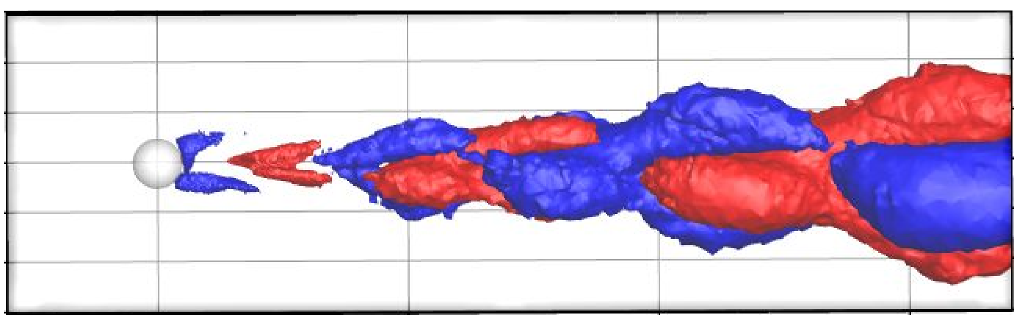
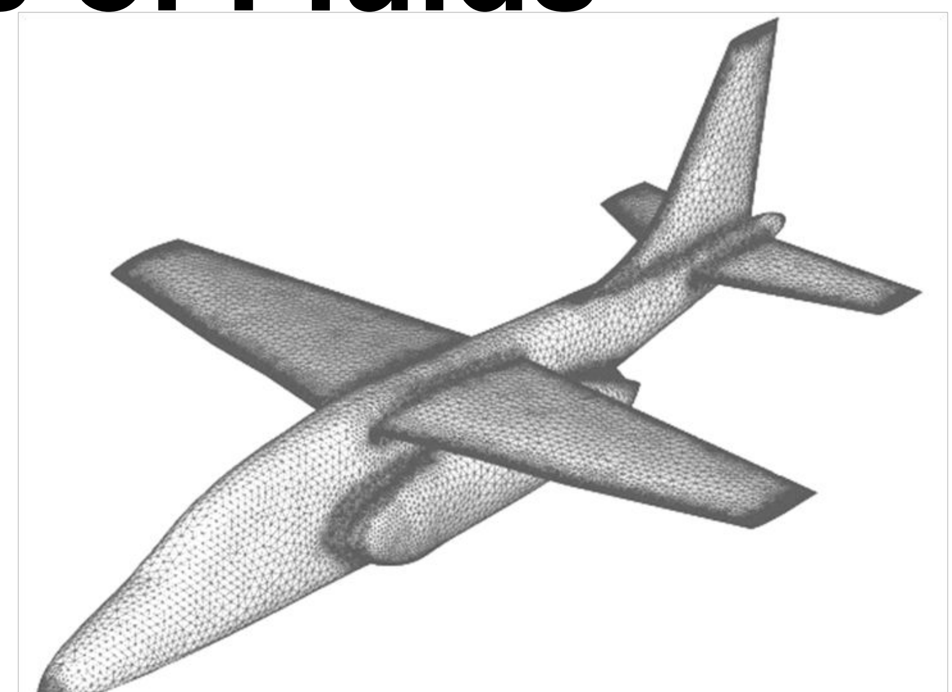
Poznan University of Technology
Institute of Combustion Engines and Transport
Virtual Engineering Group, Flutter Lab

http://stanton.ice.put.poznan.pl/flutter-laboratory/

Global Modes and Reduced Order Models of Fluids



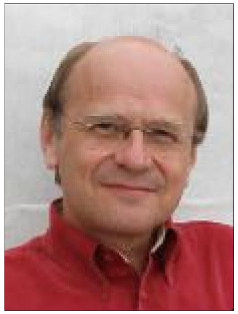


Witold STANKIEWICZ
Marek MORZYŃSKI
Robert ROSZAK

ERCOFTAC Spring Festival, May 12-13 2011, Gdańsk

Poznan University of Technology

Virtual Engineering Group



Marek MORZYŃSKI

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- **P. Posadzy**
- **K. Kotecki**
- **H. Hausa**
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- **P. Przybyła**

and others

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Collaboration:
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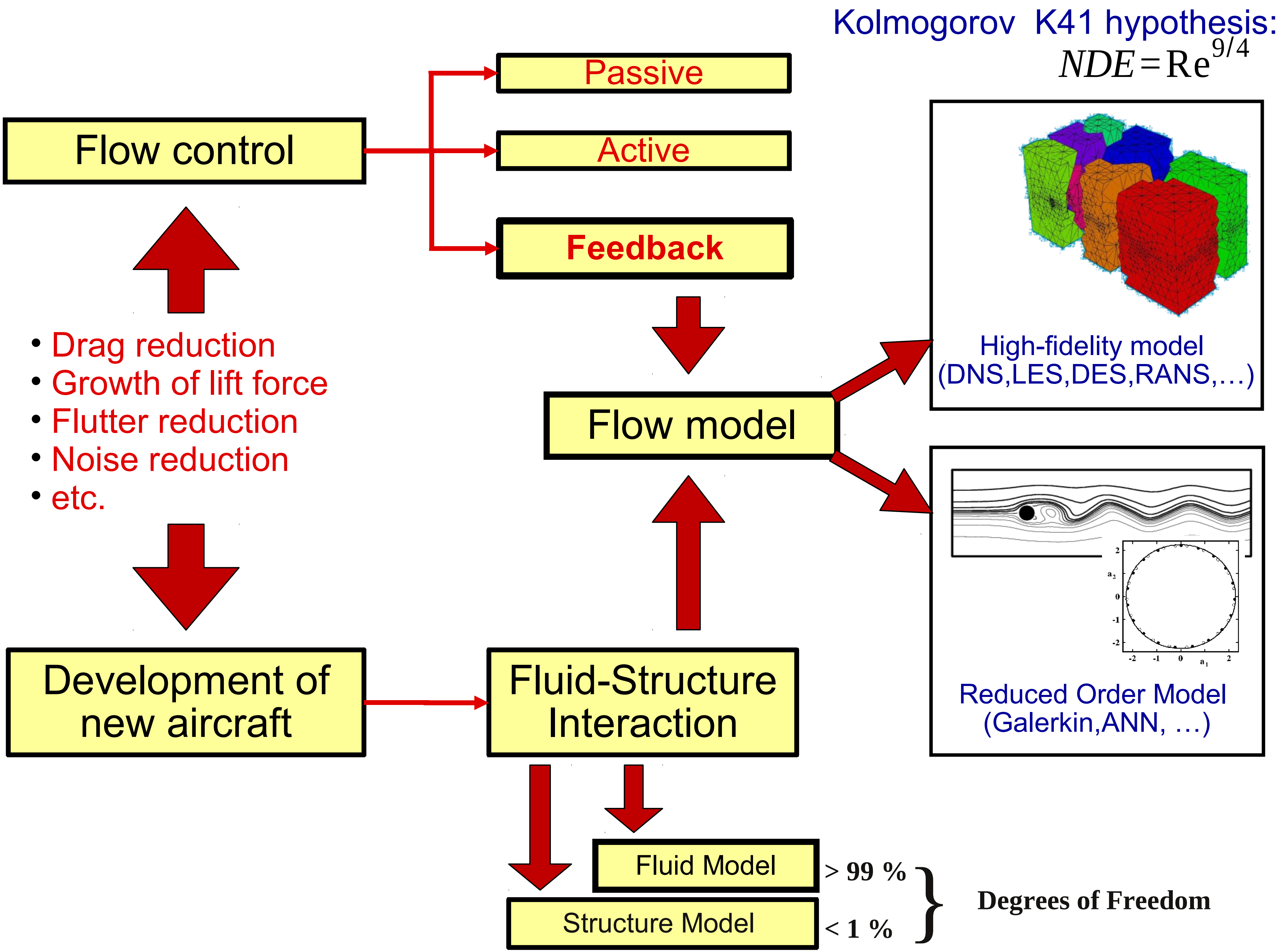
Gilead TADMOR (NEU Boston),
Frank Thiele (TU Berlin)
Institute of Aviation, Warsaw
and others



