12th ERCOFTAC NPC MEETING
May 29-30, Stockholm, Sweden

Stability analysis for a wind turbine blade

Technical University of Denmark

Gabriel G.M. Hernandez
Jens. N. Sørensen
W. Z. Shen

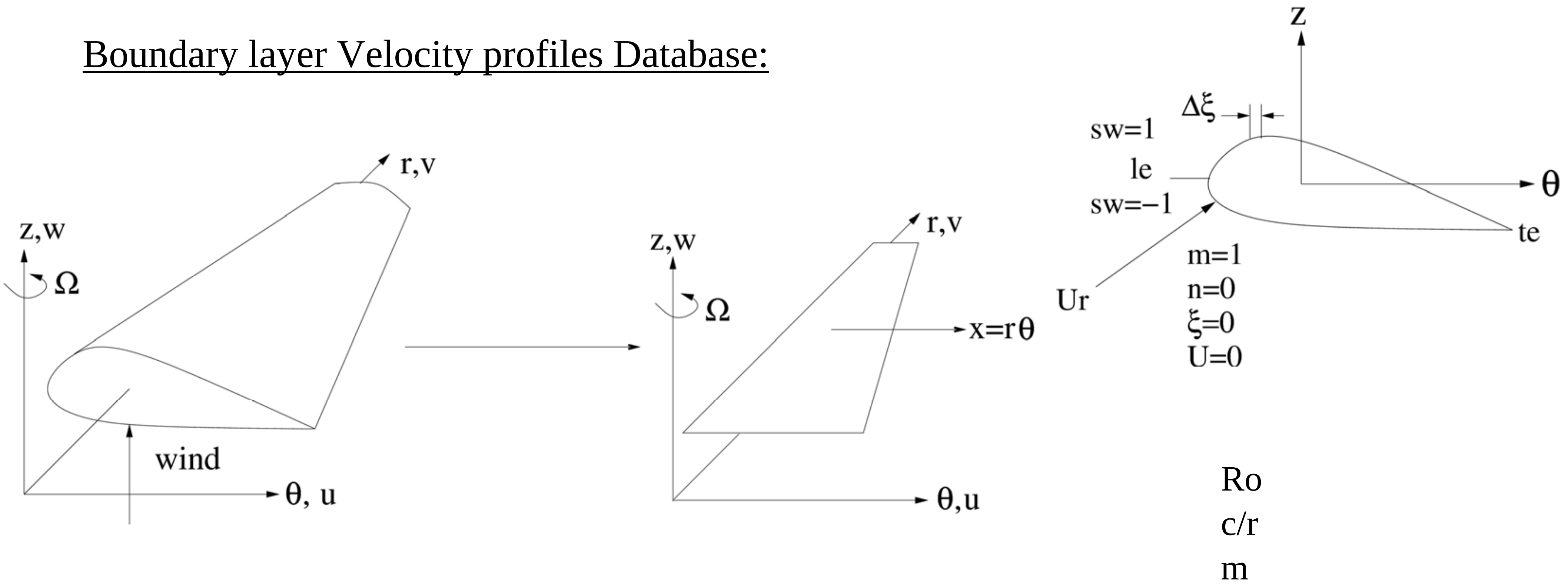
Problem Definition:

- Transition prediction for improving design of wind turbine airfoils.
- Demand a method not expensive in terms of computing time.
- Database approach easy to integrate in any numerical code.

