ADIGMA

Grid generation and adaptation in complex geometries

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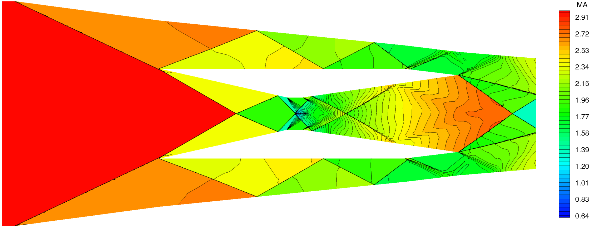
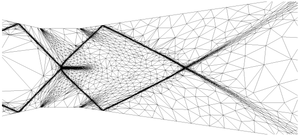
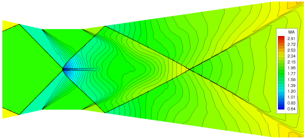
Plan of the Presentation

Motivation

Scram Jet in supersonic flow - $M = 3.0$



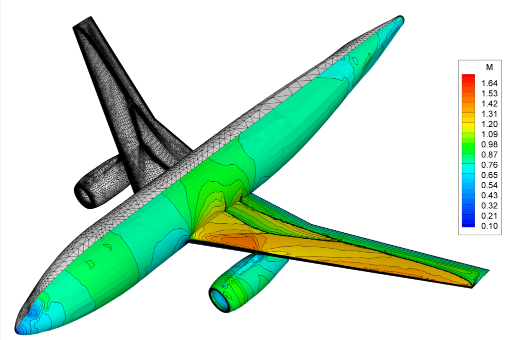


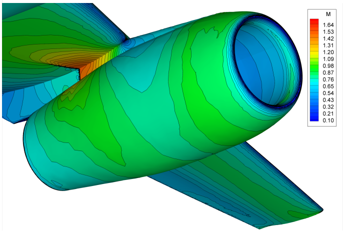


Mach number field (after 7 adaptation steps - 50611 nodes)

Motivation

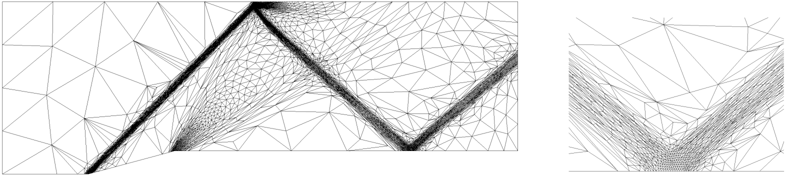
DLR F6 in transonic flow - $M = 0.76\ \alpha = 0.5°$



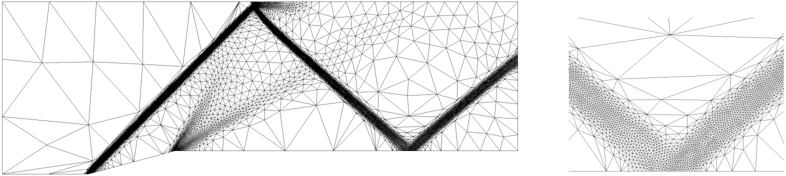


Mach number field (after 3 adaptation steps - 287915 nodes)

Anisotropic Adaptation

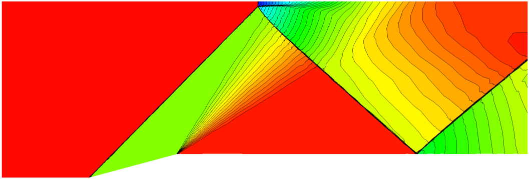


Anisotropic grid after 3 adaptations (6403 grid nodes)

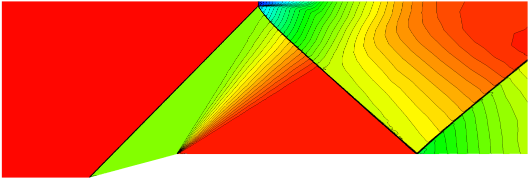


Isotropic grid after 3 adaptations (28078 grid nodes)

Anisotropic Adaptation



Mach number field calculated on the anisotropic grid (6403 grid nodes)



Mach number field calculated on the isotropic grid (28078 grid nodes)

Anisotropic Adaptation

- The adaptation is based on regeneration of the grid
- Initial grid is generated for user-specified grid spacing
- Error estimator provides to the generator a *Control Space* which describes the grid spacing
- Grid generator creates a new grid according to the spacing
- The solution from previous iteration of the adaptation is interpolated on the new grid and the solution process is restarted

Grid spacing - anisotropic concept

- Grid-spacing is typically defined as a scalar
- Anisotropic adaptation needs additional information on directionality which can be provided by second order symmetric positive definite tensor. Such tensor can be used as a metric tensor:

$$l^2 = \mathbf{e}^T \cdot \mathcal{M} \cdot \mathbf{e}$$

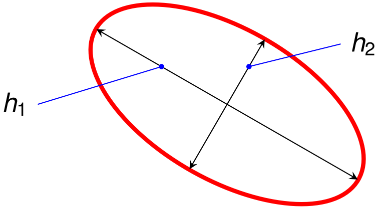
  where **e** is a direction vector and l its length (assuming that $\mathcal{M}$ is constant along **e**)
- Assuming that for computational domain exists a continuous metric tensor field it can form a Riemann space where grid is generated in such a way that edge lengths are of unit length.

Grid spacing - anisotropic concept

- Metric tensor $\mathcal{M}$ can be interpreted as an ellipse (ellipsoid in 3D)
- Since it is SPD it can be decomposed using eigenvalue problem:

$$\mathcal{M} = R \cdot \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \cdot R^{-1} = R \cdot \begin{bmatrix} \frac{1}{h_1^2} & 0 \\ 0 & \frac{1}{h_2^2} \end{bmatrix} \cdot R^{-1}$$

- R is a matrix which columns define unit directions of ellipse main axes



Error Estimation - Hessian based

- Error estimator is based on the interpolation error
- For 2nd order solver the interpolation error is proportional to the Hessian of the current solution:

$$\mathcal{H} = \begin{bmatrix} \frac{\partial^2 \phi}{\partial x^2} & \frac{\partial^2 \phi}{\partial x\,\partial y} & \frac{\partial^2 \phi}{\partial x\,\partial z} \\ \frac{\partial^2 \phi}{\partial x\,\partial y} & \frac{\partial^2 \phi}{\partial y^2} & \frac{\partial^2 \phi}{\partial y\,\partial z} \\ \frac{\partial^2 \phi}{\partial x\,\partial z} & \frac{\partial^2 \phi}{\partial y\,\partial z} & \frac{\partial^2 \phi}{\partial z^2} \end{bmatrix} = R \cdot \Lambda \cdot R^{-1}$$

- The metric field used for generation of the new grid is constructed from the decomposed Hessian:

$$\mathcal{M} = C^{-1} \;\; |\mathcal{H}| = C^{-1} \; R \cdot \begin{bmatrix} |\lambda_1| & 0 & 0 \\ 0 & |\lambda_2| & 0 \\ 0 & 0 & |\lambda_3| \end{bmatrix} \cdot R^{-1}$$

