually undergo transition right downstream these rugosities. This transition has been extensively investigated by von Doenhoff and Braslow<sup>1</sup> in the late 60’s who summarized their findings in the well known diagram depicted on figure (a). The aim of the present study is to recover this transition diagram out of informations only stemming from global stability analyses and direct numerical simulations.

The stability of the flow over rugosities of aspect ratios  $\eta = d/h$  ranging from 1 to 3 is investigated. The stationary solution for  $(Re_h, \eta, \delta_{99}) = (1250, 1, 2)$  is depicted on figure (b) in green. It consists in two recirculation bubbles and of a 4-vortex system stemming from the upstream recirculation zone. This flow turns out to be globally unstable with a branch of modes being in the upper half complex plane. The spatial structure of the leading global mode is shown on figure (b) in red and blue. The critical Reynolds numbers predicted by such analysis for the different cases considered compare well with the upper bound for transition in the von Doenhoff-Braslow diagram. The Hopf bifurcation encountered is expected to be subcritical, thus probably linking the lower bound of the diagram with the depth of the expected hysteresis cycle. This hypothesis will be illustrated by direct numerical simulations.



(a) von Doenhoff and Braslow Transition diagram



(b) Streamwise real part of the leading unstable mode (red and blue) and base flow's recirculation zones (green) for  $d/h = 1$ .

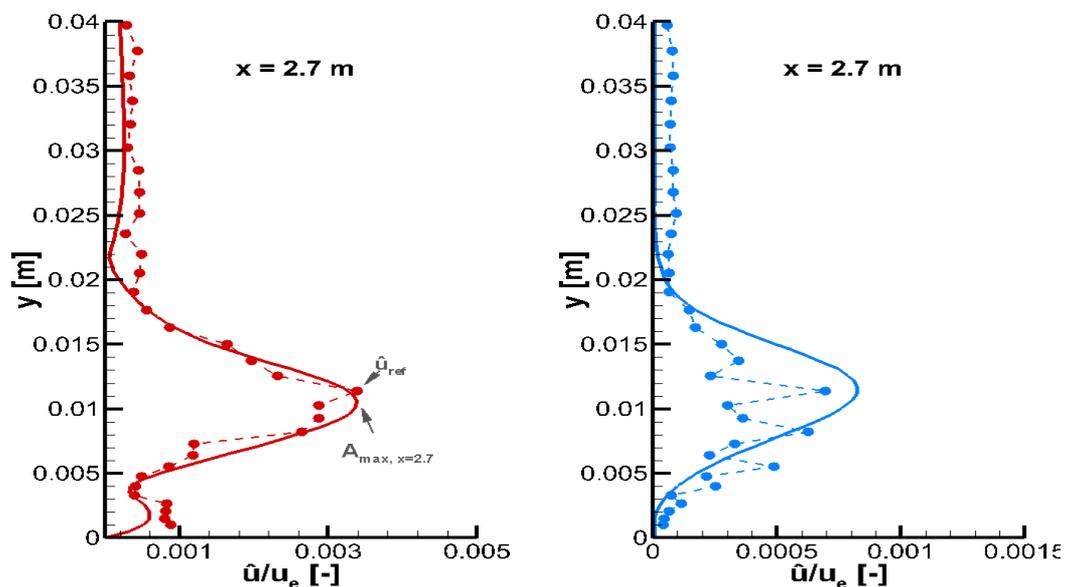
<sup>1</sup>A. von Doenhoff & A. Braslow: The effect of distributed surface roughness on laminar flow. *Boundary layer and flow control*, 1961.

## Experimental Investigation of 3D Instability Modes in the Wake of a Circular Hump in a Boundary Layer

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Flow visualizations, PIV and hot-film measurements have been performed in a flat-plate boundary layer that contains a circular hump-shaped roughness element in our laminar-flow water channel facility. The hump generates steady streamwise streaks in the boundary layer in excellent agreement with according CFD computations for the same parameters. With increasing Reynolds number these streaks become increasingly unstable and it is observed that laminar-turbulent transition starts with amplification of periodic small-amplitude disturbances followed by vortex-shedding.

Detailed measurements were performed around  $Re_h=340$ , based on roughness height, where the transitional regime extends over several roughness diameters such that the transition process is best observable and most accessible to measurements. Using two simultaneous hot-film probes allowed detecting spanwise symmetric (varicose) and asymmetric (sinuous) disturbances and measuring their amplification rates and mode shapes for comparison with linear stability theory, cf. Fig. 1. The LST solver developed by O. Schmidt uses an ansatz with two-dimensional eigenmodes in the cross-flow plane and the comparison in Fig. 1 is therefore for constant spanwise coordinate  $z$ . More comparisons will be shown during the presentation together with investigations of the influence on the instabilities of the hump shape and active forcing at constant frequency.



**Figure 1:** Comparison of symmetric (left) and antisymmetric (right) velocity signals in the wake of the roughness element with eigenfunction amplitudes from LST

# Encapsulated Selective Frequency Damping method

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## Abstract

In [1], Akervik *et al.* present a new method, called Selective Frequency Damping (SFD), to reach the steady-state of an unsteady system by damping the unstable temporal frequencies. As it is easy to implement into an existing code and does not need an initial guess of the solution, this method appeared to be an efficient alternative to classical Newton's methods.

We present an alternative formulation of the SFD method which enables the use of any existing unsteady code as a "black box" in order to find the steady state of a given problem. To find the steady-state of  $\dot{q} = f(q)$ , we can introduce the function  $\Phi$  such as the numerical solution of this problem at the step  $(n + 1)$  is given by  $q^{n+1} = \Phi(q^n)$ .

To implement the original SFD method [1], a linear forcing term is added into the solver. From a programming point of view it means that  $\Phi$  is modified. The encapsulated SFD presented here does not require any modification of the initial solver  $\Phi$ . The only work required to implement this method is to create a routine to evaluate at each time step

### Implicit formulation:

$$\begin{cases} q^{n+1} = \Phi(q^n) - \chi(q^{n+1} - \bar{q}^{n+1})\Delta t \\ \bar{q}^{n+1} = \bar{q}^n + \frac{q^{n+1} - \bar{q}^{n+1}}{\Delta} \Delta t \end{cases} \quad (1)$$

### Explicit formulation:

$$\begin{cases} q^{n+1} = \Phi(q^n) - \chi(q^n - \bar{q}^n)\Delta t \\ \bar{q}^{n+1} = \bar{q}^n + \frac{q^n - \bar{q}^n}{\Delta} \Delta t \end{cases} \quad (2)$$

where  $\bar{q}$  is a temporally filtered solution,  $\Delta$  is the filter width,  $\chi$  is the control coefficient and  $\Delta t$  is the time-step.

The convergence of (1) (or (2)) toward the steady-state is guaranteed if all the eigenvalue magnitudes of (1) (or (2)) are strictly smaller than one. If it is the case, the method is said to be stable.

In order to analyse the stability of the encapsulated SFD method, it is applied to the one-dimensional scalar problem  $u^{n+1} = \alpha u^n$ . This analysis allows us to highlight the roles of the control coefficient  $\chi$  and the filter width  $\Delta$  in the convergence (or not) of the method toward the steady-state. The behaviour of the eigenvalue magnitudes is studied for both the implicit and the explicit scheme. We show that the implicit formulation has better stability properties.