Velocity profiles with rotational effects ---> Stability analysis

Implementation on EllipSys3D code <--- Amplification Factor

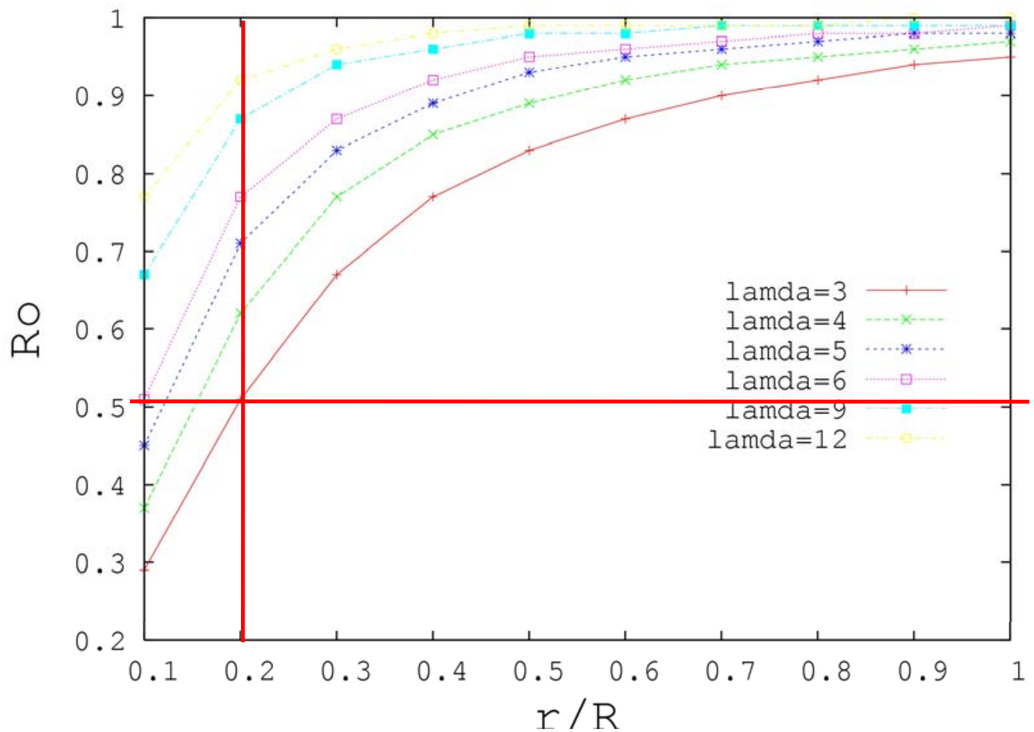
Boundary layer Velocity profiles Database:

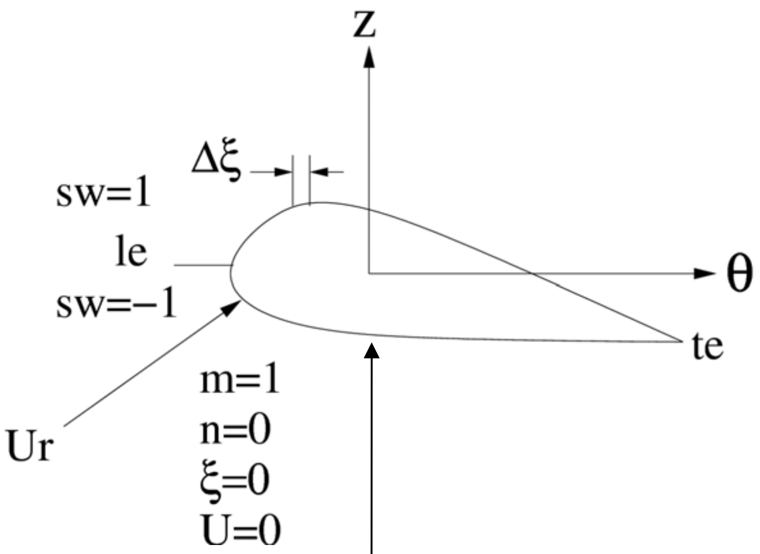


- The velocity profiles have been mapped and stored.

- Unique relation exist if the dimensionless pressure gradients m and n, with the Rossby number are specified. (True for a rotating flat plate). In a similar way another parameters can be selected: dimensionless wall shear stress in both directions and the Rossby number of the shape factors.

- Relations among the parameters have been obtained.