Error Estimation - Hessian based

Hessian reconstruction

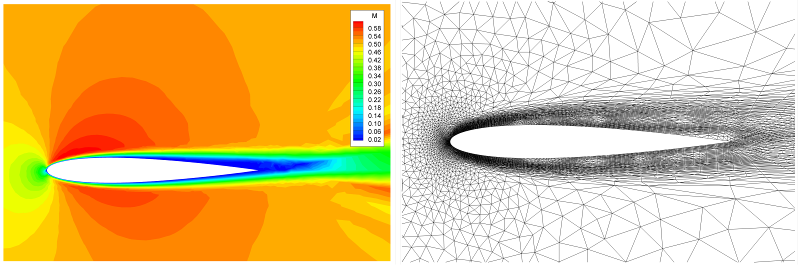
- Based on Green formula:

$$\widetilde{\nabla\phi} = \frac{1}{\Omega} \oint_{\partial\Omega} \phi \, \mathbf{n} \, dS$$

- It is done in three steps:
  - Calculate gradient $\widetilde{\nabla\phi}$
  - Calculate gradient for every component of $\widetilde{\nabla\phi}$ and using those gradients assemble the Hessian
  - Force symmetry condition on the calculated Hessian

Error Estimation - Hessian based

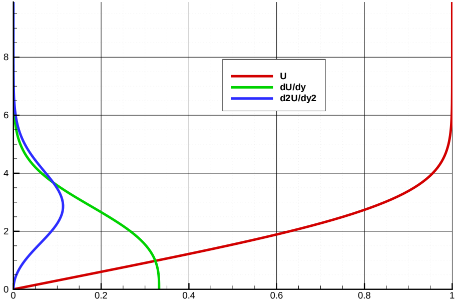
results for viscous flows



Mach number field for NACA–0012 airfoil (Re=5000) and a grid obtained using anisotropic adaptation with Hessian–based error estimator

Error Estimation - Hessian based

viscous flows – laminar boundary layer



Velocity profile for laminar boundary layer (Blasius solution)

Second derivative of the velocity indicates the region refined by Hessian–based estimator

Error Estimation - Gradient based

- Necessary to improve viscous flow adaptation
- Uses gradient of the magnitude of the fluid velocity:

$$\begin{aligned} \phi &= |\mathbf{u}| \\ \mathcal{M} &= \nabla\phi \otimes \nabla\phi \end{aligned}$$

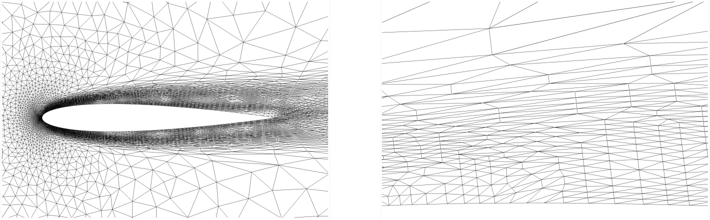
- The metric tensor has following eigenvalues:

$$\Lambda = diag(\nabla\phi \cdot \nabla\phi, 0, 0)$$

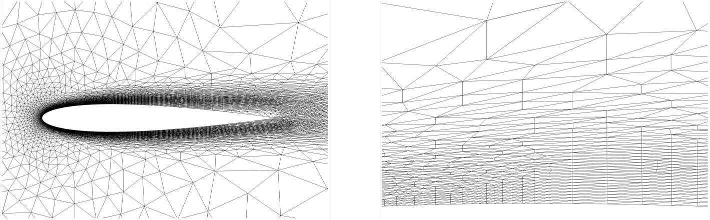
- The metric tensor is then blended with Hessian-based metric tensor

Error Estimation - comparison

viscous flow grid examples



grid generated using hessian-based approach



grid generated using gradient-based approach

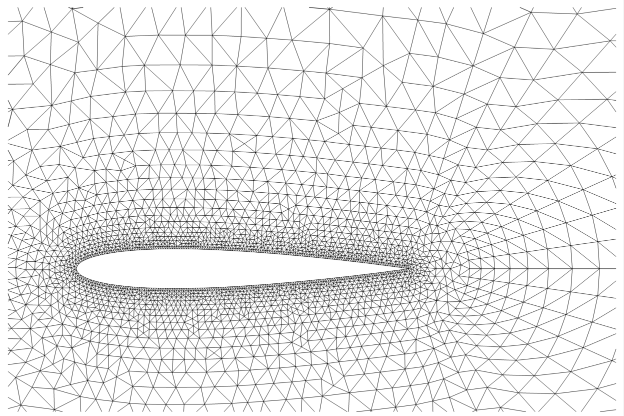
NACA-0012

in laminar flow - $M = 0.5$ $\alpha = 2.0^\circ$ $Re = 5000$

- Steady subsonic 2D flow
- Laminar Navier-Stokes equations
- Solver based on Residual Distribution LDA scheme

NACA-0012

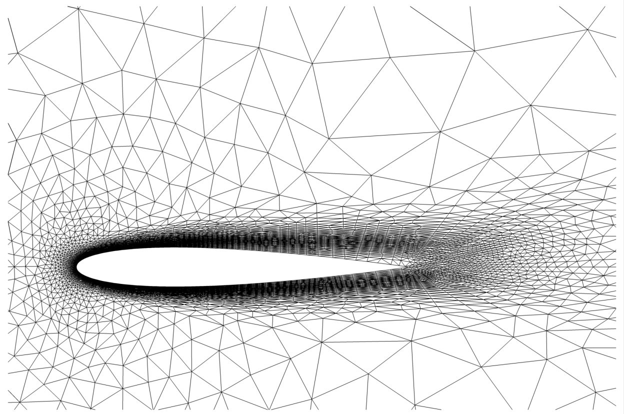
in laminar flow - $M = 0.5$ $\alpha = 2.0^{\circ}$ $Re = 5000$



Initial grid

NACA-0012

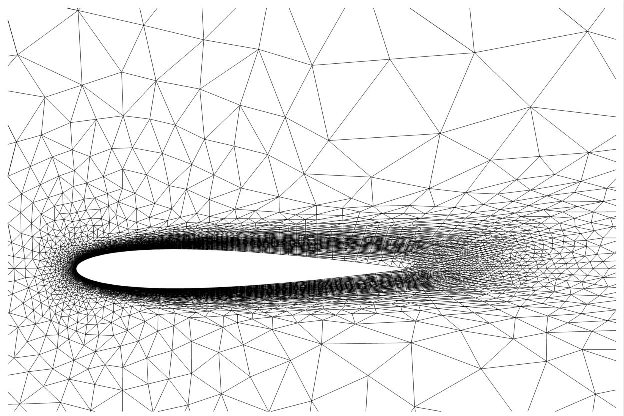
in laminar flow - $M = 0.5$ $\alpha = 2.0^{\circ}$ $Re = 5000$



Grid after 1 adaptation step

NACA-0012

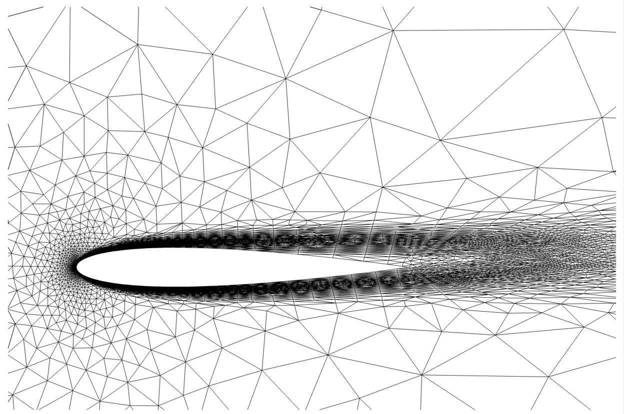
in laminar flow - $M = 0.5$ $\alpha = 2.0^\circ$ $Re = 5000$