Kolmogorov K41 hypothesis:
$NDE = \mathrm{Re}^{9/4}$
Flow control
Passive
Active
Feedback
• Drag reduction
• Growth of lift force
• Flutter reduction
• Noise reduction
• etc.
Flow model
High-fidelity model
(DNS,LES,DES,RANS,…)
Reduced Order Model
(Galerkin,ANN, …)
Development of new aircraft
Fluid-Structure Interaction
Fluid Model
Structure Model
> 99 %
< 1 %
Degrees of Freedom

Flow Modelling

$$\dot{u} + \nabla(u \otimes u) + \nabla p - \frac{1}{\mathrm{Re}} \Delta u = R$$ (Discretised) Navier-Stokes Equations

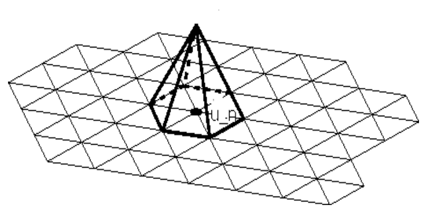
$$\left(w_i, R^{[N]}\right)_\Omega = 0$$ Weighted Residual

$$w_i = \begin{cases} 1 & \text{if inside } \Omega^i \\ 0 & \text{if outside } \Omega^i \end{cases} \qquad \left(w_i, R^{[P]}\right)_\Omega = 0$$ → Finite Volume Method

$$u^{[N]} = \sum_{i=0}^{N} a_i \cdot u_i \qquad w_i = u_i \qquad \left(u_i, R^{[P]}\right)_\Omega = 0$$ → Galerkin Method

Expansion modes:

Local



→ Finite Element Method

Global

→ Reduced Order Models

$$\dot{a}_i = \frac{1}{\mathrm{Re}} \sum_{j=0}^{N} l_{ij} a_j + \sum_{j=0}^{N} \sum_{k=0}^{N} q_{ijk} a_j a_k$$

Possible mode bases

Low-dimensional Galerkin models

MATHEMATICAL

- Carrier Field Ansatz
- Generalized streamfunctions
- Wavelets

Continuity equation

Dimension: AVERAGE

PHYSICAL

- Stokes modes,
- Singular Stokes modes,
- **Global stability analysis modes**

Continuity equation
Navier-Stokes equations

Dimension: AVERAGE-LOW

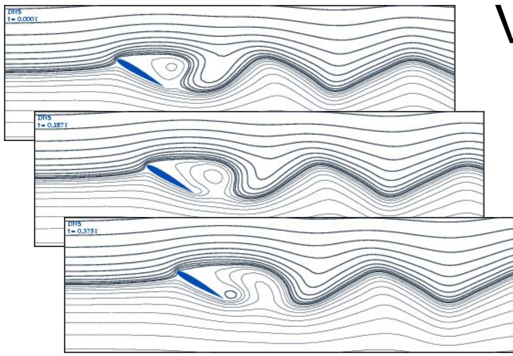
EMPIRICAL

- **Proper Orthogonal Decomposition** (Karhunen–Loeve, Principal Component Analysis)
- Dynamic Mode Decomposition

Continuity equation
Navier-Stokes equations
Unsteady flow data

Dimension: LOW

Proper Orthogonal Decomposition



Velocity fields from M time steps, each N degrees of freedom

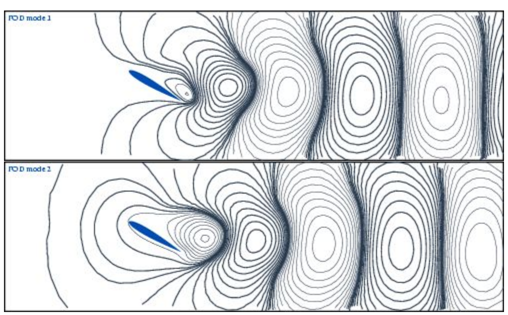
$$u_0 = \frac{1}{M}\sum_{i=1}^{M} v_i$$

$$v_i' = v_i - u_0$$

$$S = [v_1', v_2', \ldots, v_M']$$

Correlation matrix

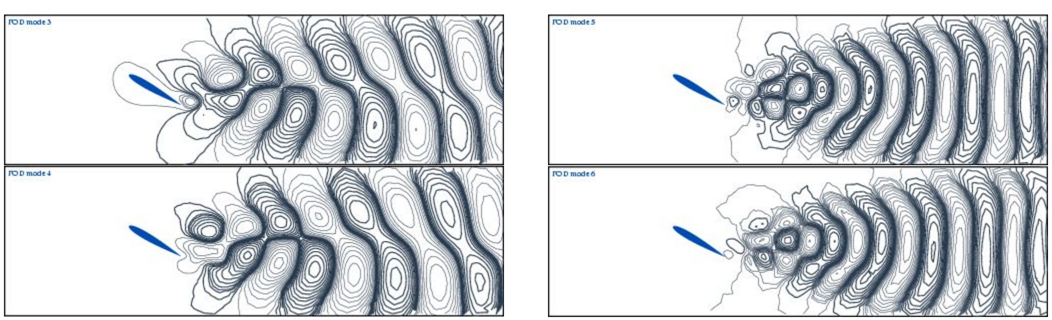
$$C = \frac{1}{M} S S^T$$



Eigenvectors u_j

Eigenvalues

Coefficients $\alpha_{ij} = u_j^T v_i$



Minimalisation of mean-square error of process representation

Global Stability Analysis

Navier-Stokes Equation

$$\dot{V}_i + V_{i,j}V_j + P_{,i} - \frac{1}{\mathrm{Re}}V_{i,jj} = 0$$

Snapshot = base flow + small disturbance

$$V_i = \overline{V}_i + V_i' \qquad P = \overline{P} + P'$$

$$V_i'(x,y,t) = \widetilde{V}_i(x,y)e^{-\lambda t} \qquad P'(x,y,t) = \widetilde{P}(x,y)e^{-\lambda t}$$