We finally present the steady-state of flow problems obtained by applying the encapsulated SFD into a code which implements the spectral/*hp* element method.

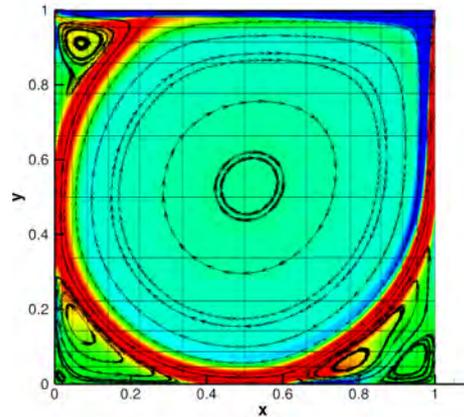


Figure 1: Steady-state of a driven cavity flow ( $Re = 10000$ ).

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# Elastic-like instabilities in non-Newtonian parallel flows

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Mixing control in fluid environments having very low Reynolds numbers is a crucial need for many practical purposes ranging from biochemistry analysis in microfluidic devices, where mixing has to be rapid and efficient, to lab-on-a-chip applications, where mixing has to be reduced to avoid spurious effects as in microfluidic rheometer applications. The ability to control fluid mixing properties is clearly subjected to a deep understanding of physical mechanisms able to originate such a mixing for very small Reynolds numbers. In this respect, a simple model to capture mesoscopic effects of order-disorder transitions of an underlying non-Newtonian fluid microstructure subjected to shear is proposed and its behaviors investigated (numerically and by means of asymptotic perturbative methods) in relation to the possible emergence of fluid elastic-like instabilities occurring for arbitrarily small flow inertia (i.e. zero Reynolds numbers). A crucial ingredient for instabilities to emerge has been identified in the finite-time response of the network structure to strain where the order-disorder transition corresponds to a change from low-to-high fluid viscosity (and not viceversa). Our results generalizes the concept of "elastic instabilities" in viscoelastic fluids to a more general and larger class of non-Newtonian fluids.

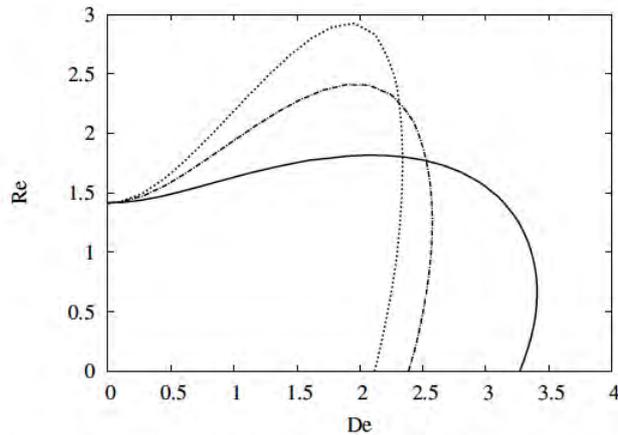


FIGURE 2. Marginal curves in the  $Re-De$  plane with  $\Gamma \equiv De$ , for different values of  $n$ . Here,  $n = 1.3$  (—),  $n = 1.7$  (---) and  $n = 2$  (···).

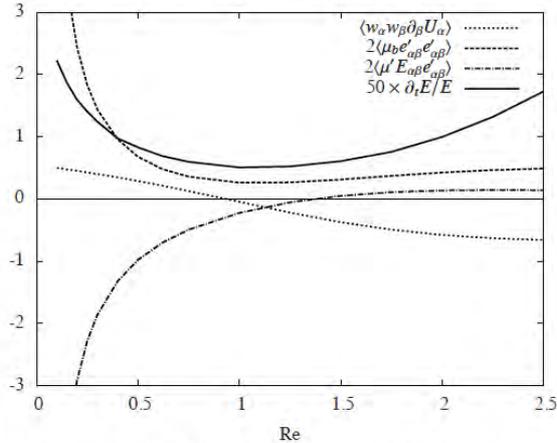


FIGURE 3. Energy balance for  $De = \Gamma = 4$  and  $n = 1.3$ . Along the ordinates we report the different contributions to the total rate of the normalized power,  $\partial_t E/E$ , from (4.1). The symbols represent production (···), dissipation (---), elastic-like (-·-) and total (—), respectively. The last has been scaled by a factor of 50 for readability.

## YOU THINK YOU KNOW HOW TO CYCLE? THAT'S RICH

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The numerical discovery of unstable travelling waves in pipe flow, together with glimpses of them in experiments (Hof *et al.*, 2004), has spurred interest in obtaining a description of turbulent flow in terms of a handful of key exact solutions of Navier–Stokes equations. Starting with the first determination of recurrent flows by Kawahara & Kida (2001), a considerable numbers of such solutions have by now been reported for pipe and plane Couette flows (Cvitanović & Gibson, 2010; Willis *et al.*, 2012; Kreilos & Eckhardt, 2012; Chandler & Kerswell, 2012). The question is - what is one to do with these solutions? Periodic orbit theory answers that.

Within the dynamical framework that has emerged in the last decade, turbulence is viewed as a walk through a forest of such solutions. The resulting visualizations show the role exact solutions play in shaping turbulence: the observed coherent structures are the physical images of the flow's least unstable invariant solutions, with turbulent dynamics arising from a sequence of transitions between these states. The long-term goals of this research program are to develop this vision into quantitative, predictive description of moderate-*Re* turbulence, and to use this description to control flows and explain their statistics.

The idea that chaotic dynamics is built upon unstable periodic orbits (in fluid dynamics parlance: 'recurrent flows') can be traced to Poincaré ("What makes periodic orbits so valuable is that they are the only breach, so to speak, through which we can try to enter a place up to now deemed unapproachable."), but it first became a theory in the Gutzwiller quantization over classical unstable periodic orbits, and in Ruelle's work on hyperbolic systems. The periodic orbit theory is developed in ChaosBook.org Cvitanović *et al.* (2012), which we shall follow in this lecture. There are many steps one has to master before the theory can be implemented.

First, one has to learn how to visualize the  $\infty$ -dimensional state space dynamics of turbulent flows, in a novel way (would celestial mechanician study solar system by tracking the sum of kinetic energies of all planets?). Then one has to quotient symmetries, and that is harder for nonlinear flows than for -let's say- quantum mechanics; turbulence breaks all symmetries, so they are hidden from the view. Next one has to compute hierarchical sets of equilibria, travelling waves periodic orbits, relative periodic orbits and their unstable manifolds, and chart the highly convoluted strange attractor on which turbulence lives. Once the geometry is under control, one has to abandon individual trajectories in favor of densities, and the Navier-Stokes PDEs in favor of Perron-Frobenius operators and Koopman operators. That sets the stage for predicting long time statistics by means of cycle-averaging formulas. And finally, one has to know when to stop; in practice one can compute only a finite number of periodic orbits. That requires Fokker-Planck operators and their adjoints, and further computations.

The theory will never be as popular as holidays on Côte d'Azur, but is our best bet for a dynamical (as opposed to statistical) theory of turbulence close to onset.

