The Leonardo Da Vinci Competition

Experimental and Numerical Study on the Small Scale Features of Turbulent Entrainment

Extended Abstract of PhD Thesis

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Abstract

The submitted work is the first of its kind to study the properties of the turbulent/nonturbulent interface in a flow without strong mean shear, with emphasis on the small-scale aspects. The main tools used are a three-dimensional particle tracking system (3D-PTV) allowing to measure and follow in a Lagrangian manner the field of velocity derivatives and direct numerical simulations (DNS). The differences between small-scale strain and enstrophy are striking and point to the definite scenario of turbulent entrainment via the viscous forces originating in strain. In analogy to the well known fundamental turbulence property of *inviscid interaction* of vorticity and strain, a second genuine feature could be identified, namely, the *viscous interaction* of vorticity and strain. The study solves a long lasting riddle in turbulence research, as it brings direct evidence that the entrainment process is a viscous process.

INTRODUCTION

Turbulent entrainment is a process of continuous transitions from laminar to turbulent flow through the boundary (hereafter referred to as *interface*) between the two coexisting regions of laminar and turbulent state. This process is one of the most ubiquitous phenomena in nature since, in fact, most turbulent flows are partly turbulent. Some examples are boundary layers, all free shear turbulent flows (jets, plumes, wakes, mixing layers), penetrative convection and mixing layers in the atmosphere and ocean, gravity currents, avalanches and clear-air turbulence. In addition, the process plays a key role in a number of applications in technology (e.g., combustion chambers, chemical technology, jets and wakes of aircrafts, missiles, ships and submarines and many others). The main open questions relate to the role played by the large scales of the flow as compared to the smaller scales, i.e. it is still unknown what exactly determines the dependence of the overall entrainment rate on the global flow parameters and what is the relation to the fine structure of the flow in the proximity of the interface. A deeper understanding is of fundamental importance, e.g., for the development of turbulent transport and mixing models.

The first physically qualitative distinction between turbulent and non-turbulent regions, made by Corrsin and Kistler [2], is that turbulent regions are *rotational*, whereas the non-turbulent ones are (practically) potential, thus employing one of the main differences between turbulent flow and its random *irrotational* counterpart on the 'other' side of the *interface* separating them. The main mechanism by which non-turbulent fluid becomes turbulent as it 'crosses' the interface is believed to involve viscous diffusion of vorticity ($\nu\omega_i\nabla^2\omega_i$) at the interface [2]. The same authors also conjectured that the stretching of vortex lines in the presence of a local gradient in vorticity at the interface leads to a steepening of this gradient since the rate of production of vorticity is proportional to the vorticity present. The mentioned processes are associated with the small scales of the flow. However, at large Reynolds numbers the entrainment rate and the propagation velocity of the interface relative to the fluid are known to be independent of viscosity (see [8], [9] and [10]). Therefore the slow process of diffusion into the ambient fluid must be accelerated by interaction with velocity fields of eddies of all sizes, from viscous eddies to the energy-containing eddies, so that the overall rate of entrainment is set by the large-scale parameters of the flow. This means that although the spreading is brought about by small eddies (viscosity), its rate is governed by larger eddies. The total area of the interface, over which the spreading is occurring at any instant, is determined by these larger eddies [9].

OBJECTIVES

The main goal of this research is a systematic study of key questions regarding turbulent entrainment, up to recently inaccessible to experimental research. Such key questions are: (i) what is the role of the large- and small-scale features in this process and (ii) what is the relation between them. Among other things this involves study of the field of velocity and velocity derivatives in a Lagrangian setting. Aspects of interest are the large and small-scale structure and properties of the entrainment interface, on one hand, and the interaction between the small-scale structure of the interface and the properties of the turbulent flow field in the proximity of the entrainment interface, on the other hand. The special emphasis is on the processes involving the field of vorticity, ω_i, and its production/destruction by inertial $\omega_i\omega_j s_{ij}$ and viscous $\nu\omega_i\nabla^2\omega_i$ processes in the proximity of the interface (s_{ij} are the components of the fluctuating rate-of-strain tensor, ν is the kinematic viscosity). Studying the production of vorticity requires access to the field of strain as well (and thereby also to the dissipation, $2\nu s_{ij}s_{ij}$, strain production, $-s_{ij}s_{jk}s_{ki}$, and its viscous destruction, $\nu s_{ij}\nabla^2 s_{ij}$).