Disturbance equation

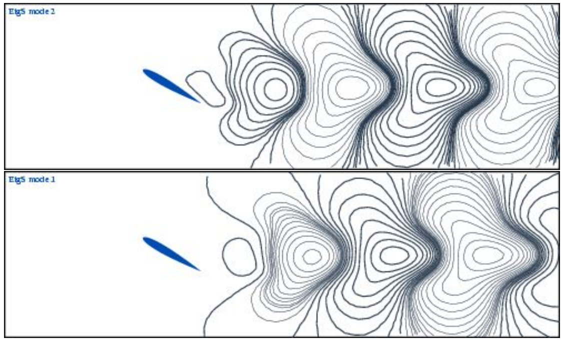
$$-\lambda\widetilde{V}_i + \widetilde{V}_{i,j}\overline{V}_j + \overline{V}_{i,j}\widetilde{V}_j + \widetilde{P}_{,i} - \frac{1}{\mathrm{Re}}\widetilde{V}_{i,jj} = 0$$

Discretisation (e.g. FEM)

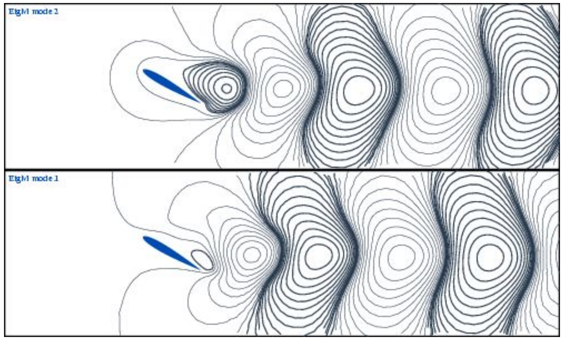
Generalised eigenvalue problem

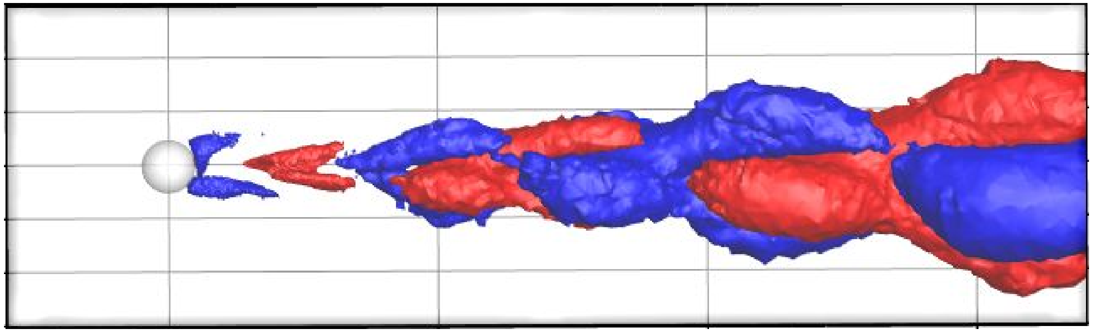
$$Ax - \lambda Bx = 0$$

Base flow = STEADY SOLUTION

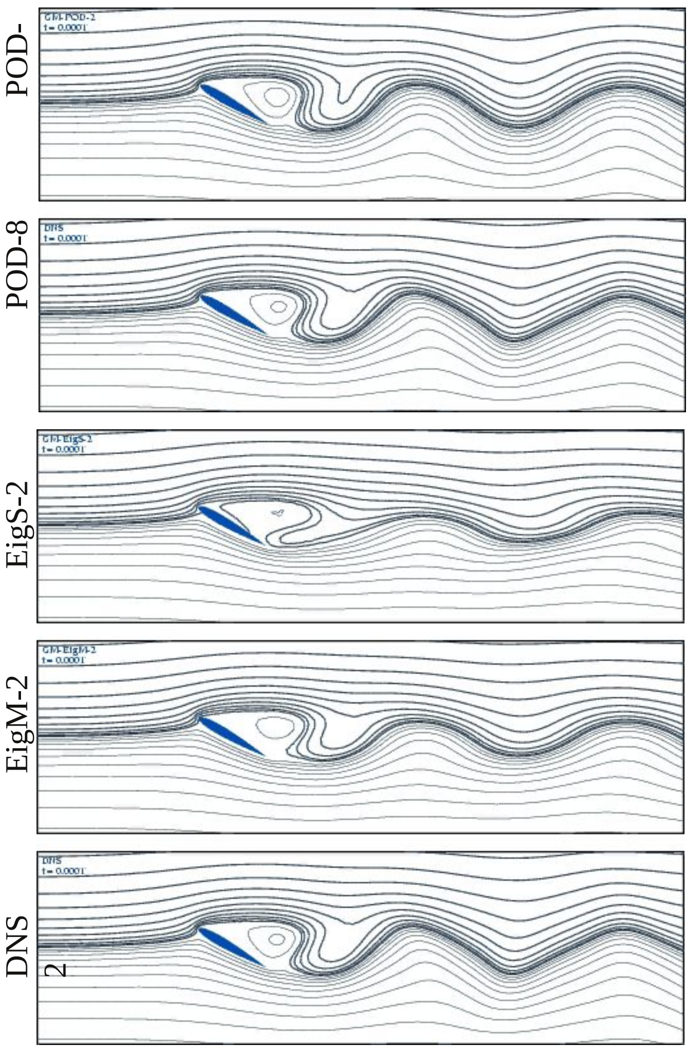


Base flow = MEAN FLOW

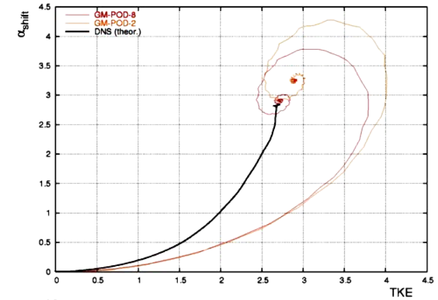


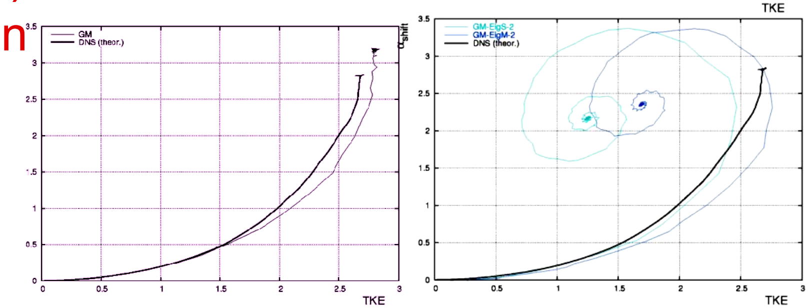


ROM limitations: transitional flow

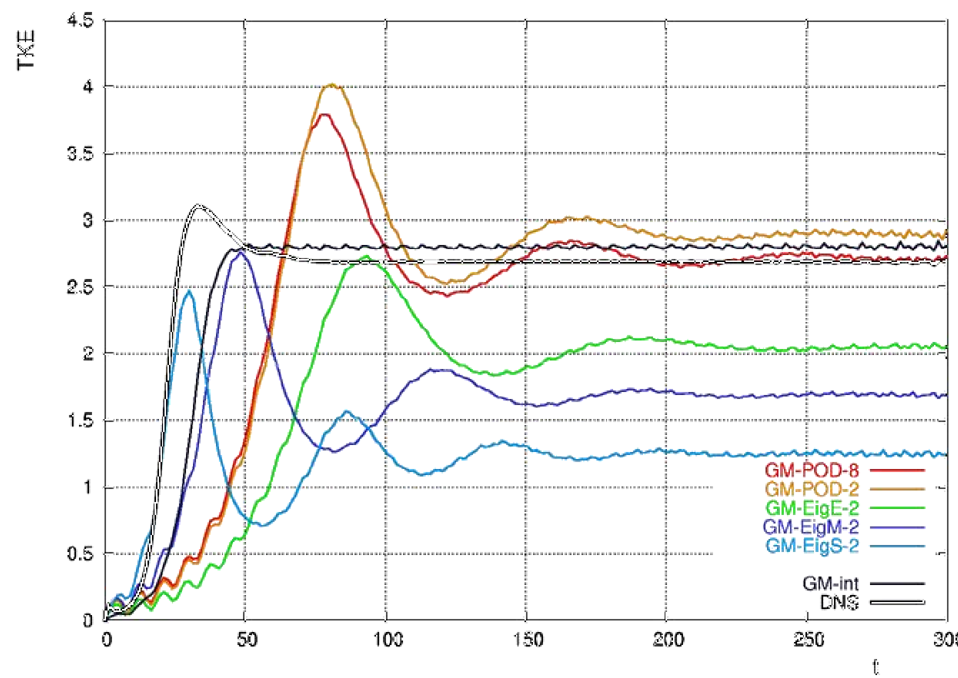


Empirical (POD) GM:
- structurally unstable for 2POD modes
- Shift-mode required
- Poor transients reconstruction (**narrow dynamic range**)
- Good LCO reconstruction





Physical GM:
- Poor reconstrucion of LCO
- Accurate for fixed-point dynamics

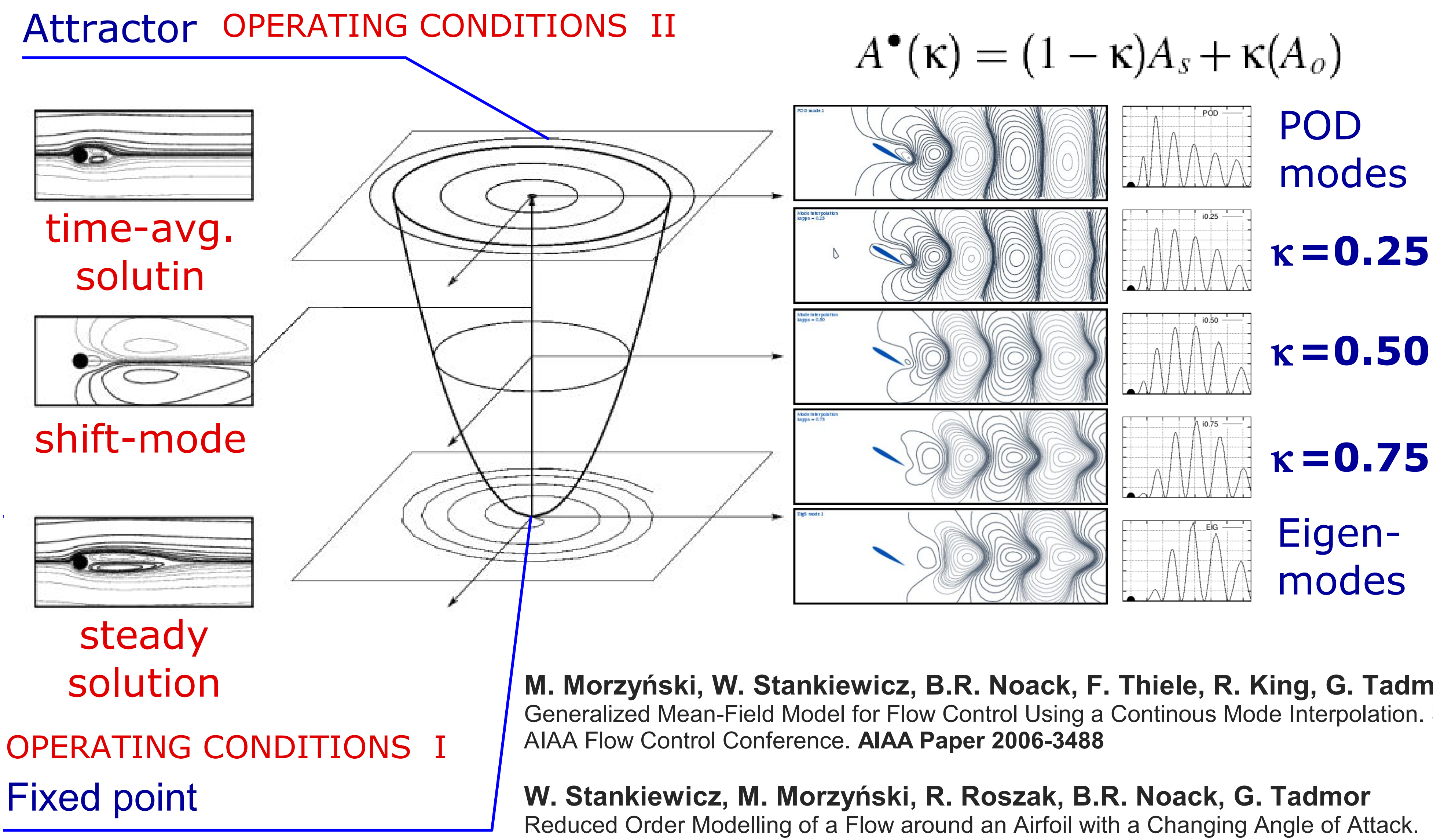


Solutions:
- Hybrid Galerkin Modes
- Parametrisation of Mode Basis

Broadband AE-ROM

Continuous Mode Interpolation – Parametrized Mode Basis

$$A^{\bullet}(\kappa) = (1-\kappa)A_s + \kappa(A_o)$$



M. Morzyński, W. Stankiewicz, B.R. Noack, F. Thiele, R. King, G. Tadmor
Generalized Mean-Field Model for Flow Control Using a Continous Mode Interpolation. 3rd AIAA Flow Control Conference. **AIAA Paper 2006-3488**

W. Stankiewicz, M. Morzyński, R. Roszak, B.R. Noack, G. Tadmor
Reduced Order Modelling of a Flow around an Airfoil with a Changing Angle of Attack. KKMP 2008. **Archives of Mechanics, vol.60, 2008**

Need of ROM in design and control

10 Mio-element mesh: computational cost of 1 step of AE analysis on 16-core PC cluster: t = 80s + 2x 10s + 30s + 4/50s

Aircraft development and certification requires flutter analysis

AIAA 2008, Rossow, Kroll
Aero Data Production **A380 wing**

50 flight points
100 mass cases
10 a/c configurations
5 maneuvers
20 gusts (gradient lengths)
4 control laws