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# From periodic to chaotic self-sustaining process in boundary-layer flows

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T. Kreilos<sup>‡</sup> and B. Eckhardt<sup>‡</sup>

Near-wall coherent structures such as streaks and quasi-streamwise vortices are a ubiquitous feature of transitional and turbulent wall-bounded shear flows. Their regeneration process is intimately connected with the occurrence of *bursting* events, *i.e.* strong intermittent ejections of low-speed fluid from the wall. We focus on coherent structures as well as bursting events in the framework of subcritical transition. A recent idea specific to subcritical instabilities is to analyse the laminar-turbulent separatrix, the invariant phase-space region separating trajectories that relaminarise from those experiencing turbulent dynamics. Relative attractors on this separatrix are called edge states. They correspond to an (unstable) equilibrium regime and are thus crucial for understanding the structure of the phase space and identifying the physical mechanisms by which the flow can sustain non-trivial dynamics.

The flow under consideration is the asymptotic suction boundary layer (ASBL), an incompressible boundary-layer flow above a permeable flat plate subject to constant wall suction. The flow is parallel which simplifies theoretical consideration and reduces computational costs in comparison with spatially developing flows.<sup>1</sup>

We focus on spanwise extended numerical domains of size  $(L_x, L_y, L_z) = (6\pi \delta^*, 15 \delta^*, 50 \delta^*)$ . In this case we obtain three different states (two of them are actually equivalent under a reflection symmetry), which are localised in the spanwise direction and periodic in time.<sup>2</sup> In all three cases the states are dominated by a pair of high- and low-speed streaks which undergo a burst before being translated in the spanwise direction. Depending on the direction of the shift we distinguish between the two symmetry-related states that repeatedly shift towards the right R or towards the left L (see figure 1(a)), and the state that alternates regularly between shifting left and right LR. As the domain length is decreased to  $4\pi$ , the periodicity of the edge state is lost (see figure 1(b)). The resulting dynamics on the edge is chaotic, though also consisting of calm and bursting phases. The latter still correspond to shifts in the spanwise direction. However, the direction and the distance of those shifts is no longer fixed but varies in an unpredictable fashion. Preliminary results of slowly lowering the box length from  $6\pi$  down to  $4\pi$  reveal interesting dynamics in-between, with period doubling, intermediate states like LLLRRR and pockets of regularity.

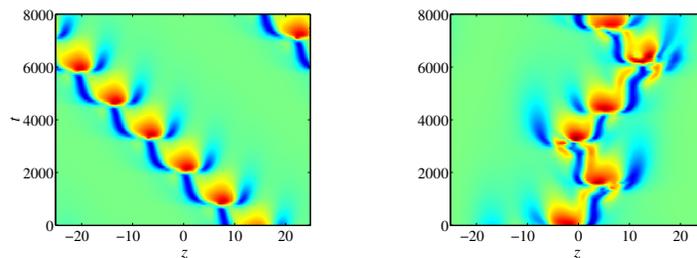


Figure 1: Space-time diagrams for streamwise velocity fluctuations  $u'$  averaged in streamwise direction at a fixed wall-parallel plane, for Reynolds number  $Re = U_\infty \delta^* / \nu = 500$ . (a) Left-going periodic edge state (L) for  $L_x = 6\pi$ . (b) Chaotic edge state for  $L_x = 4\pi$ .

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<sup>1</sup>Duguet *et al.*, *Phys. Rev. Lett.* **108**, 044501 (2012).

<sup>2</sup>Khapko *et al.*, *J. Fluid Mech.* **717**, R6 (2013).

# Pockets of turbulence in plane Couette flow

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Studies of the transition to turbulence in linearly stable shear flows, such as pipe flow and plane Couette flow, have revealed a route to turbulence that passes through the formation of a chaotic saddle. We here present the analysis of the state-space structures and the bifurcations that lead to the creation of this saddle.

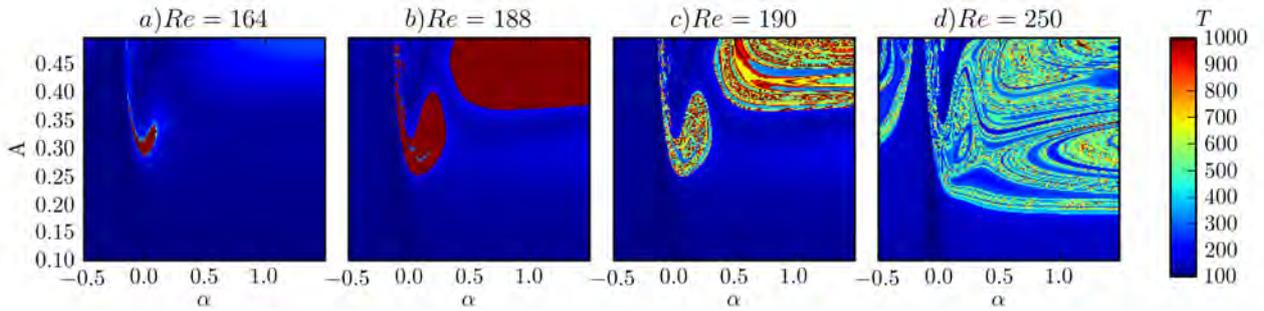


Figure 1: Two-dimensional slices of the state-space of transitional plane Couette flow. Coordinates are chosen such that the x-axis is an interpolation between the lower- and upper-branch Nagata-Busse-Clever states. The y-axis is the amplitude of the interpolated state. The structure of state-space is shown by the lifetimes  $T$  of initial conditions before and after the crisis.

We study plane Couette flow in a small periodic domain of size  $2\pi \times 2 \times \pi$  in the downstream, wall-normal and spanwise directions with imposed shift-and-reflect symmetry. The Nagata state is the lower-branch state in a saddle-node bifurcation at Reynolds number  $Re = 163.8$ , with exactly one unstable direction; the corresponding upper-branch solution is stable. The latter one is hence an attractor that coexists with the laminar attractor. Figure 1(a) illustrates the situation by showing the lifetimes of initial conditions in a two-dimensional slice of the state-space at  $Re = 164$ . The small dark-red region corresponds to the basin of attraction of the upper-branch, the blue region to states that decay to the laminar state. The fixed-point undergoes a series of bifurcations that result in a chaotic attractor.

At  $Re = 188.7$ , the chaotic attractor has expanded so much that it collides with the lower-branch state, leading to a boundary crisis. At this point, the basin of attraction opens up and the chaotic attractor turns into a chaotic saddle. By comparing the situation before and after the crisis, we still see that the shape of the former attractor is still visible.

Inside the chaotic saddle the distribution of lifetimes follows an exponential scaling, with a characteristic lifetime  $\tau$ . Calculating  $\tau$  for various Reynolds numbers, we observe a non-monotonic variation, which we are able to associate to the creation of “pockets of turbulence”: in a small region in state-space a stable orbit and a hyperbolic point are created in a saddle-node bifurcation; this pocket grows, opens up and merges with other pockets, leading to an increase in lifetimes.

The crisis-bifurcation discussed above gives a first example of such a pocket. As a second example we study a pair of periodic orbits which are created in a saddle-node bifurcation at  $Re = 249.01$ . Again, the upper-branch orbit is an attractor, the lower-branch orbit has one unstable direction. The situation is hence similar to the one described above for the Nagata-solutions, only that now the saddle node bifurcation occurs inside the chaotic saddle. The attractor is destroyed in a new crisis at  $Re = 250.13$ , leaving behind a larger chaotic saddle than before.

The creation of invariant solutions in saddle-node bifurcations and the destruction in crisis bifurcations provides one mechanism underlying the generally observed increase of turbulent lifetimes with  $Re$ .