Parameter Range:

$$\lambda_g = [3-12]$$

$$\lambda = \left[\frac{\lambda_g}{ns} - \lambda_g\right]$$

$$c/r = [0.50-0]$$

$$Ro = [0.50-0.95]$$

Pressure

Gradient:

$$\alpha = [0-20°]$$

$$m = \frac{\xi}{U}\frac{dU}{d\xi}\frac{1}{S_t}$$

$$n = \frac{c}{r} Ro^2 \xi$$

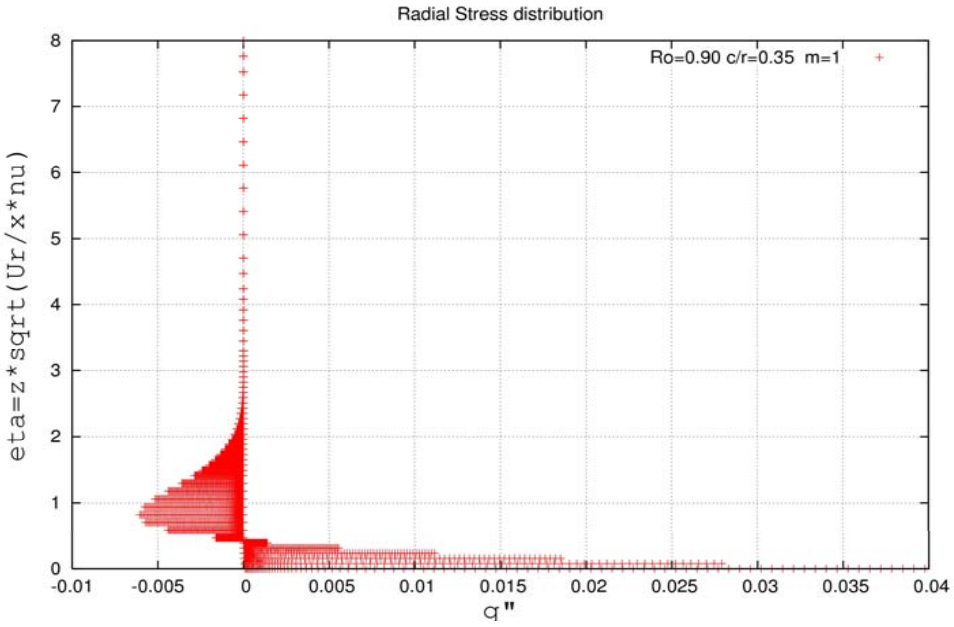
$$Ro = \frac{\lambda}{\sqrt{\lambda^2 + (1-a)^2}}$$