⇓

~20,000,000 CFD simulations

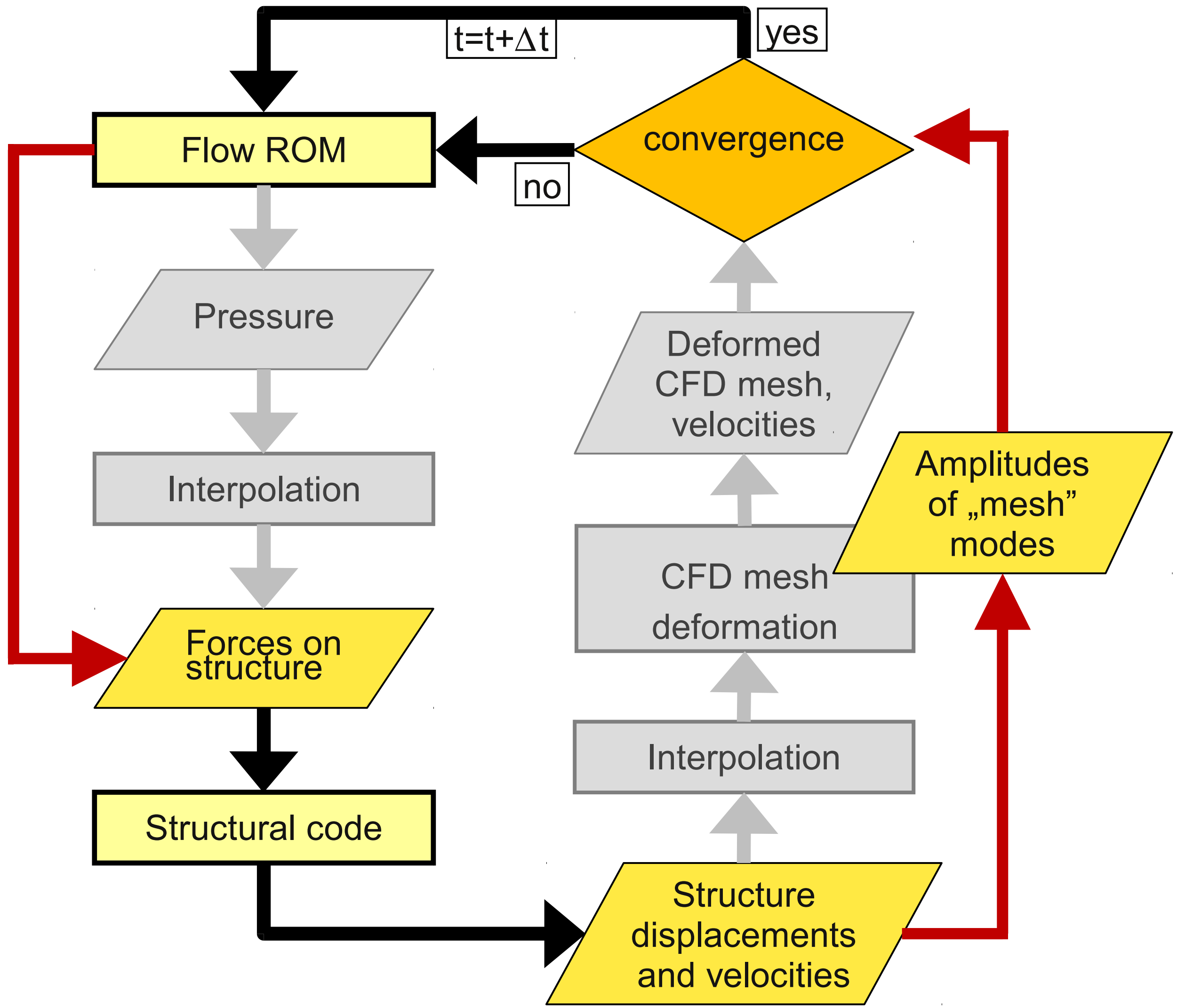
⇓

Engineering experience for current configurations and technologies

⇓

~100,000 simulations

Fluid – Structure Interaction algorithm



Motion of the boundary and mesh

$$\dot{u} + \nabla(u \otimes u) + \nabla p - \frac{1}{\mathrm{Re}}\Delta u = 0 \quad + \quad M\ddot{x} + C\dot{x} + Kx = F(\ddot{x}, \dot{x}, x, t)$$

Eulerian approach Lagrangian approach

Arbitrary Lagrangian-Eulerian Approach (ALE) binds with each other the velocity of the flow **u** and the velocity of the (deforming) mesh $\mathbf{u}_{grid}$. For incompressible Navier-Stokes equations the mesh velocity modifies the convective term:

$$\dot{\mathbf{u}} + \nabla \cdot \left((\mathbf{u} - \mathbf{u}_{grid}) \otimes \mathbf{u} \right) + \nabla \mathbf{p} - \frac{1}{Re} \triangle \mathbf{u} = 0$$

With boundary conditions: $\mathbf{u} = \mathbf{u}_{grid}$

The fluid mesh can move independently of the fluid particles.

Donea J., Huerta A., Ponthot J.-Ph. and Rodriguez-Ferran A. , Arbitrary Lagrangian-Eulerian Methods, Encyclopedia of Computational Mechanics, Edited by Erwin Stein, Rene de Borst and Thomas J.R. Hughes. Volume 1: Fundamentals. John Wiley and Sons, Ltd., 2004.

Projection of convective term

$$\dot{u} + \nabla\left((u - u_{grid}) \otimes u\right) + \nabla p - \frac{1}{\mathrm{Re}}\Delta u = 0$$

Arbitrary Lagrangian-Eulerian Approach

1. DECOMPOSITION

$$u_{grid} = \sum_{i=1}^{N_G} a_i^G \cdot u_i^G$$

2. GALERKIN PROJECTION

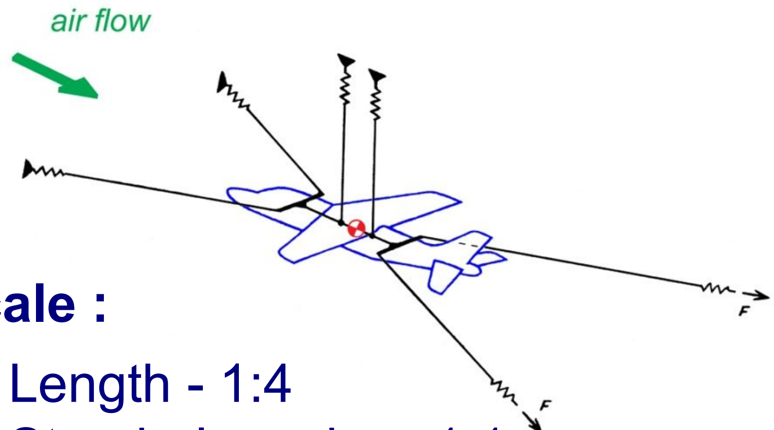
$$-\left(u_i, \nabla \cdot \left((u - u_{grid}) \otimes u\right)\right)_\Omega = -\left(u_i, \nabla \cdot \left(u \otimes u\right)\right)_\Omega + \left(u_i, \nabla \cdot \left(u_{grid} \otimes u\right)\right)_\Omega =$$

$$= \sum_{j=0}^{N}\sum_{k=0}^{N} q_{ijk} a_j a_k - \sum_{j=1}^{N_G}\sum_{k=0}^{N} q_{ijk}^G a_j^G a_k$$

$$q_{ijk}^G = -\left(u_i, \nabla \cdot \left(u_j^G \otimes u_k\right)\right)_\Omega$$

Flutter Laboratory
IoA and PUT
experiment and computations

Flutter Lab



Scale :