Grid after 2 adaptation steps

NACA-0012

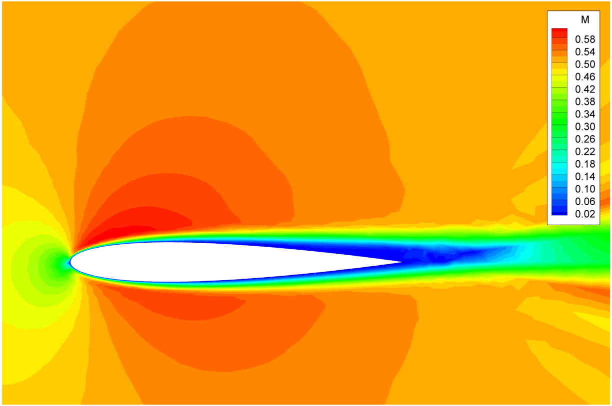
in laminar flow - $M = 0.5$ $\alpha = 2.0^{\circ}$ $Re = 5000$



Grid after 3 adaptation steps

NACA-0012

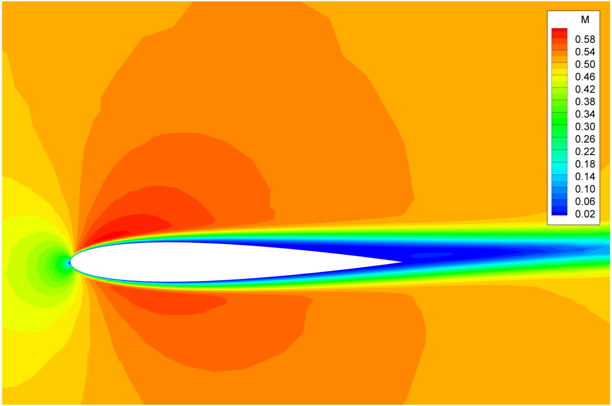
in laminar flow - $M = 0.5$ $\alpha = 2.0^{\circ}$ $Re = 5000$



Mach number field (initial grid)

NACA-0012

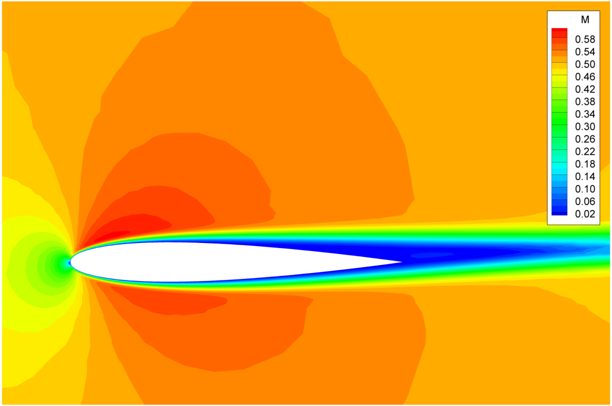
in laminar flow - $M = 0.5$ $\alpha = 2.0^\circ$ $Re = 5000$



Mach number field (after 1 adaptation step)

NACA-0012

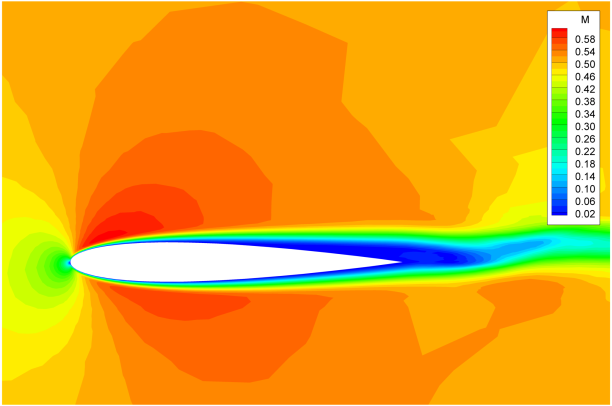
in laminar flow - $M = 0.5$ $\alpha = 2.0°$ $Re = 5000$



Mach number field (after 2 adaptation steps)

NACA-0012

in laminar flow - $M = 0.5$ $\alpha = 2.0°$ $Re = 5000$



Mach number field (after 3 adaptation steps)

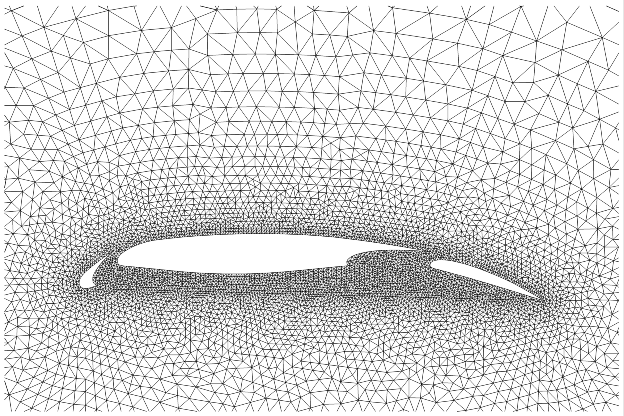
Multi element airfoil L1T2

in turbulent flow - $M = 0.197$ $\alpha = 20.18^\circ$ $Re = 3.52x10^6$

- Steady subsonic 2D flow
- RANS equations with Spalart–Almaras turbulence model
- Solver based on Residual Distribution LDA scheme (VKI – THOR code)