# Oblique laminar-turbulent interfaces in transitional shear flows

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*The onset of transition to turbulence in subcritical wall-bounded flows is characterised by large-scale localised structures such as turbulent spots or turbulent stripes. Interestingly, the laminar-turbulent interfaces associated with these structures always display obliqueness with respect to the mean direction of the flow. We will attempt to explain this phenomenon using an assumption of scale separation between large and small scales, and we can show analytically why the corresponding laminar-turbulent interfaces are always oblique with respect to the mean direction of the flow in the case of plane Couette flow. This mechanism can be easily extended to other flows such as Plane Poiseuille flow or Taylor-Couette flow.*

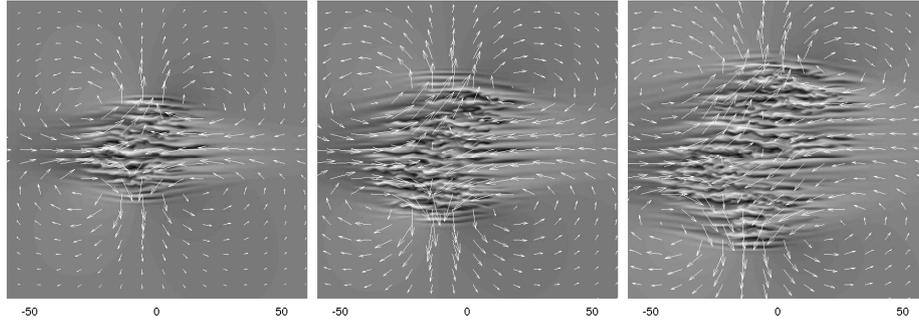


Figure 1: Obliquely growing spot in plane Couette flow at  $R = 360$  (grey: streamwise velocity in the midplane) and associated  $y$ -integrated large-scale flow  $(\bar{U}_x, \bar{U}_z)$  ( $x, z$ ) (arrows). From left to right:  $t = 200, 300$  and  $400$ . Simulations in a periodic domain with  $\Lambda = 500$  and  $1536 \times 33 \times 2048$  spectral modes. Only the subdomain  $[-60 : 60] \times [-60 : 60]$  is displayed here.

An example of formation of large-scale hydrodynamical patterns of co-existing laminar and turbulent flow is found at the onset of transition of plane Couette flow, the flow between two counter-sliding plates. Surprisingly, the disordered phase breaks the symmetries associated with the geometry and adopts an orientation oblique with respect to the mean flow [1]. Especially the origin of the obliqueness of the associated interfaces has long remained mysterious. We assume that the plates move with opposite velocities  $\pm U$  in the streamwise direction  $x$ . We also define the half-gap  $h$  between the plates in the  $y$  direction, the spanwise direction  $z$ , and the Reynolds number  $R$  as  $Uh/\nu$ , where  $\nu$  is the kinematic viscosity of the fluid, supposed Newtonian and incompressible. Lengths and velocities are non-dimensionalised by respectively  $h$  and  $U$ . Direct numerical simulations were carried out with the same pseudo-spectral code as in Ref. [2], in a periodic domain of size  $\Lambda \times 2 \times \Lambda$ , with  $\Lambda = 500$  and with high spectral resolution.

We base our analysis on the existence of two distinct characteristic scales and assume clear spectral separation. The small scales correspond to the coherence of the turbulent fluctuations inside a turbulent patch (streaks) while the large scales correspond to the diffusive tails of the streaks, so that the scale separation grows like  $O(R)$ . Using the scale separation hypothesis we can separate the flow field  $\mathbf{u}$  into small scales  $\tilde{\mathbf{u}} = H\mathbf{u}$  and large scales  $\mathbf{U} = L\mathbf{u}$  using adequate plane-isotropic Gaussian low-pass and high-pass filters, respectively  $H$  and  $L$ . Denoting  $y$ -averaging with a bar  $(\bar{\cdot})$ , we deduce from the incompressibility of the flow a two-dimensional divergence-free condition in the  $xz$ -plane for the large-scale flow,  $\partial_x \bar{U}_x + \partial_z \bar{U}_z = 0$ .

The growth of a turbulent patch is shown in Fig. 1 for  $Re = 360$  along with the corresponding large scale flow  $(\bar{U}_x, \bar{U}_z)$ . The streamwise ends of a such a turbulent patch are characterised by so-called overhang regions where locally turbulent flow on one wall faces nearly laminar flow near the other wall. Those regions correspond to a mismatch in the flow rates  $\bar{U}_x \neq 0$ , whereas  $\bar{U}_x = 0$  everywhere else. As a consequence  $\partial_x \bar{U}_x \neq 0$  in the overhang regions, hence  $\bar{U}_z \neq 0$ , and the large-scale flow is locally oblique with respect to the streamwise direction. In order to understand how the large-scale affects the shape of the laminar-turbulent interface, we use the decomposition introduced earlier and apply successively the filters  $L$  and  $H$  to the wall-normal momentum equation. The scale separation hypothesis results in a simplified system, similar to a transport equation for the small scales by the large scales. This indicates that newly nucleated streaks at the tips of the spots will be advected by the large-scale flow, which we know has a non-zero angle with respect to the streamwise direction. As a consequence, the growth of the spots will be distorted by the presence of the large-scale flow and proceed obliquely as well [3].

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# Unstable structures in rotating plane Couette flow - quantitative measurements of flow velocities

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Rotating plane Couette flow (RPCF) is governed by two non-dimensional parameters, the Reynolds number ( $Re = U_w h / \nu$ ) and a rotation number ( $\Omega = 2\Omega_z h^2 / \nu$ ) (see figure 1(a) for the definition of the geometry and parameters). For cyclonic rotation ( $\Omega < 0$ ) the flow is stabilized by the Coriolis effect, but for anti-cyclonic rotation ( $\Omega > 0$ ) the flow can be shown to be destabilized in a certain part of the parameter plane, where the critical Reynolds number is explicitly expressible, in terms of  $Re$  and  $\Omega$ , as in Ref. [1]

$$Re_{crit} = \Omega + 107\Omega^{-1} \quad (1)$$

For  $Re$  and  $\Omega$  parameter values above the neutral curve, RPCF shows a number of different instabilities (see Ref. [2]). For low Reynolds numbers the flow remains laminar, even in the case of anti-cyclonic rotation where various types of organized structures in the form of streamwise roll cells may appear. In order to obtain quantitative information of such flows, the RPCF apparatus at KTH has been instrumented with a Particle Image Velocimetry (PIV) system that allows “simultaneous” measurements of the streamwise and spanwise velocity components in planes parallel to the moving walls. As an example we show results from  $Re = 100$  and  $\Omega = 8$ , figure 1(b), where we observe 3D wavy structures that are similar to the stationary disturbances predicted in Ref. [3]. In the presentation, we will discuss flow structures and dynamics for various values of the parameters.