- Length - 1:4
- Strouhal number 1:1

Aircraft structure model

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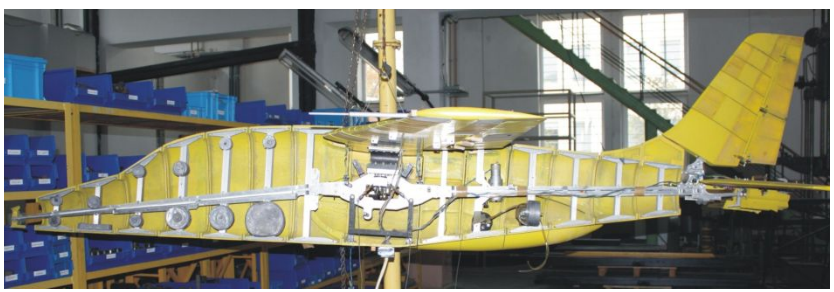

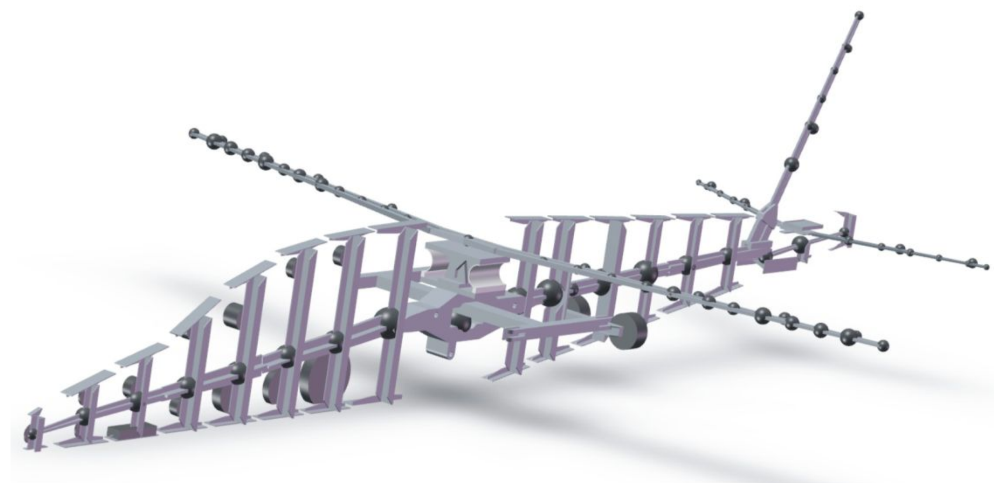

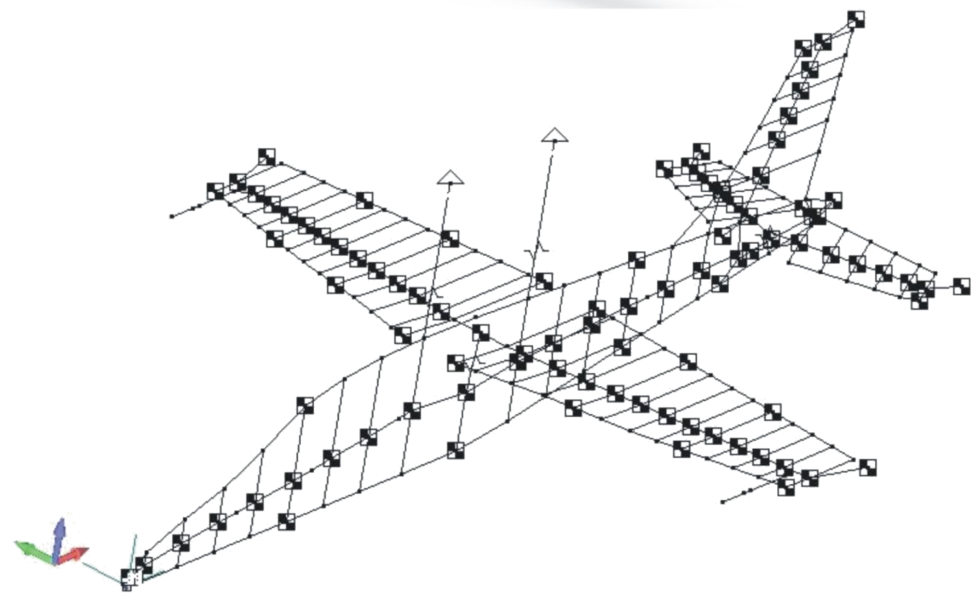

3D model of an aircraft structure (SolidWorks CAD geometry and Finite Element Model)

CFD model of an aircraft

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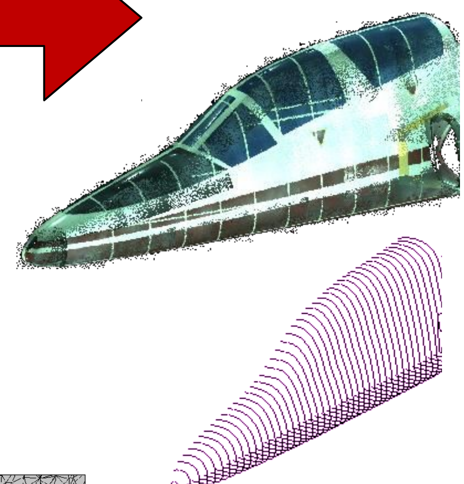
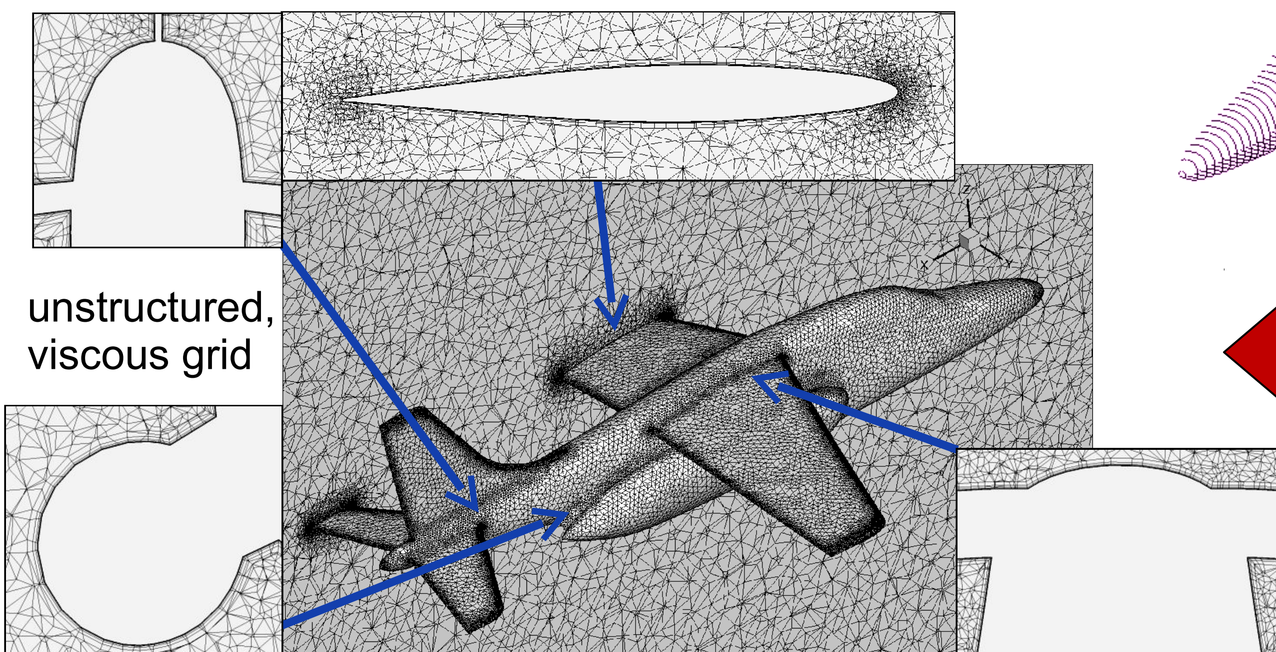
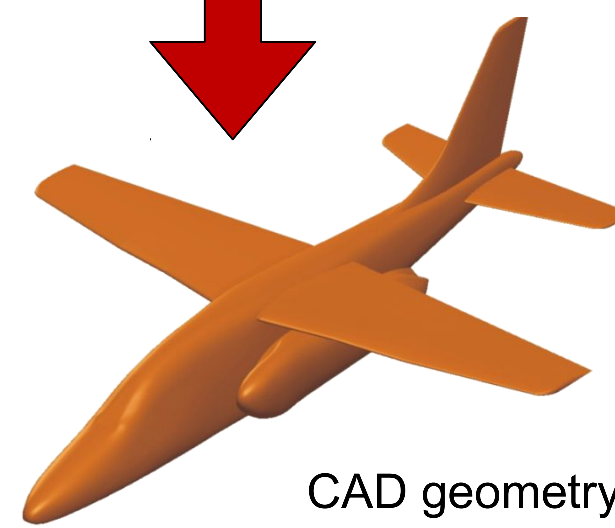
3D scanning