RESEARCH ROUTE

On the experimental side, before the Lagrangian measurements were conducted, several preliminary steps were carried out, namely (i) further development of the 3D-PTV technique, (ii) a feasibility study of measuring the Laplacian of vorticity, $\nu\nabla^2\boldsymbol{\omega}$, and (iii) benchmark of detection techniques for the turbulent/non-turbulent interface. First, the experimental method of 3D-PTV was further developed by implementing a scanning system allowing for significantly larger particle densities and/or larger observation windows. Compared to classical PTV, both the observation volume and the spatial resolution could be nearly doubled ([7]). As a next step, experiments in quasi-homogeneous turbulence were conducted to assess the feasibility of measuring the Laplacian of vorticity through PTV. The same experiments were also used to analyze some simple aspects of the contribution of this term to vorticity dynamics. Despite some experimental error associated with the measurements of $\nu\nabla^2\boldsymbol{\omega}$, comparison with DNS demonstrates that the results are in agreement with the simulation, at least on the qualitative level ([6]. In view of preparation for PTV measurements in proximity of a turbulent/non-turbulent interface, a preliminary study on the identification of the interface and its large scale propagation was carried out. Namely, Particle Image Velocimetry (PIV) and dye visualization experiments were conducted to benchmark detection techniques of the turbulent/non-turbulent interface generated by an oscillating grid. Among other things, the theoretically predicted $t^{1/2}$ law of propagation of the turbulent front in time, t, could be verified with both experimental methods by using simple level-based detection techniques ([3]). Finally, small scale enstrophy and strain dynamics in proximity of a turbulent/non-turbulent interface are analyzed by using PTV and DNS ([4] and [5]) and some selected results of this analysis are presented herein.

METHODOLOGY

The methodology is based on the measurement and numerical simulation of the full set of quantities (all the three components of the velocity field and all its nine derivatives, especially vorticity and strain, their production and viscous destruction) in a Lagrangian setting, i.e. along particle trajectories.

Experimentally, a turbulent/non-turbulent inter-

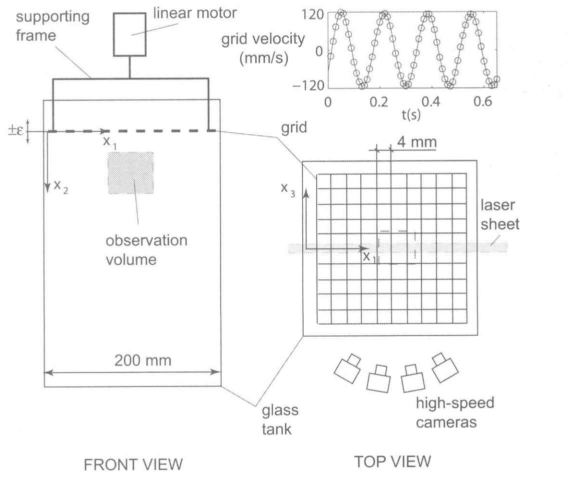

Figure 1: Schematic of the experimental setup. A time sample of the grid velocity obtained from the encoder signal is shown in the upper right corner.

face was realized by using the oscillating planar grid described in [3]. A schematic of the experimental setup is shown in Figure 1. The grid is a fine woven screen installed near the upper edge of a water filled glass tank, oscillating at a frequency of 6 Hz and an amplitude of 4 mm. The scanning method of 3D particle tracking velocimetry (3D-PTV) used here is presented in [7]. The number of tracked particles is about $6{\cdot}10^3$ in a volume of $2{\times}2{\times}1.5$ cm^3 and the interparticle distance is about 1 mm. In both experiment and simulation, the Taylor microscale Reynolds number is Re_λ=50.