Multi element airfoil L1T2

in turbulent flow - $M = 0.197$ $\alpha = 20.18^\circ$ $Re = 3.52x10^6$

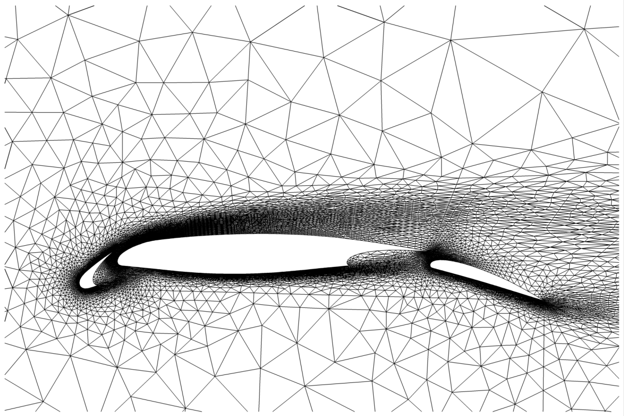


Initial grid

ERCOFTAC Spring Festival, GDANSK 12.05.2011

Multi element airfoil L1T2

in turbulent flow - $M = 0.197$ $\alpha = 20.18°$ $Re = 3.52x10^6$



Grid after 1 adaptation step

Multi element airfoil L1T2

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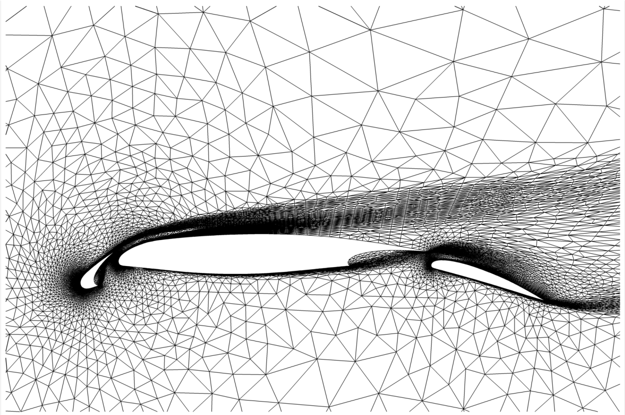
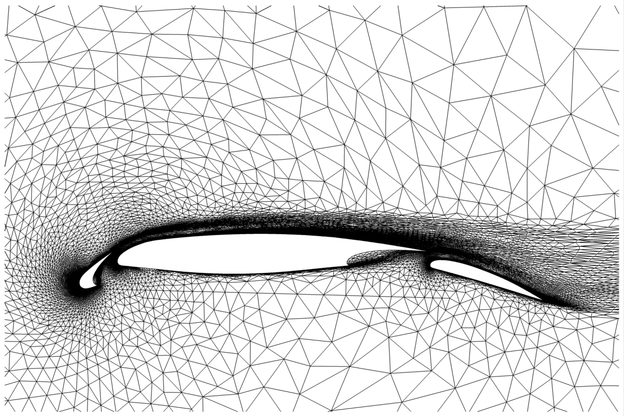


Grid after 4 adaptation steps

Multi element airfoil L1T2

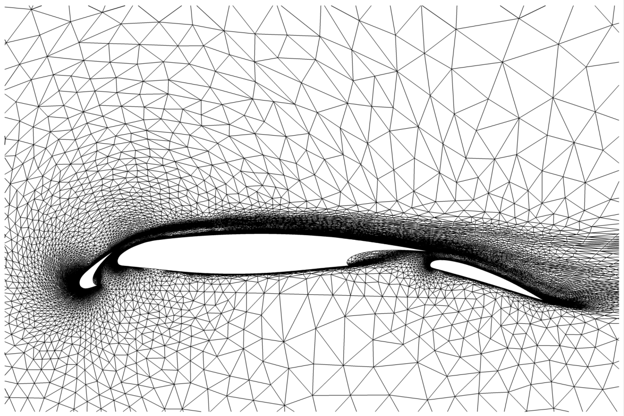
in turbulent flow - $M = 0.197$ $\alpha = 20.18^\circ$ $Re = 3.52x10^6$



Grid after 6 adaptation steps

Multi element airfoil L1T2

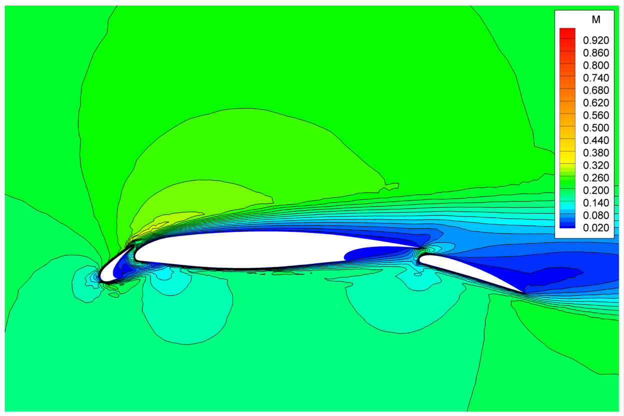
in turbulent flow - $M = 0.197$ $\alpha = 20.18°$ $Re = 3.52x10^6$



Grid after 8 adaptation steps

Multi element airfoil L1T2

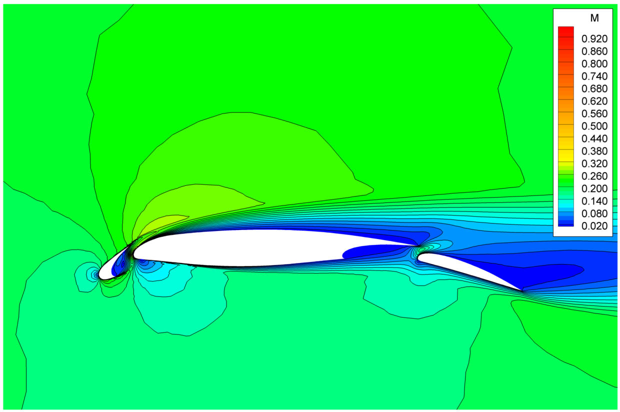
in turbulent flow - $M = 0.197$ $\alpha = 20.18^\circ$ $Re = 3.52x10^6$



Mach number field (initial grid)

Multi element airfoil L1T2

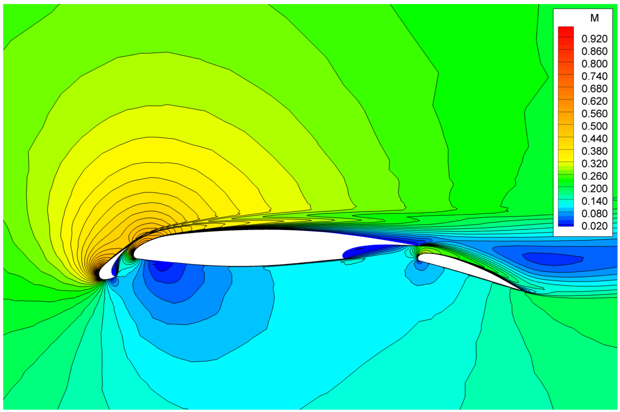
in turbulent flow - $M = 0.197$ $\alpha = 20.18^\circ$ $Re = 3.52x10^6$



Mach number field (after 1 adaptation step)

Multi element airfoil L1T2

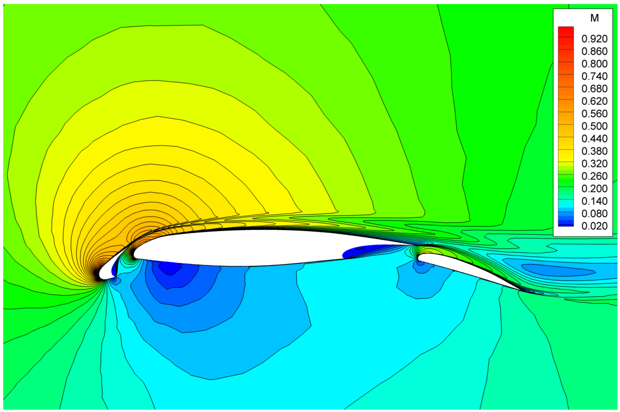
in turbulent flow - $M = 0.197$ $\alpha = 20.18^\circ$ $Re = 3.52x10^6$



Mach number field (after 4 adaptation steps)

Multi element airfoil L1T2

in turbulent flow - $M = 0.197$ $\alpha = 20.18^\circ$ $Re = 3.52x10^6$



Mach number field (after 6 adaptation steps)