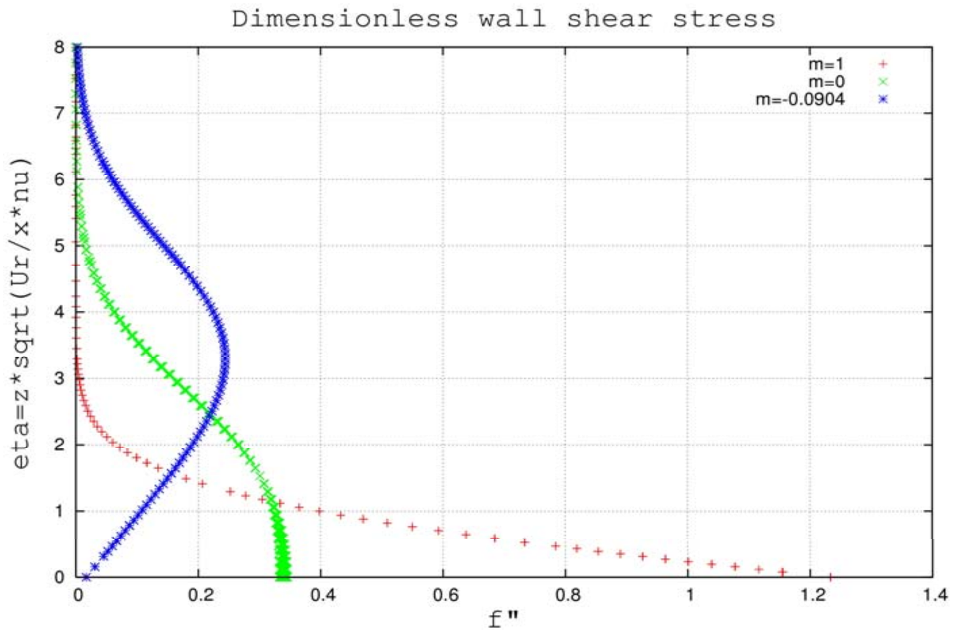
$$r/R = \frac{\lambda}{\lambda_g}$$

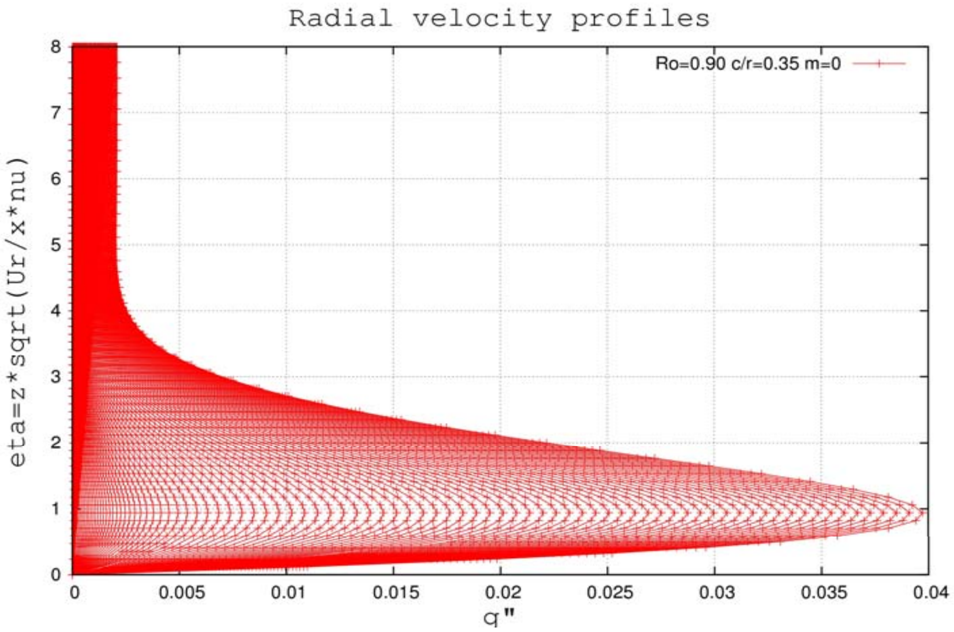
$$f''(0) \equiv \frac{1}{2} C_{f,u} \sqrt{\mathrm{Re}_x}$$

$$g''(0) \equiv \frac{1}{2} C_{f,v} \sqrt{\mathrm{Re}_x}$$

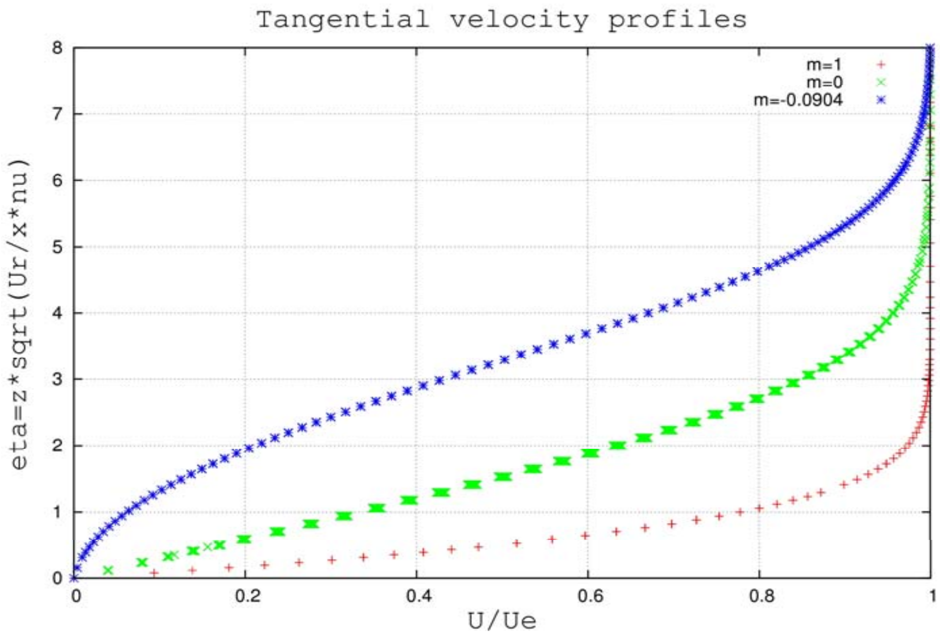
Input for the stability Analysis:







V



U

Orr-Sommerfeld equation:

-- Based on parallel flow assumption.
-- Linear stability is used in the derivation the equation.

- Disturbance shape equation:

$$r' = r(y)e^{[i(\alpha x+\beta z-\omega t)]} \qquad \alpha = \frac{2\pi}{\lambda x} \qquad \beta = \frac{2\pi}{\lambda z} \qquad \omega = \frac{c}{\alpha}$$

- Spatial vs temporal approach.

$$\varphi'''' - 2\alpha^2\varphi'' + \alpha^4\varphi - iR\left[(\alpha U - \omega)(\varphi'' - \alpha^2)\varphi - \alpha U''\varphi\right] = 0$$

$$\varphi'''' - 2(\alpha^2+\beta^2)\varphi'' + (\alpha^2+\beta^2)^2\varphi - iR\left[(\alpha U + \beta V - \omega)(\varphi'' - (\alpha^2+\beta^2)\varphi) + (\alpha U'' + \beta V'')\varphi\right] = 0$$

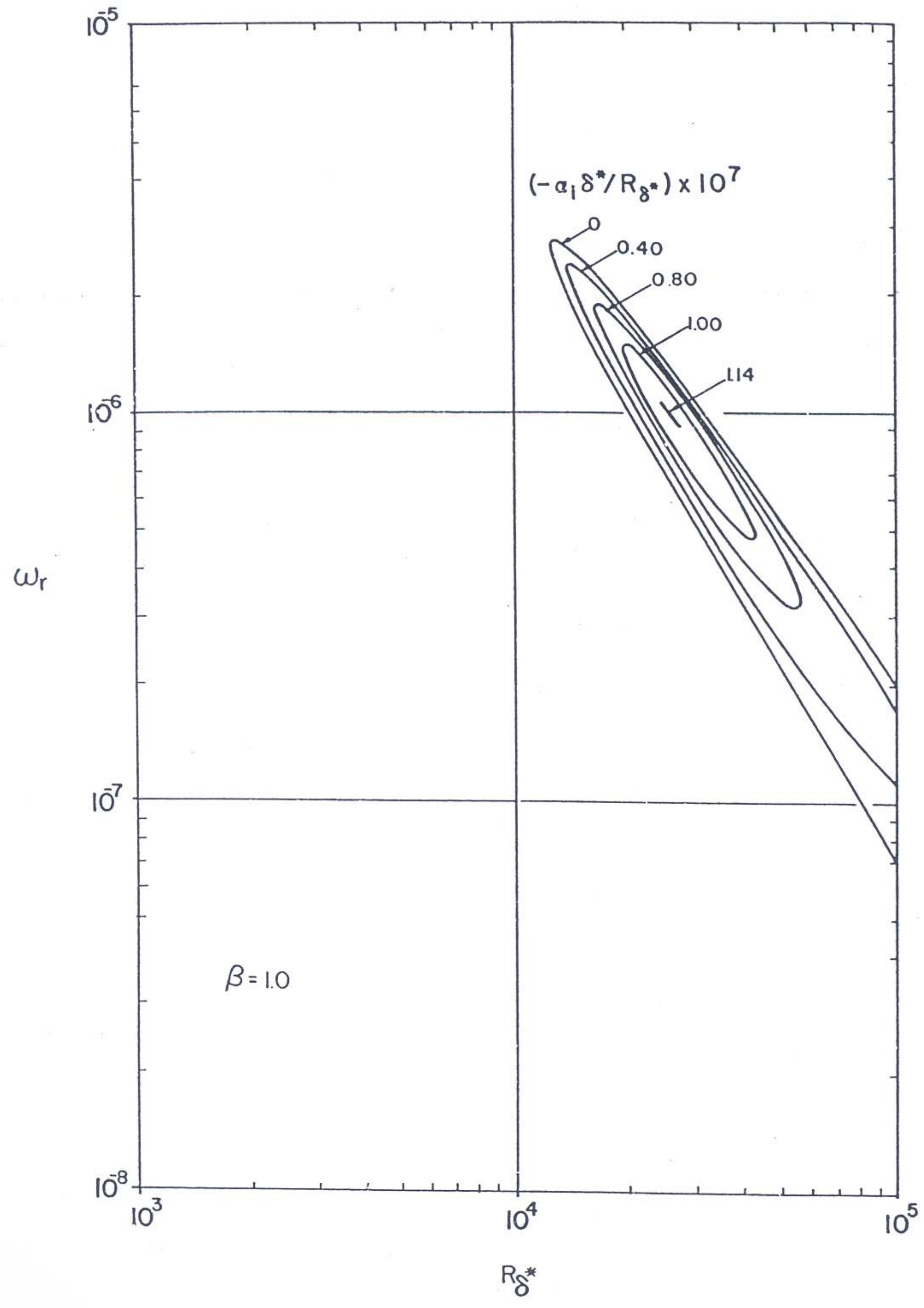
Boundary Conditions:

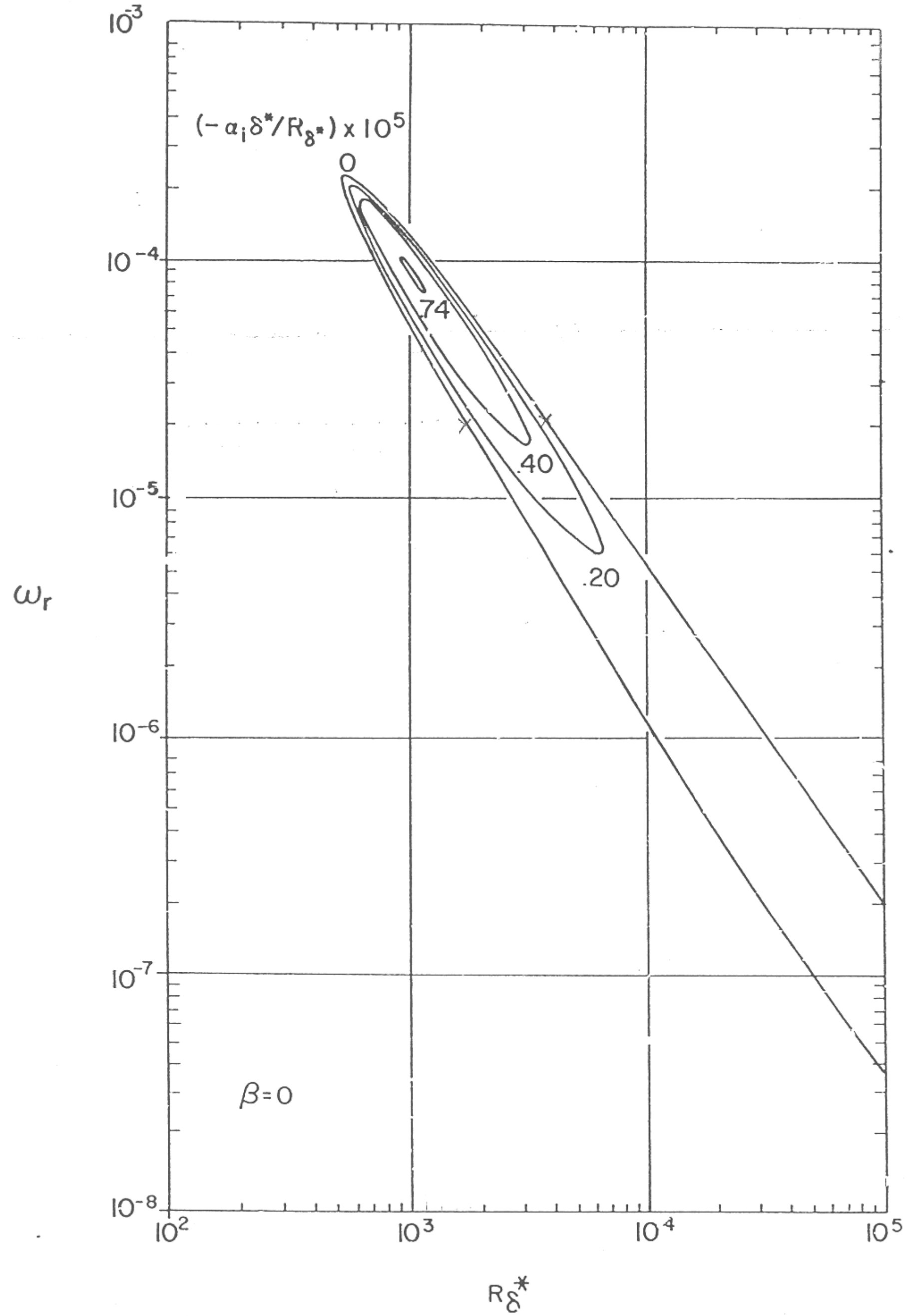
$$y = 0, \varphi = 0, \varphi' = 0$$

Eigenvalue Problem