Direct numerical simulation (DNS) was performed in a box (side-lengths $5L_1$, $3L_2$, $5L_3$) of initially still fluid. Random (in space and time) velocity perturbations are applied at the boundary x_2=0. The computational domain is finite in the x_2 direction, as $x_2 \leq 3L_2$. Shear-free conditions $\partial u_1/\partial x_2 = \partial u_3/\partial x_2 = u_2 = 0$ are imposed at the boundary $x_2 = 3L_2$. The Navier Stokes equations are solved by using a finite difference method for the spatial discretization and the time advancement is computed by a semi-implicit Runge-Kutta method. The resolution is $256{\times}256{\times}256$ gird points in x_1,x_2 and x_3 directions and the local Kolmogorov length scale is about twice the grid spacing. The paths of 4000 fluid particles have been calculated using, $\frac{D\mathbf{x}}{Dt} = \mathbf{u}(\mathbf{x}, t)$, where $\mathbf{x}$ is the position of the fluid particle at time t. For the time integration of the particle position a 3rd order explicit Runge-Kutta scheme was used. The velocity and other quantities of interest were interpolated to the trajectory point using a bilinear interpolation in space. Further details on both experiment and simulation are described in [4] and [5].

OUTCOME

The selected major results shown herein involve the evolution of small scale quantities in a Lagrangian frame of reference, that is, fluid tracers crossing the turbulent/non-turbulent interface are analyzed. For each particle trajectory, the point in time, t^*, when the interface is crossed was identified using a fixed threshold of enstrophy, $\omega^2 = \omega_i\omega_i$, for details see [3], [4], [5] and references therein. In Figure 2 we show the

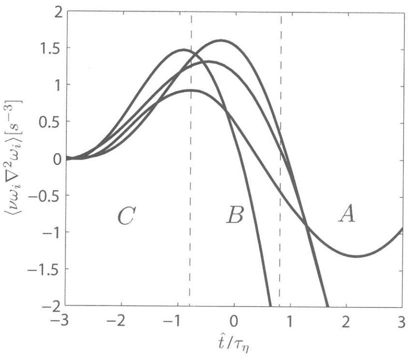

Figure 2: Individual Lagrangian trajectories of $\nu\omega_i\nabla^2\omega_i$ obtained from PTV.

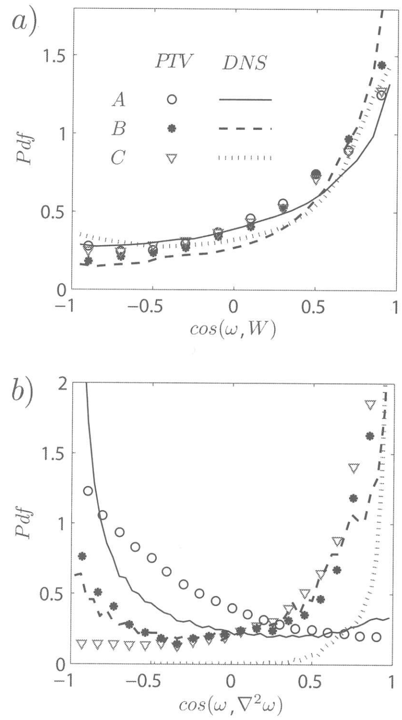

Figure 3: PDF's of the cosine between vorticity, $\boldsymbol{\omega}$, and the vortex stretching vector, $\mathbf{W}$ (a) and the cosine between vorticity, $\boldsymbol{\omega}$, and the Laplacian of vorticity, $\nabla^2\boldsymbol{\omega}$ (b), for the three regions.