3D model for CFD analyses (Surface geometry and Unstructured Finite Element Mesh)





CAD geometry

Full configuration aircraft

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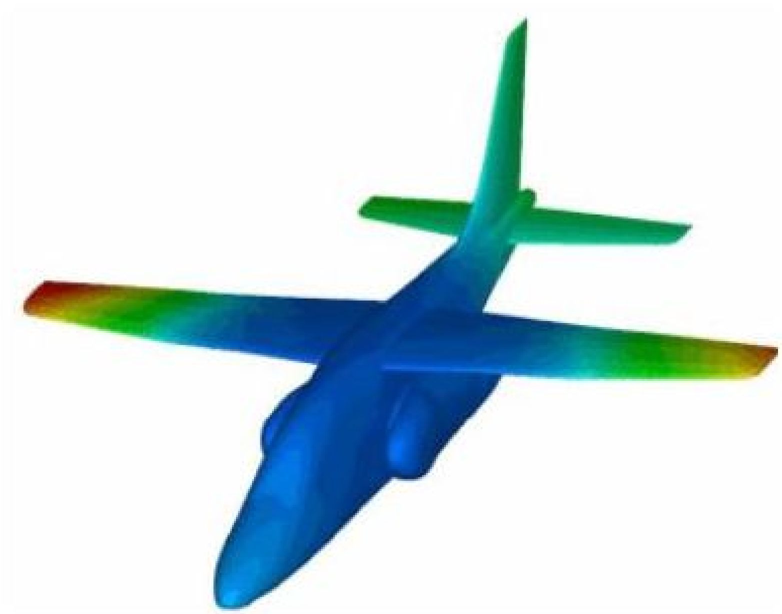
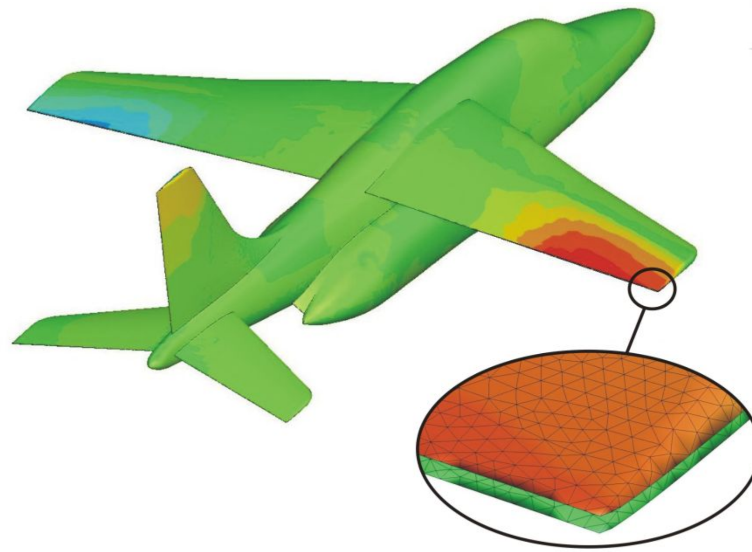
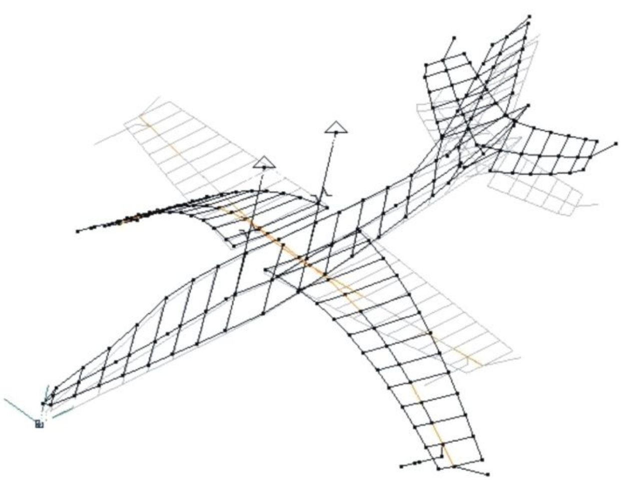
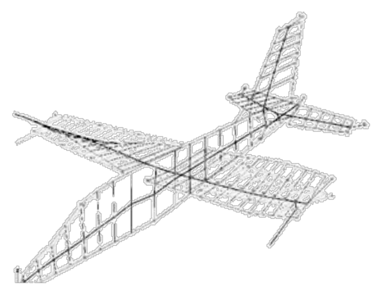
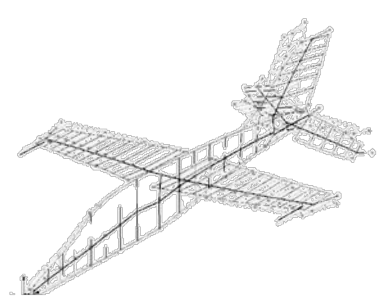