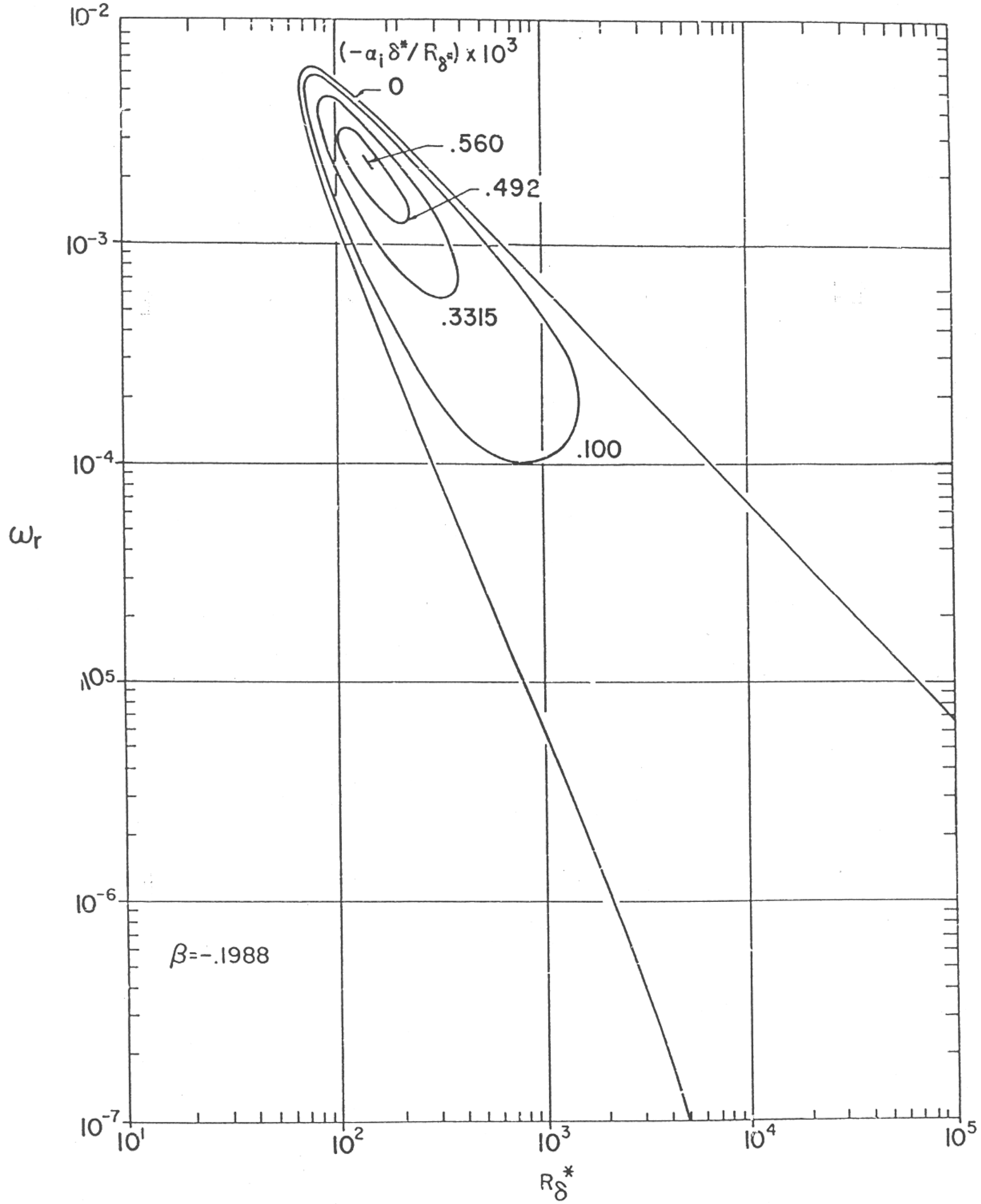
$$F(\alpha,\omega,R) = 0 \qquad F(\alpha,\beta,\omega,R) = 0$$