Multi element airfoil L1T2

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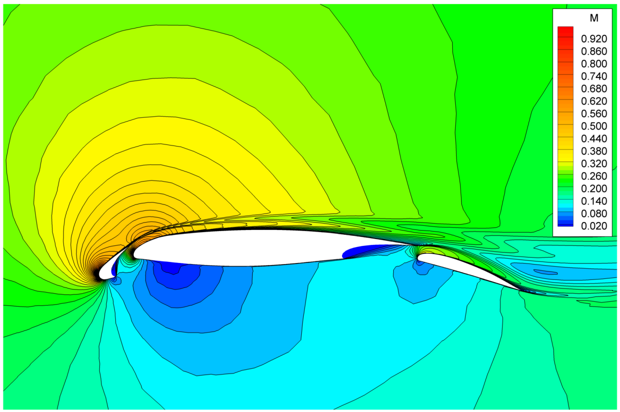
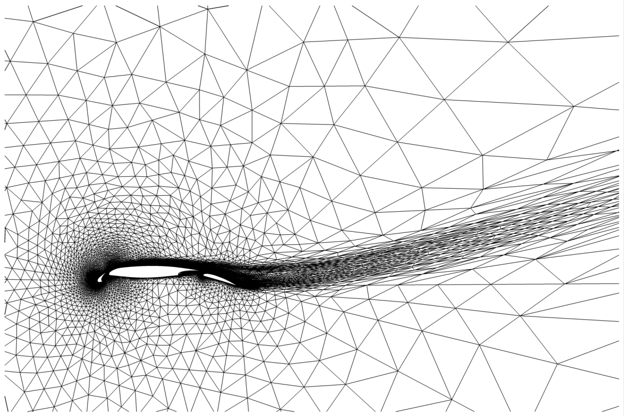


Mach number field (after 8 adaptation steps)

Multi element airfoil L1T2

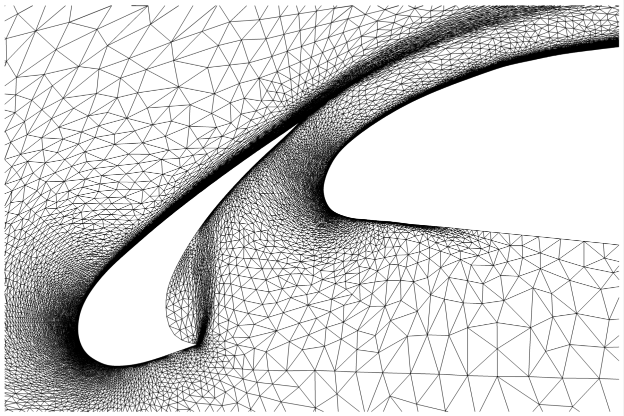
in turbulent flow - $M = 0.197$ $\alpha = 20.18°$ $Re = 3.52x10^6$



Grid after 8 adaptation steps - wake

Multi element airfoil L1T2

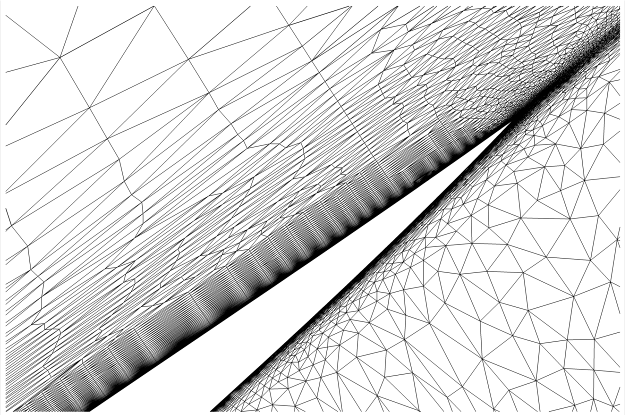
in turbulent flow - $M = 0.197$ $\alpha = 20.18^\circ$ $Re = 3.52x10^6$



Grid after 8 adaptation steps - slat

Multi element airfoil L1T2

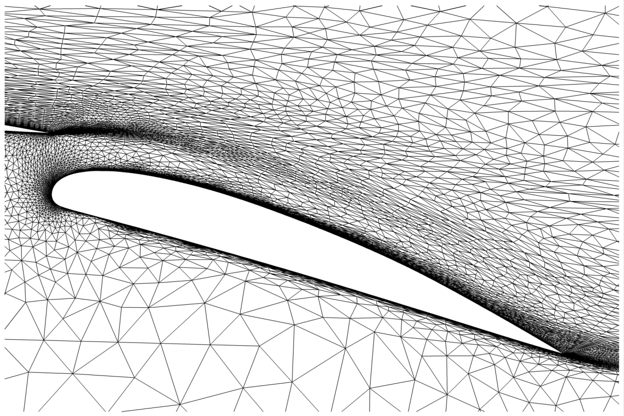
in turbulent flow - $M = 0.197$ $\alpha = 20.18^{\circ}$ $Re = 3.52x10^{6}$



Grid after 8 adaptation steps slat boundary layer grid

Multi element airfoil L1T2

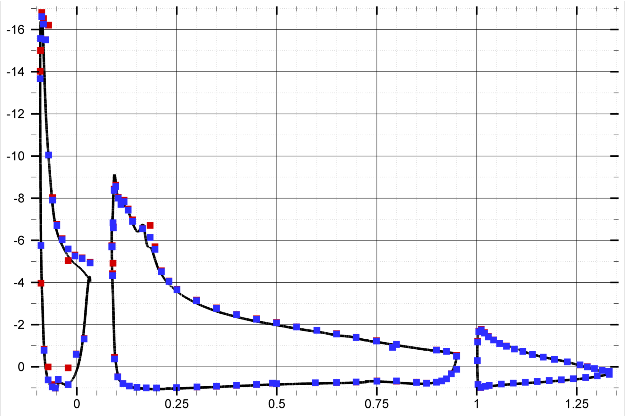
in turbulent flow - $M = 0.197$ $\alpha = 20.18^\circ$ $Re = 3.52x10^6$



Grid after 8 adaptation steps - flap

Multi element airfoil L1T2

in turbulent flow - $M = 0.197$ $\alpha = 20.18°$ $Re = 3.52x10^6$

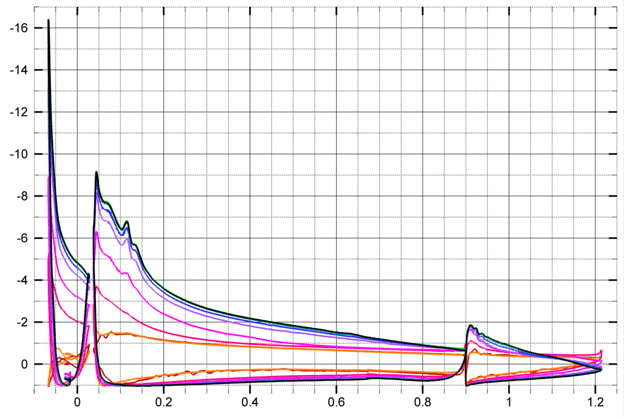


Cp distribution - comparison with experiment

black lines - computation, ■ - experiment 1, ■ - experiment 2

Multi element airfoil L1T2

in turbulent flow - $M = 0.197$ $\alpha = 20.18^\circ$ $Re = 3.52x10^6$



Cp distribution for all adaptation steps

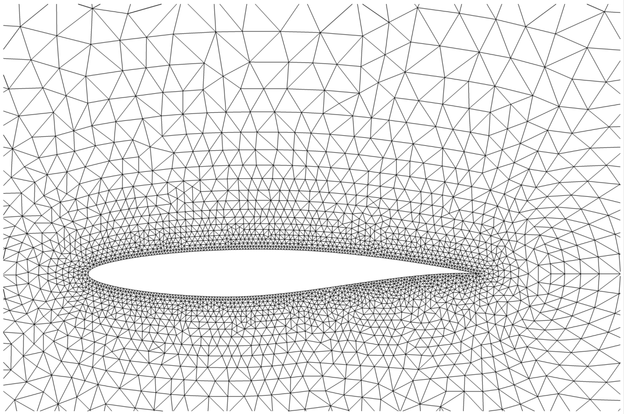
RAE - 2822

in turbulent flow – $M = 0.73$ $\alpha = 3.19^\circ$ $Re = 6.5x10^6$

- Steady transonic 2D flow
- RANS equations with Spalart–Almaras turbulence model
- Solver based on Residual Distribution LDA/N scheme (VKI – THOR code)