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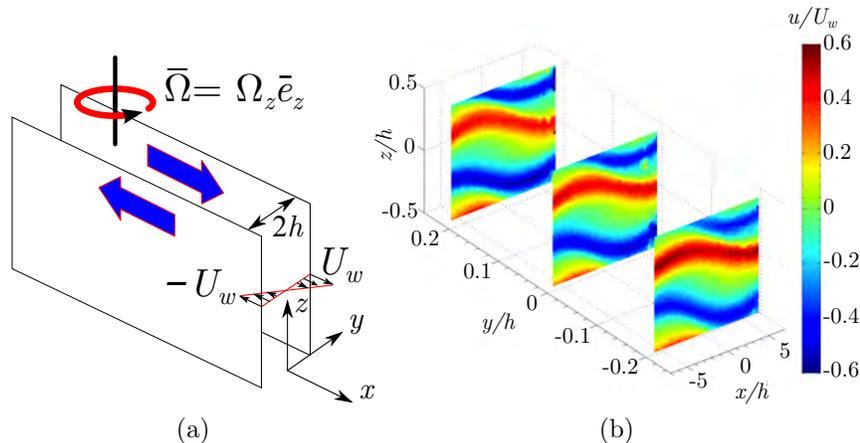


Figure 1: (a) Geometric description and parameters, (b) The streamwise velocity  $u(x, y_i, z)$  for  $y_i/h = -0.21, 0, 0.21$  shows 3D wavy structures at  $Re = 100$  and  $\Omega = 8$ .

# Laminar-turbulent transition of rotating-disk flow - an experimental and numerical approach

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A main objective of this experimental and numerical work is to investigate laminar-turbulent transition of rotating-disk flow. The observation that transition occurs at a quite specific Reynolds number led Lingwood (1995) to suggest that the transition is due to an absolute instability. In experiments typically 28 to 32 stationary vortices are observed and appear to dominate the flow within the convectively unstable region (see e.g. Gregory et al. 1955) since such vortices are excited by minute roughnesses on the disk surface, which reinforce the disturbance pattern continuously. However Lingwood (1995) found that *travelling* waves made the flow absolutely unstable above  $R = 507$  ( $R$  is Reynolds number based on the radius). The numerical simulation has the advantage that the convective stationary disturbances can be avoided and thereby enable detailed studies of the absolute instability, whereas for experimental studies it is hard to isolate the stationary disturbances from the travelling and hence the absolute instability. The figures below show developments of the stationary convectively unstable vortices by visualizing the azimuthal velocity component and there is a good agreement between experimental and numerical data. We will discuss how Lingwood's absolute instability mechanism can be observed in our experiments and simulations of the transition process.

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Gregory, N., Stuart, J.T. & Walker, W. S. (1955) On the stability of three-dimensional boundary layers with application to the flow due to a rotating disk. *Phil. Trans. R. Soc. Lond. A* **248**, 155-199 .

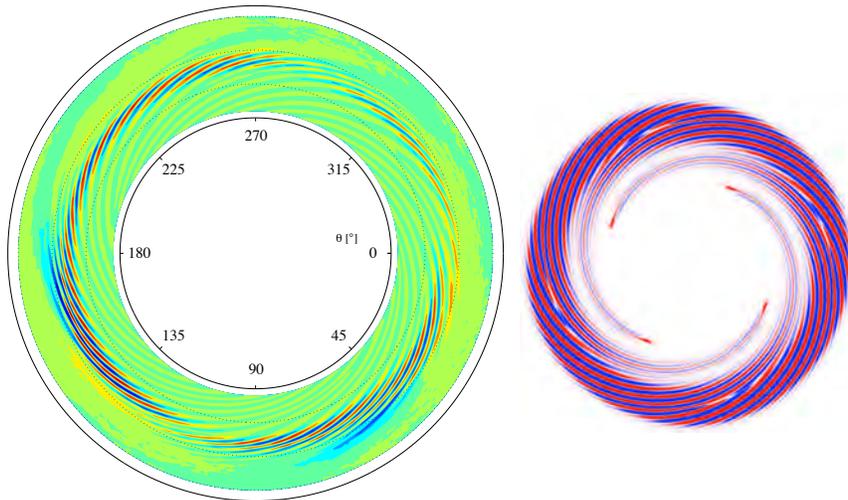


Figure 1: Experimental (*left*) and numerical (*right*) results for the stationary disturbances. In the simulation the disturbance is triggered at four positions and the wave number at  $R = 450$  becomes equal to 28 which is close to what is also observed in the experiment. Edge Reynolds number is 731 for experiments.

# Experimental Investigation of Swept-Wing Boundary Layer Receptivity to Modest Freestream Turbulence

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The path from crossflow instability to turbulent breakdown in swept-wing boundary layers is strongly influenced by freestream turbulence and surface roughness. The foundational experiments of Deyhle and Bippes<sup>1</sup> demonstrate that the growth of traveling crossflow waves outpaces that of stationary crossflow waves when the turbulence intensity is greater than about 0.2%. Recent experiments<sup>2</sup> indicating the disturbance environment can affect instability growth at lower levels as well have reinvigorated the study of freestream turbulence effects on crossflow-instability-dominated transition. The receptivity of a 45-degree swept-wing boundary layer to freestream turbulence and surface roughness is examined for  $Tu < 0.2\%$ , in the presence of regular arrays of discrete cylindrical roughness elements. The initial unsteady-mode amplitudes are observed to increase with  $Tu$ ; this trend is in agreement with the results of experiments conducted at higher levels of freestream turbulence<sup>3</sup>. However, the surface roughness arrays also enhance the amplitude of the unsteady disturbances. Although receptivity of the stationary mode is largely unaffected by turbulence in this range, subsequent development of the stationary and unsteady disturbances of Figure 1 indicate a significant dependence on low-intensity turbulence. Increasing  $Tu$  from 0.02% promotes transition and attenuates the growth of stationary crossflow waves.

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<sup>1</sup>Deyhle and Bippes, *J. Fluid Mech.* **316** (1996).

<sup>2</sup>Hunt and Saric, *AIAA Paper* 2011-3881 (2011).

<sup>3</sup>Kurian et al., *Phys. Fluids* **23**, 034107 (2011).

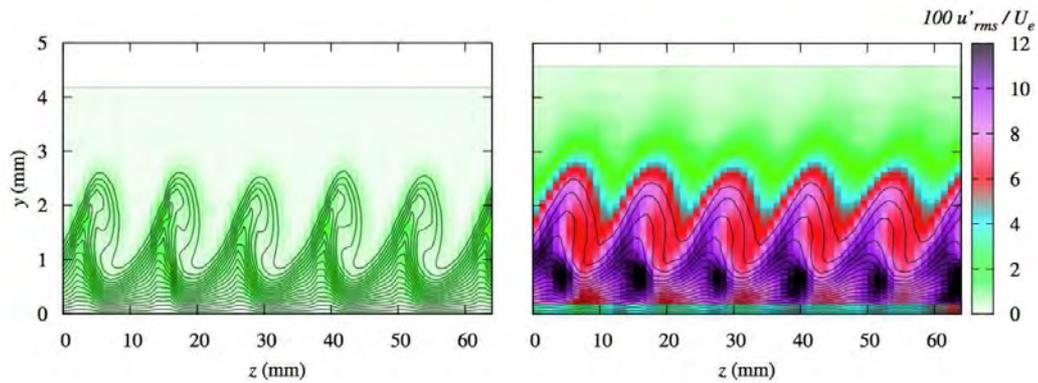


Figure 1: Steady and fluctuating velocity contours measured at  $Re_x = 10^6$  with critically-spaced roughness of height  $k = 12 \mu\text{m}$ . Left:  $Tu = 0.02\%$ , right:  $Tu = 0.19\%$ .

