Global Modes and Reduced Order Models of Fluids

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ERCOFTAC Spring Festival, May 12-13 2011, Gdańsk
Poznan University of Technology
Virtual Engineering Group

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Flow model

- Passive
- Active

Feedback

Flow control

- Drag reduction
- Growth of lift force
- Flutter reduction
- Noise reduction
- etc.

Development of new aircraft

Flow model

Fluid-Structure Interaction

Reduced Order Model (Galerkin, ANN, …)

High-fidelity model (DNS, LES, DES, RANS, …)

Kolmogorov K41 hypothesis:

\[ NDE = Re^{9/4} \]

Degrees of Freedom

\( > 99 \% \)
\( < 1 \% \)
Flow Modelling

\[ \dot{u} + \nabla(u \otimes u) + \nabla p - \frac{1}{\text{Re}} \Delta u = R \]  

(Discretised) Navier-Stokes Equations

\[ \left( w_i, R^{[N]} \right)_\Omega = 0 \]  

Weighted Residual

\[ w_i = \begin{cases} 
1 & \text{if inside } \Omega^i \\
0 & \text{if outside } \Omega^i 
\end{cases} \]  

\[ \left( w_i, R^{[P]} \right)_\Omega = 0 \]  

Finite Volume Method

\[ u^{[N]} = \sum_{i=0}^{N} a_i \cdot u_i \]  

\[ w_i = u_i \]  

\[ \left( u_i, R^{[P]} \right)_\Omega = 0 \]  

Galerkin Method

Expansion modes:

Local

Finite Element Method

Global

Reduced Order Models

\[ \dot{a}_i = \frac{1}{\text{Re}} \sum_{j=0}^{N} l_{ij} a_j + \sum_{j=0}^{N} \sum_{k=0}^{N} q_{ijk} a_j a_k \]
Possible mode bases

**MATHEMATICAL**
- Carrier Field Ansatz
- Generalized streamfunctions
- Wavelets

**PHYSICAL**
- Stokes modes,
- Singular Stokes modes,
- Global stability analysis modes

**EMPIRICAL**
- Proper Orthogonal Decomposition (Karhunen–Loève, Principal Component Analysis)
- Dynamic Mode Decomposition

**Low-dimensional Galerkin models**

- Continuity equation
- Dimension: AVERAGE

- Navier-Stokes equations
- Dimension: AVERAGE-LOW

- Unsteady flow data
- Dimension: LOW
Proper Orthogonal Decomposition

Velocity fields from M time steps, each N degrees of freedom

\[ u_0 = \frac{1}{M} \sum_{i=1}^{M} v_i \]

\[ v'_i = v_i - u_0 \]

\[ S = [v'_1, v'_2, \ldots, v'_M] \]

Eigenvectors \( u_j \)

Eigenvalues

Coefficients \( \alpha_{ij} = u_j^T v_i \)

Correlation matrix

\[ C = \frac{1}{M} SS^T \]

Minimalisation of mean-square error of process representation
Global Stability Analysis

Navier-Stokes Equation
\[
\dot{V}_i + V_{i,j} V_j + P_i - \frac{1}{\text{Re}} V_{i,jj} = 0
\]

Snapshot = base flow + small disturbance
\[
V_i = \bar{V}_i + V'_i \quad \quad P = \bar{P} + P'
\]
\[
V'_i(x, y, t) = \tilde{V}_i(x, y)e^{-\lambda t} \quad \quad P'(x, y, t) = \tilde{P}(x, y)e^{-\lambda t}
\]

Disturbance equation
\[
-\lambda \tilde{V}_i + \tilde{V}_{i,j} V_j + \bar{V}_{i,j} \tilde{V}_j + \tilde{P}_i - \frac{1}{\text{Re}} \tilde{V}_{i,jj} = 0
\]

Base flow = STEADY SOLUTION

Base flow = MEAN FLOW

Discretisation (e.g. FEM)

Generalised eigenvalue problem
\[
A \mathbf{x} - \lambda B \mathbf{x} = 0
\]
ROM limitations: transitional flow

Empirical (POD) GM:
- structurally unstable for 2POD modes
- Shift-mode required
- Poor transients reconstruction (narrow dynamic range)
- Good LCO reconstruction

Physical GM:
- Poor reconstruction of LCO
- Accurate for fixed-point dynamics

Solutions:
- Hybrid Galerkin Modes
- Parametrisation of Mode Basis
Broadband AE-ROM