RAE - 2822

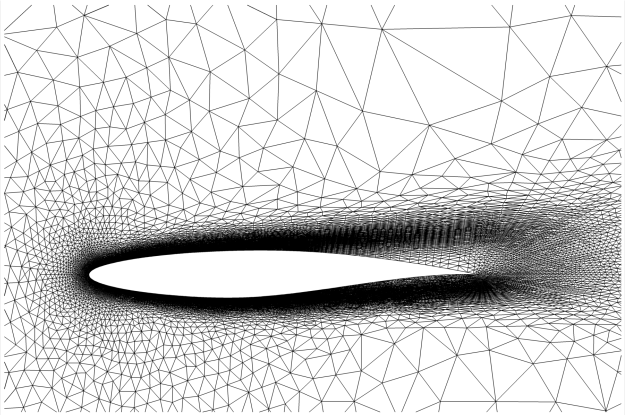
in turbulent flow – $M = 0.73$ $\alpha = 3.19^{\circ}$ $Re = 6.5x10^{6}$



Initial grid

RAE - 2822

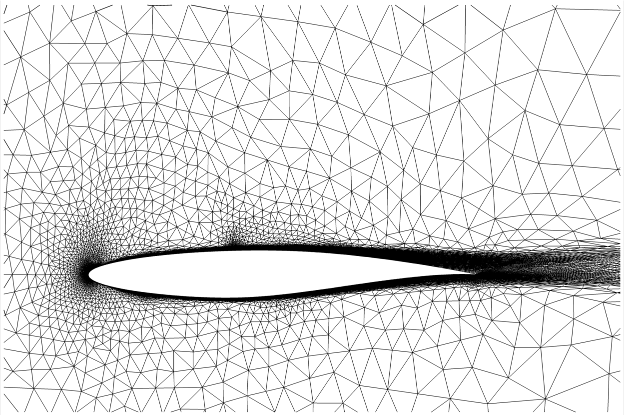
in turbulent flow – $M = 0.73$ $\alpha = 3.19^\circ$ $Re = 6.5x10^6$



Grid after 1 adaptation step

RAE - 2822

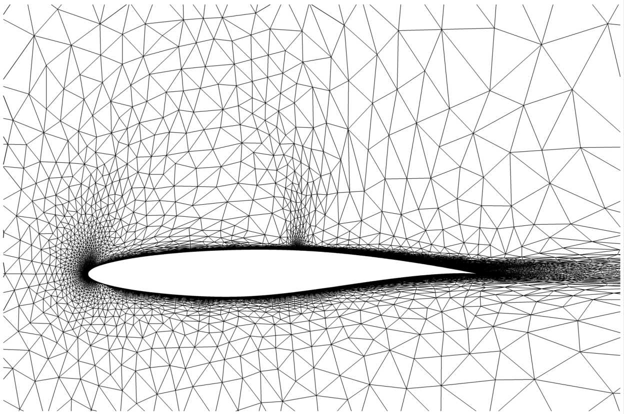
in turbulent flow – $M = 0.73$ $\alpha = 3.19°$ $Re = 6.5x10^6$



Grid after 3 adaptation steps

RAE - 2822

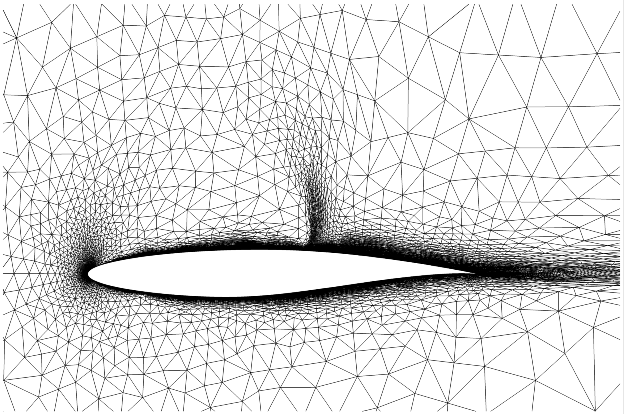
in turbulent flow – $M = 0.73$ $\alpha = 3.19°$ $Re = 6.5x10^6$



Grid after 5 adaptation steps

RAE - 2822

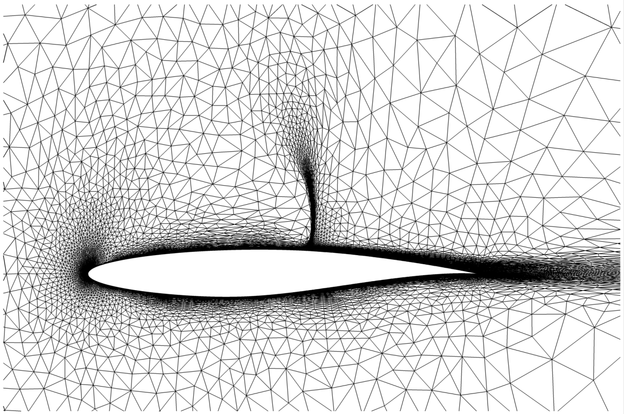
in turbulent flow – $M = 0.73$ $\alpha = 3.19°$ $Re = 6.5x10^6$



Grid after 8 adaptation steps

RAE - 2822

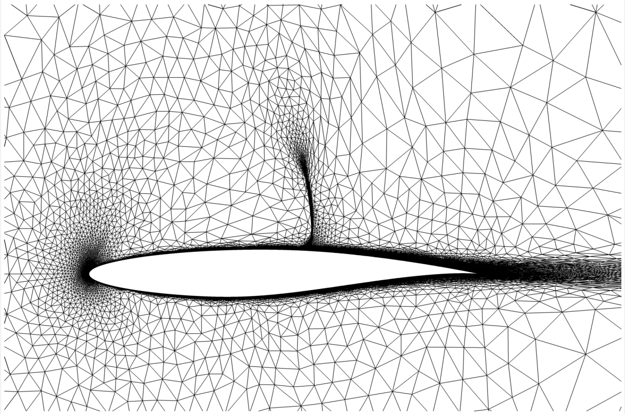
in turbulent flow – $M = 0.73$ $\alpha = 3.19^\circ$ $Re = 6.5x10^6$



Grid after 10 adaptation steps

RAE - 2822

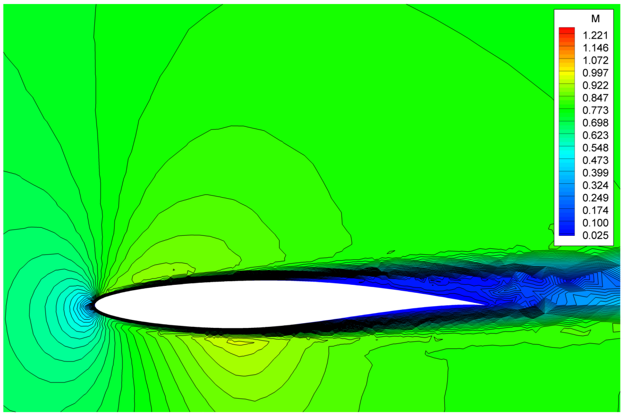
in turbulent flow – $M = 0.73$ $\alpha = 3.19^{\circ}$ $Re = 6.5x10^{6}$