# Optimal initial perturbations in streamwise corner-flow

Oliver T. Schmidt, Seyed M. Hosseini, Ulrich Rist, Ardeshir Hanifi & Dan S. Henningson

Localised optimal initial perturbations are studied to gain an understanding of the global stability properties of streamwise corner-flow. A self-similar and a modified base-flow are considered. The latter mimics a characteristic deviation from the self-similar solution, commonly observed in experiment. Power-iterations in terms of subsequent direct and adjoint linearized Navier-Stokes solution sweeps are employed to converge optimal solutions for two optimization times. The optimal response manifests as a wave packet that initially gains energy through the Orr mechanism and continues growing exponentially thereafter. The study at hand represents the first global stability analysis of streamwise corner-flow and confirms key observations made in theoretical and/or experimental work on the subject. Namely, the presence of an inviscid instability mechanism in the near-corner region and a destabilizing effect of the characteristic mean-flow deformation found in experiment. Additionally, the same procedure has been applied to the subcritical regime in the corner-flow to investigate the presence of transient growth. Similarly, two optimization times have been selected in order to compare the structures of the optimal initial perturbation between the subcritical and supercritical regime. The optimal response manifests itself in the form of coexisting streaks and corner modes.

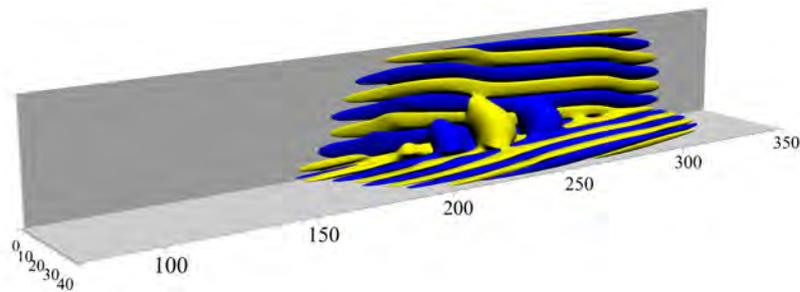


Figure 1. Isosurfaces of the streamwise perturbation velocity of the flow response to the optimal initial condition in the subcritical regime. Isosurfaces are drawn for  $\pm 0.1 \|u'\|_\infty$ .

# A discrete mode representation of the spatial transient growth mechanism found in streamwise corner-flow

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The flow in a streamwise corner is known to feature an inviscid instability mechanism originating from its locally inflectional streamwise velocity profile in the near-corner-region. A direct numerical simulation is conducted where the flow is forced by harmonic perturbations at both walls. Global modes are obtained from the transient simulation data by means of direct mode decomposition (DMD). The modal structures are analyzed and reveal spatial growth over some streamwise distance in the near-corner-region, i.e. even at subcritical Reynolds numbers. It is demonstrated that this behavior can be explained by a spatial transient growth theory. An optimal perturbation based on the discrete spectrum of the underlying linear stability problem is constructed. The maximum spatial transient growth gain and the downstream location of its occurrence are found in satisfactory agreement with the direct numerical simulation data.

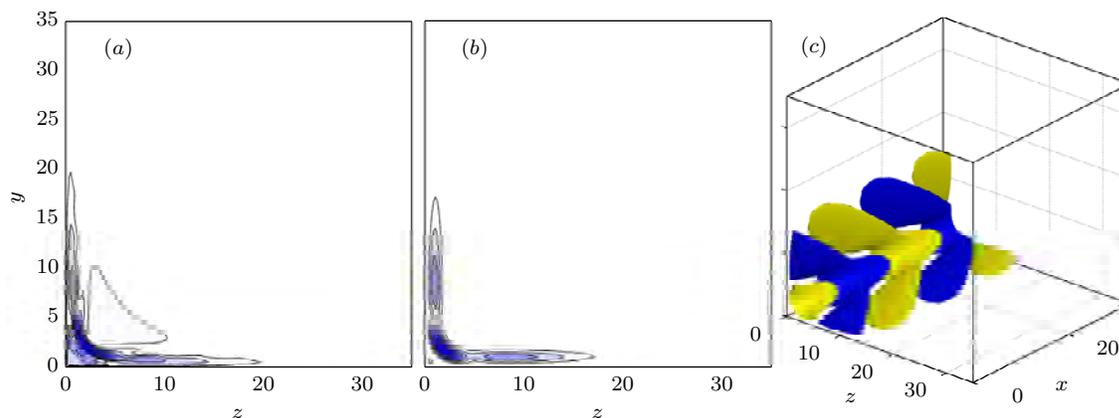


Figure 1: Discrete spectrum based optimal initial condition (a), response (b) and 3D reconstruction of the response (d).

Figure 1 depicts optimal initial condition and response for  $\omega = 0.13$ , respectively. The optimal structure shown in part (c) of the figure is in close agreement with the coherent structure observed in the direct numerical simulation. It is concluded that axial corner-flow has a natural tenancy towards spatial transient growth even if no artificially generated optimal structure is introduced into the flow. Additionally, the sensitivity of the temporal and spatial linear stability operator is analysed by means of  $\epsilon$ -pseudospectra.

# Optimal disturbances on curved surfaces for two-dimensional compressible boundary layers

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**Keywords:** boundary layer, transient growth, optimal perturbation, curvature, Görtler vortices, neutral curve

Görtler vortices, which take the form of streamwise-oriented periodic counter-rotating vortices, are found in boundary layers over concave surfaces where centrifugal forces destabilise the flow [1]. In the study of their development, Hall [2] pointed out the strong dependence of the neutral curve <sup>†</sup> to the initial perturbation, thus showing that the concept of a unique neutral curve is not readily accessible for the Görtler problem as opposed to usual parallel flow stability problems.

This work aims at computing the maximum spatial growth rate of Görtler vortices using an optimal approach. The appropriate parabolic set of linear partial differential equations for a two-dimensional compressible boundary layer flow over a curved surface has been derived. Computation of the optimal perturbations – the perturbations which undergo maximum energy amplification at a given chordwise position – is done by using an iterative direct/adjoint procedure [3, 4]. The optimization is realized over all possible initial perturbations, which solves the problem of the strong dependence to the initial perturbation in computing the neutral curve.

A new criterion for convergence has been set up which uses the neutral position – where the longitudinal derivative of the energy first vanishes – as the new position of the outlet station for optimization, ensuring the computation of the most rapidly amplified perturbation. Therefore the resulting neutral curve can be seen as an envelope including all possible neutral curves for different initial perturbations.

The results obtained with the optimal approach display the asymptotic agreement with previously derived analytical expressions and modal calculations for large spanwise wavenumbers. The method is validated for both incompressible and compressible flows, with neutral curves for the optimal perturbation clearly covering previous calculations for various initial disturbances and available experimental data. The stabilizing effect of compressibility is also highlighted.

The method is then extended to study a flow over a surface with variable curvature. Results emphasize the deep destabilizing effect of a concave curvature on the boundary layer.  $N$  factor curves are also computed and comparisons with previous calculations reveal an early amplification of the perturbations close to the leading-edge, which is due to a non-negligible zone of transient growth before classical linear amplification of Görtler vortices.

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<sup>†</sup>The neutral curve is the zero contour curve of growth rate and separates the domain into the region of exponentially growing perturbations and the stable region.