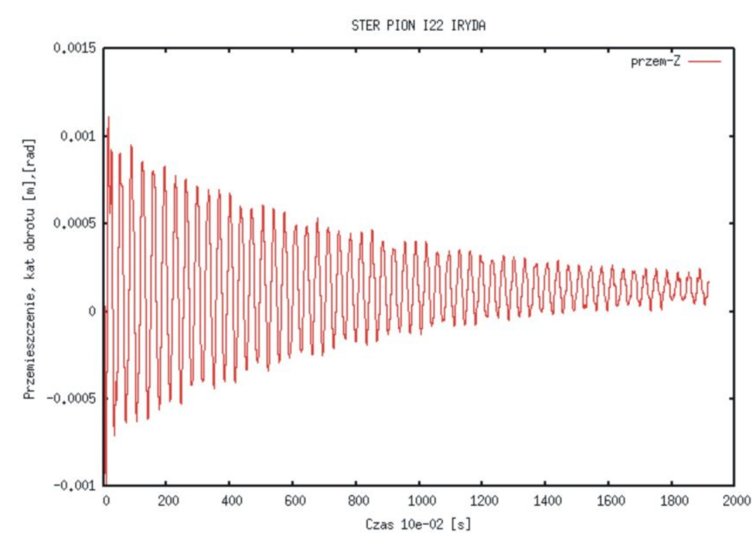


Numerical response in Aeroelastic analysis









The first of the aircraft structure's eigenmodes

Displacements of control node on the leading edge of the wing

Full configuration aircraft

Wind tunnel tests (Institute of Aviation, Warsaw) and computational flutter simulation (Poznan University of Technology)

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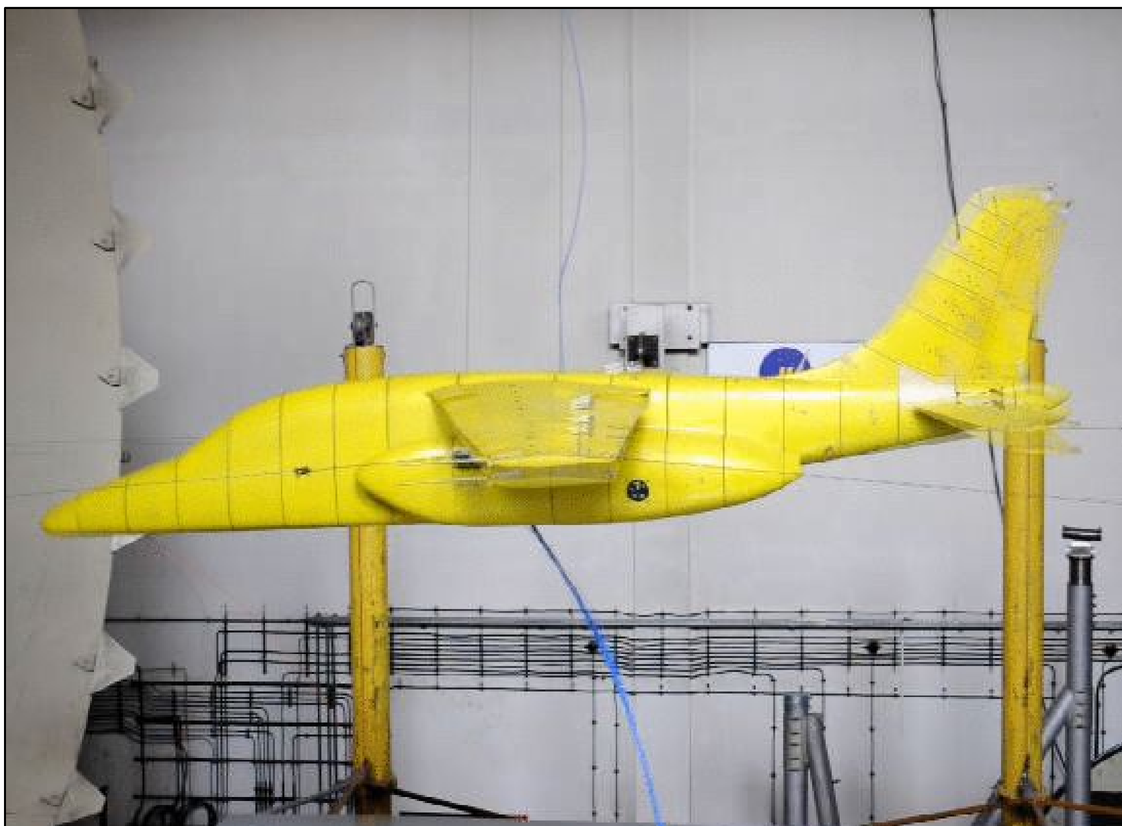

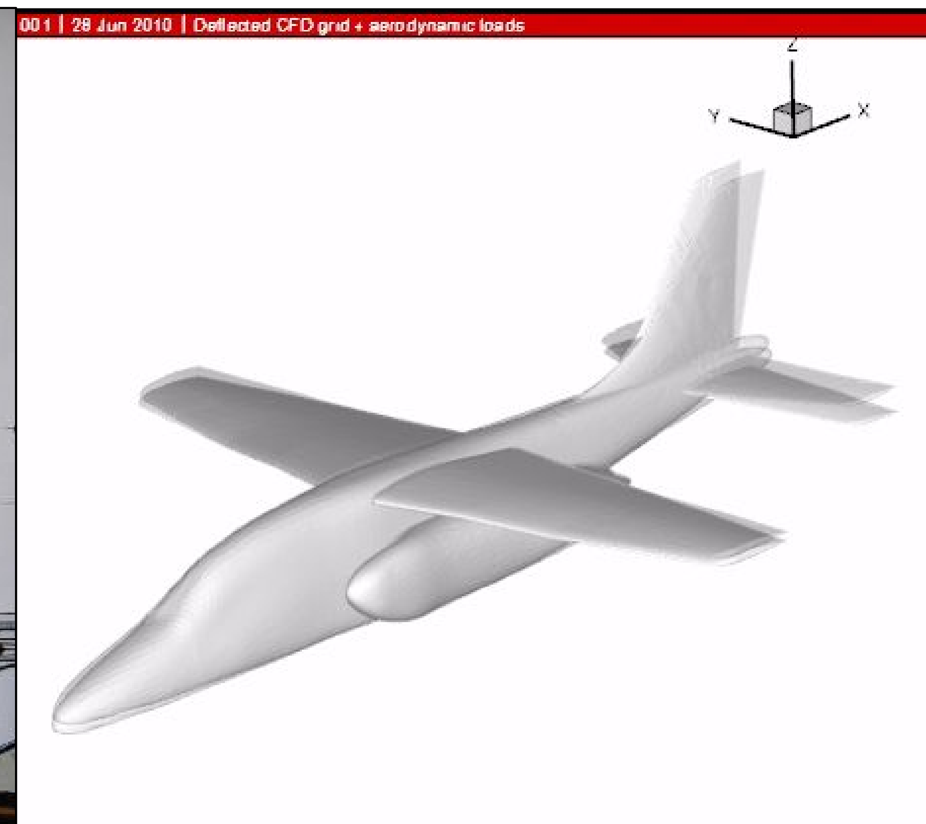

Summary

Activities of Virtual Engineering Group at Poznan University of Technology

- Low dimensional analysis
  - Global flow stability analysis
  - Reduced Order Modelling of fluid flows
  - Feedback flow control
- (Very) high dimensional analysis
  - CFD and aeroelastic analysis of industrial-relevant cases
  - Development of aeroelastic tools
- Investigations cover advanced theoretical methods as well as practical applications
- Investigations are oriented to use at highly advanced methods in practical problems (Reduced Order Aeroelastic Models of full configurations of aircraft)
- Activities not covered in the talk
  - Topological optimisation of aerostructures
  - Modal analysis in signal processing and biomechanics
  - Activities in EU and other Projects