remarkable behavior of the term $\nu\omega_i\nabla^2\omega_i$. The figure shows a set of representative individual trajectories of $\nu\omega_i\nabla^2\omega_i$, where the time axis is normalized with the Kolmogorov time scale, τ_η. The important finding is that $\nu\omega_i\nabla^2\omega_i$ is initially positive and shows a distinct maximum during the crossing of the interface. Therefore, we use the maximum of the viscous term as the exact location of the interface, defined in a physically more appealing way than the threshold-dependent time moment t^*. Based on the term $\nu\omega_i\nabla^2\omega_i$, we define three physically distinct regions of the interface with respect to the observed local maximum of $\nu\omega_i\nabla^2\omega_i$ (the regions are marked in Figure 2): (A) the *turbulent* region, in which the behavior of the viscous term is 'normal', i.e. it is mainly negative like in fully developed stationary turbulence, where the term is negative in the mean (e.g., [10]), (B) the interval between the peak and the point where $\nu\omega_i\nabla^2\omega_i$=0 is termed *intermediate* region (with the 'abnormal' viscous production) and, (C) the *non-turbulent* region from the peak to $\hat{t}/\tau_\eta$=-3. It is instructive to look at the properties of the inertial and viscous terms in the three regions, separately. We represent enstrophy production as a scalar product of the vorticity vector and the vortex stretching vector, $W_i = \omega_j s_{ij}$, as $\omega_i\omega_j s_{ij}= \boldsymbol{\omega}\cdot\mathbf{W}$. In analogy we can write the viscous term as $\nu\omega_i\nabla^2\omega_i = \nu\boldsymbol{\omega}\cdot\,\nabla^2\boldsymbol{\omega}$.

For a closer inspection of the nature of enstrophy production in the region of the interface we show PDF's of the cosine between vorticity, $\boldsymbol{\omega}$, and the vortex stretching vector, $\mathbf{W}$, as obtained from PTV and DNS in Figure 3a. We see that in region A the PDF is clearly positively skewed. The positive skewness of this PDF is a well known genuine property of turbulence and one of the main reasons for the positiveness of the mean enstrophy production, $\langle\omega_i\omega_j s_{ij}\rangle >0$ (see [10] and references therein). Interestingly, the changes throughout the regions are quite weak, as the positive skewness is slightly increased in region B, while in region C it is again comparable to region A. In contrast, the cosine of the angle between vorticity and its Laplacian, $\nabla^2\boldsymbol{\omega}$, shown in Figure 3b exhibits significant changes across the regions A,B and C. In region C we observe a very strong positive skewness of the Pdf. The observed transition from positive (alignment, region C) to negative (anti-alignment, region A) values is in agreement with the qualitatively different behavior of $\nu\omega_i\nabla^2\omega_i$ in these regions observed from the individual trajectories shown before. The results indicate that an interpretation of the viscous term $\nu\omega_i\nabla^2\omega_i$ as *interaction between strain and vorticity due to viscosity* is physically more appealing than 'simple' diffusion of vorticity due to viscosity. We emphasize that $\nu\omega_i\nabla^2\omega_i$ is the interaction of vorticity and strain since (e.g., [1]) $\nu\nabla^2\boldsymbol{\omega}= 1/\rho\ \nabla\times\mathbf{F}^s$, where $F_i^s = 2\nu\partial/\partial x_k\{s_{ik}\}$ and ρ is the fluid density.

CONCLUSIONS

For the first time, small scale enstrophy and strain dynamics were analyzed in proximity of a turbulent/non-turbulent interface without strong mean shear. Among other things, we found that both $\omega_i\omega_j s_{ij}$ and $\nu\omega_i\nabla^2\omega_i$ are responsible for the increase of ω^2 at the interface and substantiate the physical interpretation of the term $\nu\omega_i\nabla^2\omega_i$ as *viscous interaction*, in analogy to $\omega_i\omega_j s_{ij}$, commonly referred to as the *inviscid interaction* of vorticity and strain. The results obtained are key for a deeper understanding of turbulent transport and mixing processes in general. The techniques and results can be effectively implemented in many fields where entrainment and mixing are of importance. Examples are processes with combustion, chemical or biochemical reactions.

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