# Nonlinear Transient Growth – A New Tool for Analysing Nonlinear Stability

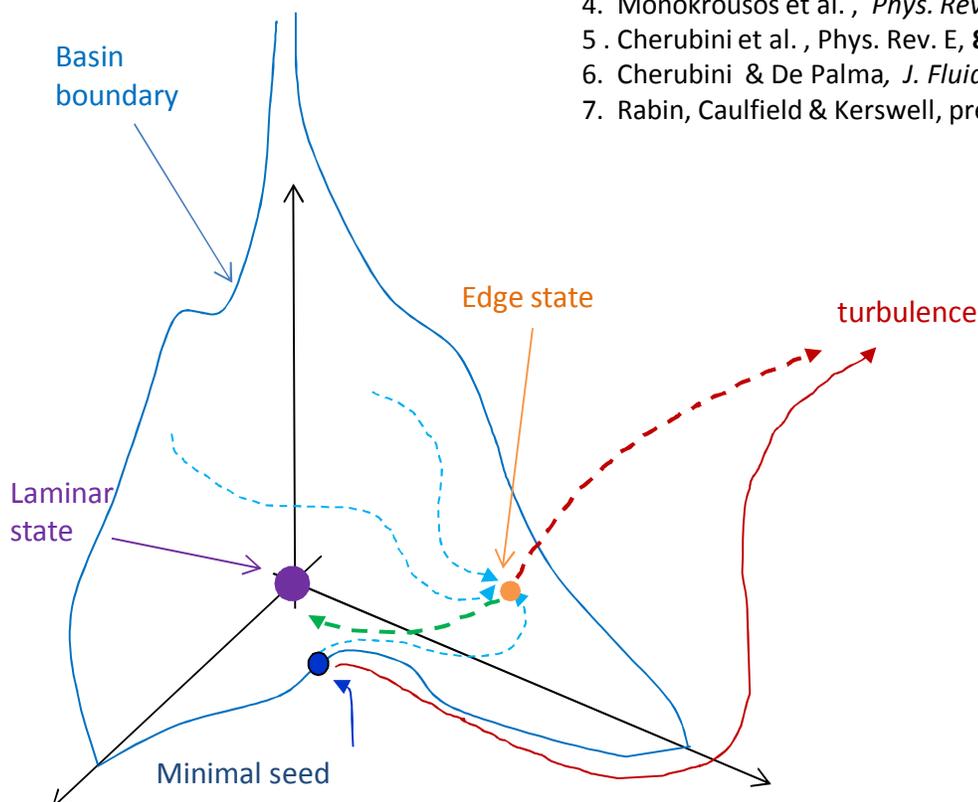
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( with Colm Caulfield (Cambridge), Chris Pringle (Bristol), Sam Rabin (Cambridge)  
& Ashley Willis (Sheffield) )

For linearly-stable shear flows, there has been a lot of recent interest in linear transient growth as a mechanism for triggering transition. Since transition is a highly nonlinear process, such a connection has required a considerable leap of faith. There is, however, a well defined (mathematical) route to connect the two processes – making the transient growth calculations fully nonlinear [1]. I'll discuss this procedure for a simple low dimensional dynamical system and show how it reveals a generic method for assessing the nonlinear stability of a state in a dynamical system [2]. After showing results for some Navier-Stokes calculations [2-6], I will then illustrate how this new tool can be used to *design* a more stable plane Couette flow by manipulating the boundaries of the flow [7].

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# New exact coherent states in plane Poiseuille flow

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## Abstract

Two new classes of travelling wave solution are found in plane Poiseuille flow by continuing the stationary (Itano & Generalis, 2009) and travelling wave (Deguchi & Nagata, 2010) hairpin vortex states in plane Couette flow. The first class arises from a saddle-node bifurcation, characterised by mirror-symmetric vertical planes perpendicular to the spanwise direction. The second class bifurcates by breaking the mid-plane symmetry of the first class. We find that both classes exist below the critical saddle-node Reynolds number known to date (Waleffe, 2003).

Both classes of the new solution are featured by two quasi-streamwise low-speed streaks within one spanwise period. The low-speed streaks are aligned with the vertical planes of mirror symmetry, with their width varying in a varicose fashion in the streamwise direction. These streaks appear close to both top and bottom channel walls for the first class and only one of the channel walls for the second class. A pair of quasi-streamwise vortices forms a  $\Lambda$ -shaped vortex: vortices are up-lifted downstream while attached by one foot to each of two neighboring varicose bulges of the streamwise low-speed streaks.

We shall compare the flow structure of the new classes with that of Waleffe's solution as well as available experimental observations and DNSs in detail. We believe that the existence of the new exact coherent states contributes towards the understanding of laminar-turbulent transition in plane Poiseuille flow.

## THE ONSET OF TRANSIENT TURBULENCE IN MINIMAL PLANE COUETTE FLOW

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**Summary** In plane Couette flow a nonlinear time-periodic solution to the Navier–Stokes equations and its stable manifold have recently been found to form the basin boundary between laminar and turbulent flows. Here we present homoclinic orbits from/to this periodic solution. Through the Smale–Birkhoff theorem their existence explains turbulent bursting represented by vortical structures different from those in the familiar regeneration cycle. A pair of the homoclinic orbits is traced down to their tangency Reynolds number at which transient turbulence appears.

### INTRODUCTION

Recent studies have shown that in transitional wall-bounded shear flows without linear instability of a laminar state the boundary between laminar and turbulent flows can be numerically identified in phase space as the edge of chaos [1]. A particularly interesting situation arises when the laminar-turbulent boundary is formed by the stable manifold of a traveling wave or periodic solution, which is then called a “simple edge state.” Edge states have now been computed for flow in channels as well as pipes, with various numerical schemes and resolutions, and there is some consensus that they are a robust feature of subcritical shear flow.

Here we study the two-dimensional unstable manifold of an edge-state periodic solution [2, 3] to the incompressible Navier–Stokes equations in plane Couette flow, using a novel computational algorithm, and find that it contains two orbits which return to the edge state along its stable manifold. We anticipate that these homoclinic orbits collide and disappear in a tangency bifurcation at lower Reynolds number. Away from this bifurcation, these orbits are transversal intersections of the stable and unstable manifolds. Thus, their presence generically implies the existence of an intricate tangle of these manifolds through the classical Smale–Birkhoff theorem. This, in turn, may explain the onset of transient turbulence.

### NUMERICAL COMPUTATION

We consider plane Couette flow at a Reynolds number of  $Re = 400$  in the minimal flow unit of dimensions  $L_x \times 2h \times L_z$ , where  $Re$  is based on half the velocity difference between the two walls,  $U$ , and half the wall separation,  $h$ . The streamwise and spanwise periods are  $(L_x/h, L_z/h) = (1.755\pi, 1.2\pi)$  [4]. We used resolutions of  $16 \times 33 \times 16$  as well as  $32 \times 33 \times 32$  grid points in the streamwise ( $x$ ), wall-normal ( $y$ ) and spanwise ( $z$ ) directions, respectively, and checked that the behaviour is qualitatively the same. The edge state in this computational domain is a time-periodic variation, which shows weak, meandering streamwise streaks [2, 3]. This gentle periodic orbit has a single unstable Floquet multiplier and thus its unstable manifold has dimension two and its stable manifold has codimension one in phase space.