Continuous Mode Interpolation – Parametrized Mode Basis

Attractor

OPERATING CONDITIONS II

- $A^*(\kappa) = (1 - \kappa)A_s + \kappa(A_o)$

- POD modes
  - $\kappa = 0.25$
  - $\kappa = 0.50$
  - $\kappa = 0.75$

- Eigenmodes

OPERATING CONDITIONS I

- time-avg. solution
- shift-mode
- steady solution

Fixed point

M. Morzyński, W. Stankiewicz, B.R. Noack, F. Thiele, R. King, G. Tadmor
Generalized Mean-Field Model for Flow Control Using a Continous Mode Interpolation. 3rd AIAA Flow Control Conference. AIAA Paper 2006-3488

W. Stankiewicz, M. Morzyński, R. Roszak, B.R. Noack, G. Tadmor
Need of ROM in design and control

Aircraft development and certification requires flutter analysis

AIAA 2008, Rossow, Kroll
Aero Data Production A380 wing

50 flight points
100 mass cases
10 a/c configurations
5 maneuvers
20 gusts (gradient lengths)
4 control laws
~20,000,000 CFD simulations

Engineering experience for current configurations and technologies
~100,000 simulations

10 Mio-element mesh: computational cost of 1 step of AE analysis on 16-core PC cluster:
\[ t = 80s + 2 \times 10s + 30s + \frac{4}{50}s \]

Fluid – Structure Interaction algorithm

Flow ROM
Convergence
Pressure
Interpolation
Forces on structure
Structural code

Deformed CFD mesh, velocities
Interpolation
Amplitudes of "mesh" modes

Structure displacements and velocities
Motion of the boundary and mesh

\[ \begin{align*}
\dot{u} + \nabla (u \otimes u) + \nabla p - \frac{1}{Re} \Delta u &= \cdot & \text{Eulerian approach} \\
M \ddot{x} + C \dot{x} + K x &= F(\ddot{x}, \dot{x}, x, t) & \text{Lagrangian approach}
\end{align*} \]

**Arbitrary Lagrangian-Eulerian Approach** (ALE) binds with each other the velocity of the flow \( u \) and the velocity of the (deforming) mesh \( u_{grid} \). For incompressible Navier-Stokes equations the mesh velocity modifies the convective term:

\[ \dot{u} + \nabla \cdot \left( (u - u_{grid}) \otimes u \right) + \nabla p - \frac{1}{Re} \Delta u = 0 \]

With boundary conditions: \( u = u_{grid} \)

The fluid mesh can move independently of the fluid particles.

Projection of convective term

\[ \dot{u} + \nabla \left( (u - u_{\text{grid}}) \otimes u \right) + \nabla p - \frac{1}{\text{Re}} \Delta u = 0 \]

Arbitrary Lagrangian-Eulerian Approach

1. DECOMPOSITION

\[ u_{\text{grid}} = \sum_{i=1}^{N_G} a_i^G \cdot u_i^G \]

2. GALERKIN PROJECTION

\[ - \left( u_i, \nabla \cdot (u - u_{\text{grid}}) \otimes u \right)_\Omega = - \left( u_i, \nabla \cdot (u \otimes u) \right)_\Omega + \left( u_i, \nabla \cdot (u_{\text{grid}} \otimes u) \right)_\Omega = \]

\[ = \sum_{j=0}^{N} \sum_{k=0}^{N} q_{ijk} a_j a_k - \sum_{j=1}^{N_G} \sum_{k=0}^{N} q_{ijk}^G a_j^G a_k \]

\[ q_{ijk}^G = - \left( u_i, \nabla \cdot (u_j^G \otimes u_k) \right)_\Omega \]
Flutter Laboratory
IoA and PUT
experiment and computations

Scale:
- Length - 1:4
- Strouhal number 1:1
Aircraft structure model

3D model of an aircraft structure (SolidWorks CAD geometry and Finite Element Model)
CFD model of an aircraft

3D model for CFD analyses
(Surface geometry and Unstructured Finite Element Mesh)

1:4 model

3D scanning

unstructured, viscous grid

CAD geometry
Full configuration aircraft

Numerical response in Aeroelastic analysis

The first of the aircraft structure's eigenmodes

Displacements of control node on the leading edge of the wing
Full configuration aircraft

Wind tunnel tests (Institute of Aviation, Warsaw) and computational flutter simulation (Poznan University of Technology)
Summary
Activities of Virtual Engineering Group at Poznan University of Technology

- Low dimensional analysis
  - Global flow stability analysis
  - Reduced Order Modelling of fluid flows
  - Feedback flow control
- (Very) high dimensional analysis
  - CFD and aeroelastic analysis of industrial-relevant cases
  - Development of aeroelastic tools
- Investigations cover advanced theoretical methods as well as practical applications
- Investigations are oriented to use at highly advanced methods in practical problems (Reduced Order Aeroelastic Models of full configurations of aircraft)
- Activities not covered in the talk
  - Topological optimisation of aerostructures
  - Modal analysis in signal processing and biomechanics
  - Activities in EU and other Projects