Grid after 13 adaptation steps

RAE - 2822

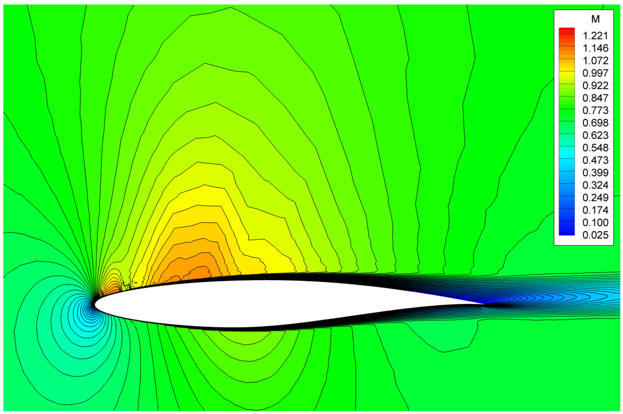
in turbulent flow – $M = 0.73$ $\alpha = 3.19^\circ$ $Re = 6.5x10^6$



Mach number field (initial grid)

RAE - 2822

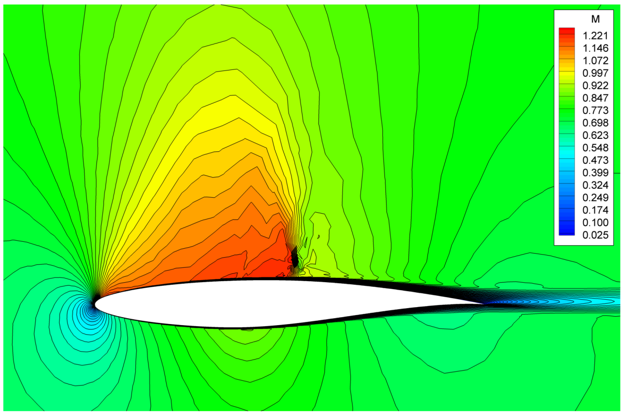
in turbulent flow – $M = 0.73$ $\alpha = 3.19^\circ$ $Re = 6.5x10^6$



Mach number field (after 1 adaptation step)

RAE - 2822

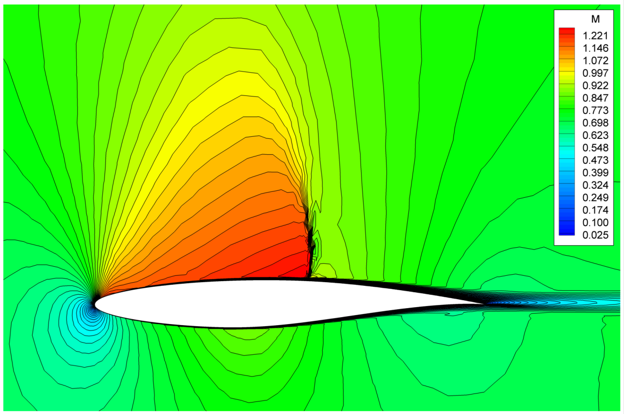
in turbulent flow – $M = 0.73\ \alpha = 3.19^{\circ}\ Re = 6.5x10^{6}$



Mach number field (after 3 adaptation steps)

RAE - 2822

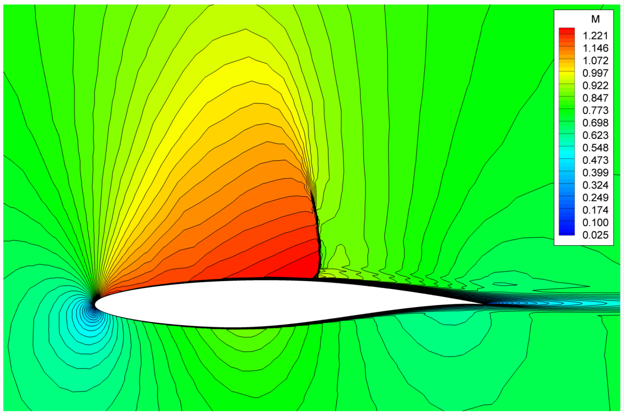
in turbulent flow – $M = 0.73$ $\alpha = 3.19^\circ$ $Re = 6.5x10^6$



Mach number field (after 5 adaptation steps)

RAE - 2822

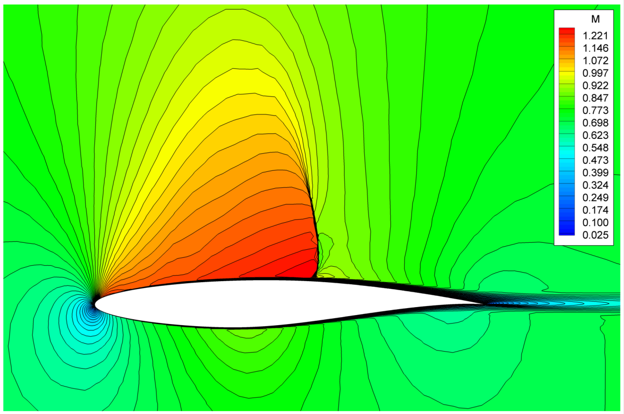
in turbulent flow – $M = 0.73$ $\alpha = 3.19°$ $Re = 6.5x10^6$



Mach number field (after 8 adaptation steps)

RAE - 2822

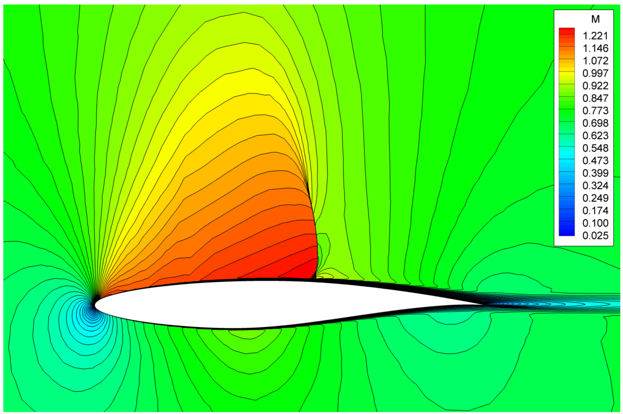
in turbulent flow – $M = 0.73$ $\alpha = 3.19^\circ$ $Re = 6.5x10^6$



Mach number field (after 10 adaptation steps)

RAE - 2822

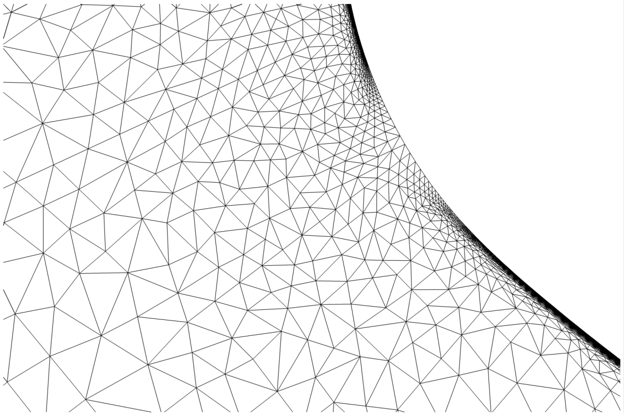
in turbulent flow – $M = 0.73$ $\alpha = 3.19°$ $Re = 6.5x10^6$