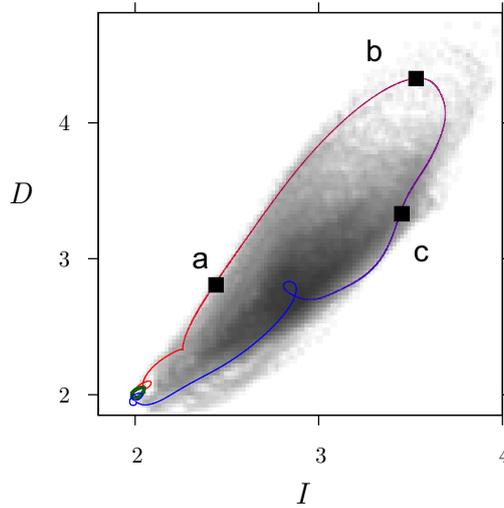
A piece of the two-dimensional unstable manifold can be computed by Newton–Krylov continuation of orbit segments. A boundary value problem is set up which specifies that the initial point of the segment must lie in the linear approximation of the manifold, whereas the final point satisfies some scalar condition, e.g., the energy dissipation rate has a fixed value. This boundary value problem is under-determined by a single degree of freedom and thus we can compute a family of orbit segments by arclength continuation [5]. Since a connecting orbit separates the manifold into two components, the consecutive orbit segments can converge to a connecting orbit during the continuation.

### HOMOCLINIC ORBITS

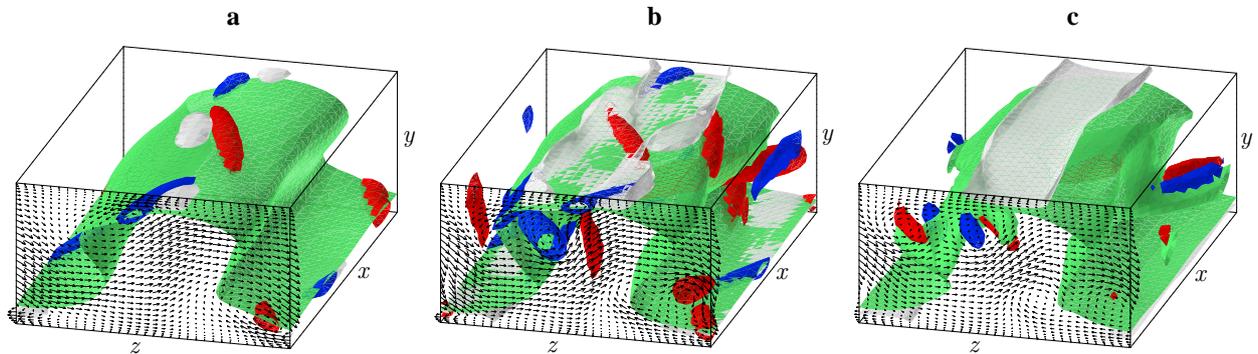
We have found two homoclinic orbits from/to the gentle periodic solution [6]. In Figure 1 one of the two homoclinic orbits is shown, which makes a large excursion during which the energy dissipation rate reaches more than twice the mean value in periodic motion and four times the value at the laminar equilibrium. In the dissipative phase of this bursting cycle, as shown in Figure 2, at the valley and the crest of the streaks small quasi-streamwise vortices are formed and move rapidly in the spanwise direction, and the larger part of the energy dissipation takes place there. Similar dissipative flow structures are also observed during bursting in a turbulent state, and they are very different from those for the near-wall regeneration cycle [4, 2].

The existence of orbits homoclinic to the edge state implies that the geometry of the laminar-turbulent boundary is rather complex and we can expect to find a manageable approximation to it only locally. At the same time, it generically implies the existence of infinitely many unstable periodic orbits which correspond to flows with arbitrarily many, arbitrarily long, near laminarization events. It is natural then to think of turbulent shear flow as governed by a large chaotic attractor which comprises both unstable periodic orbits in the turbulent regime, which reproduce the regeneration cycle [2], and ones which reproduce near-laminarization and bursting events.

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**Figure 1.** Projection of the orbit homoclinic to the gentle periodic solution (shown in green) in plane Couette flow at  $Re = 400$  onto the energy input  $I$  and the dissipation rate  $D$ , normalised by their value in laminar flow. The piece of orbit leaving the gentle periodic solution is shown in red and the one approaching it in blue. In the background, the probability density function of turbulence is shown in gray scale. The labels **a–c** correspond to the snapshots in Figure 2.



**Figure 2.** Visualizations of flow structures in one periodic box  $L_x \times 2h \times L_z$  at three phases on the orbit homoclinic to the gentle periodic solution in plane Couette flow at  $Re = 400$ , labeled as in Figure 1. The green, corrugated isosurfaces of the null streamwise ( $x$ ) velocity represent a streamwise streak. Red and blue objects are isosurfaces of  $0.4\rho(U/h)^2$  for the Laplacian of the pressure, and denote the vortex tubes of the positive and negative streamwise-vorticity component. Gray isosurfaces show the local energy dissipation at 20 times the value of laminar dissipation. Cross-stream ( $y, z$ ) velocity is also shown in the plane  $x = 0$ . The flow symmetry suggests that cross-stream velocity in the plane  $x/h = L_x/2$  is given by the reflection of that in the plane  $x = 0$  in the spanwise ( $z$ ) direction.

## TANGENCY OF HOMOCLINIC ORBITS

A pair of the two homoclinic orbits is observed to collide at their tangency Reynolds number  $Re = 241.3$  below which there is no homoclinic tangle. The Reynolds number of such a tangency bifurcation could be considered as the onset Reynolds number for a homoclinic tangle leading to transient turbulence, which should be significant for a theoretical interpretation of subcritical transition to turbulence in wall-bounded shear flow. In the presentation of this work, we shall also discuss the identification of the onset of transient turbulence in terms of tangency of homoclinic orbits.

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# Minimal transition thresholds in plane Couette flow

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Subcritical transition to turbulence requires finite-amplitude perturbations. Using a nonlinear optimisation technique [1] in a periodic computational domain of plane Couette flow of size  $4\pi \times 2 \times 2\pi$ , we identify the perturbations transitioning with least initial kinetic energy. We propose a new scaling law  $E_c = O(Re^{-2.7})$  for the energy threshold, steeper than all previously identified scalings in plane shear flows but matching well experimental estimates for pipe flow[2]. In all cases the associated perturbations are initially localised in space. The route to turbulence associated with the minimal perturbation is analysed in detail for  $Re = 1500$ . Several instabilities are found to occur in sequence: Orr mechanism, Oblique Wave interaction, lift-up, streak breakdown and spanwise spreading. The phenomenon of streak breakdown is revisited using leading finite-time Lyapunov exponents along the edge trajectory.

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# Decaying past the edge: studying decay in plane Couette flow.

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## Abstract

In recent years there has been extensive study into decaying shear flow turbulence in order to understand the effects of domain size and Reynolds number upon turbulent lifetimes. A separate research track in this field has involved finding exact solutions in shear flows using the “edge of chaos”. The edge is a hypersurface in phase-space separating initial conditions which undergo turbulent behaviour from those which simply relaminarize. However, the existence of this hypersurface raises the following question: at transitional Reynolds numbers, where turbulence is transient, how do initial conditions on the “turbulent side” of the edge pass the edge to reach the laminar state?

To attack this problem we consider small domains in plane Couette flow. Looking for a set of routes back to the laminar state, we consider the statistics of decay when trajectories are aligned to decay at a fixed point in time. This approach is used to analyse the self-sustaining process during decay. Finally, we consider the edge geometry of low-dimensional models and compare with observations from the full dynamics of plane Couette flow.

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