Mach number field (after 13 adaptation steps)

RAE - 2822

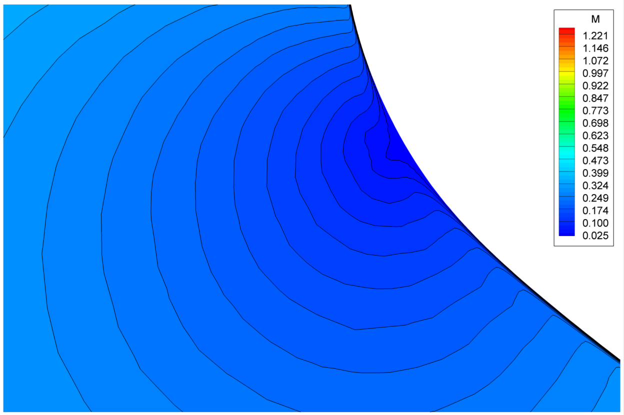
in turbulent flow – $M = 0.73$ $\alpha = 3.19^{\circ}$ $Re = 6.5x10^6$



Grid after 13 adaptation steps details near the stagnation point

RAE - 2822

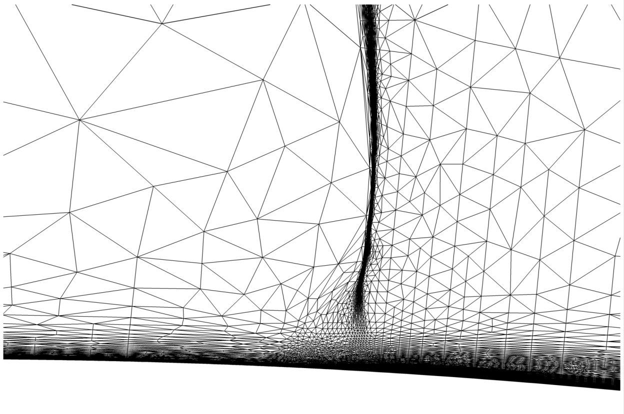
in turbulent flow – $M = 0.73\ \alpha = 3.19^\circ\ Re = 6.5x10^6$



Mach number field (after 13 adaptation steps)
details near the stagnation point

RAE - 2822

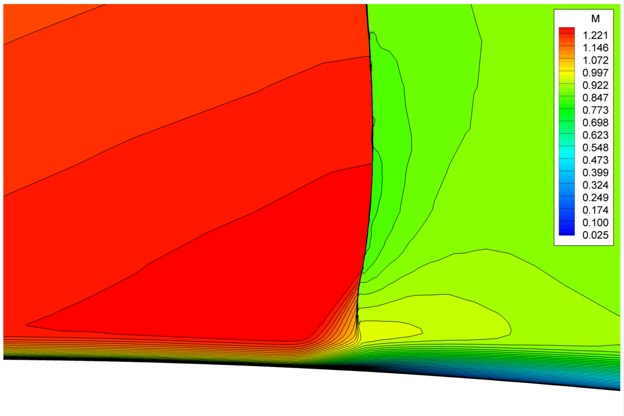
in turbulent flow – $M = 0.73$ $\alpha = 3.19^\circ$ $Re = 6.5x10^6$



Grid after 13 adaptation steps details near the shock wave foot

RAE - 2822

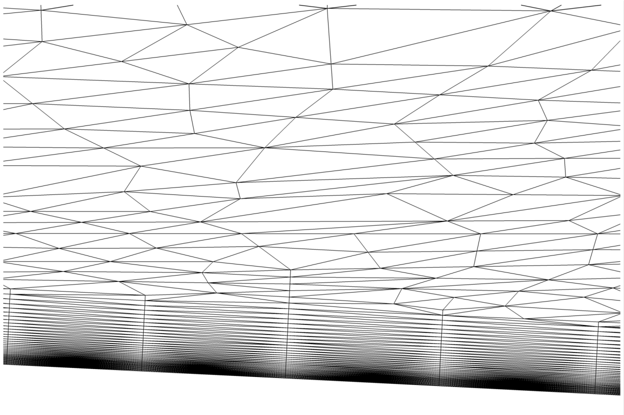
in turbulent flow – $M = 0.73$ $\alpha = 3.19°$ $Re = 6.5x10^6$



Mach number field (after 13 adaptation steps)
details near the shock wave foot

RAE - 2822

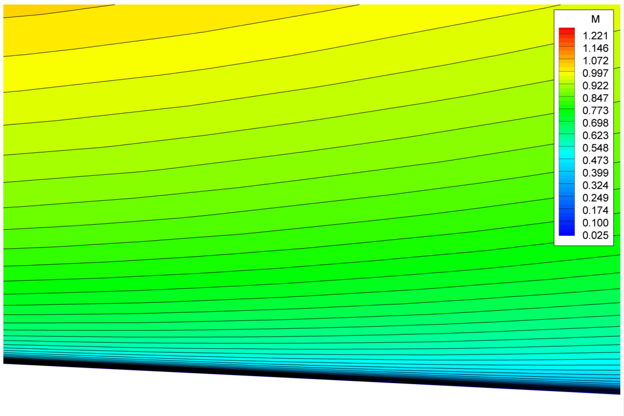
in turbulent flow – $M = 0.73$ $\alpha = 3.19^\circ$ $Re = 6.5x10^6$



Grid after 13 adaptation steps boundary layer

RAE - 2822

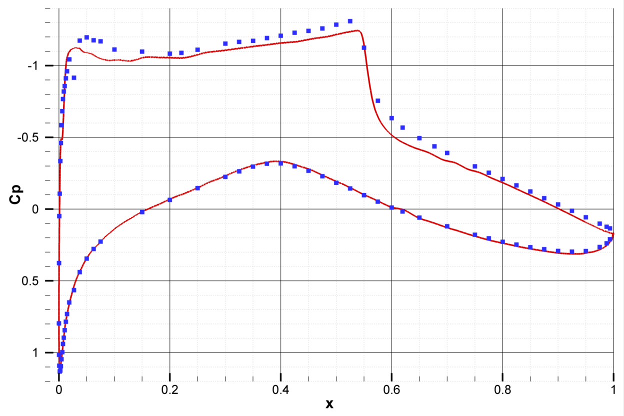
in turbulent flow – $M = 0.73$ $\alpha = 3.19^\circ$ $Re = 6.5x10^6$



Mach number field (after 13 adaptation steps)
boundary layer

RAE - 2822

in turbulent flow – $M = 0.73$ $\alpha = 3.19°$ $Re = 6.5x10^6$



Cp distribution for all adaptation steps

red lines - computation, ■ - experiment

Conclusions

- Hessian based error estimator without additional components is not sufficient for definition of the boundary layer grid spacing
- Adding the gradient-based component into Hessian-based metric allows for fully automatic adaptation of 2D high Reynolds number flows

Future Work

- Adaptation for 3D turbulent flows
- 3D volume grid generator - coupled with semistructured boundary layer grid generator.
- Improvement of the interpolator used for transferring the solution from the old grid to the new one.
- Improvement of the error estimator.

Acknowledgments

- The majority of the work presented here were done in frame of the ADIGMA project AST5-CT-2006-030719 in close cooperation with the Von Karman Institute.
- All turbulent calculations were done using THOR code developed in VKI.