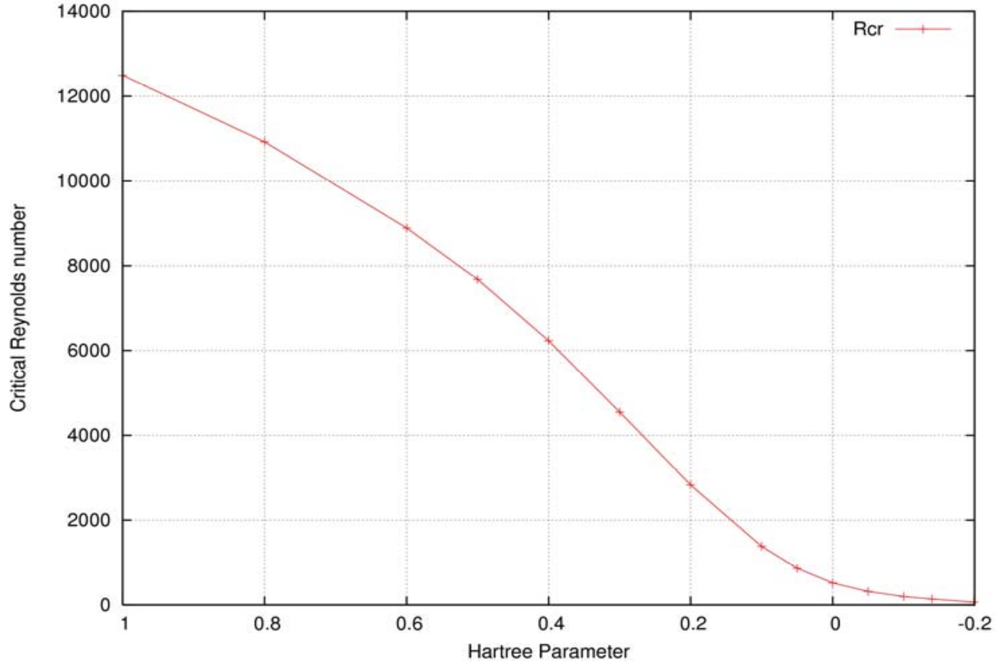
Tangential flow Database :

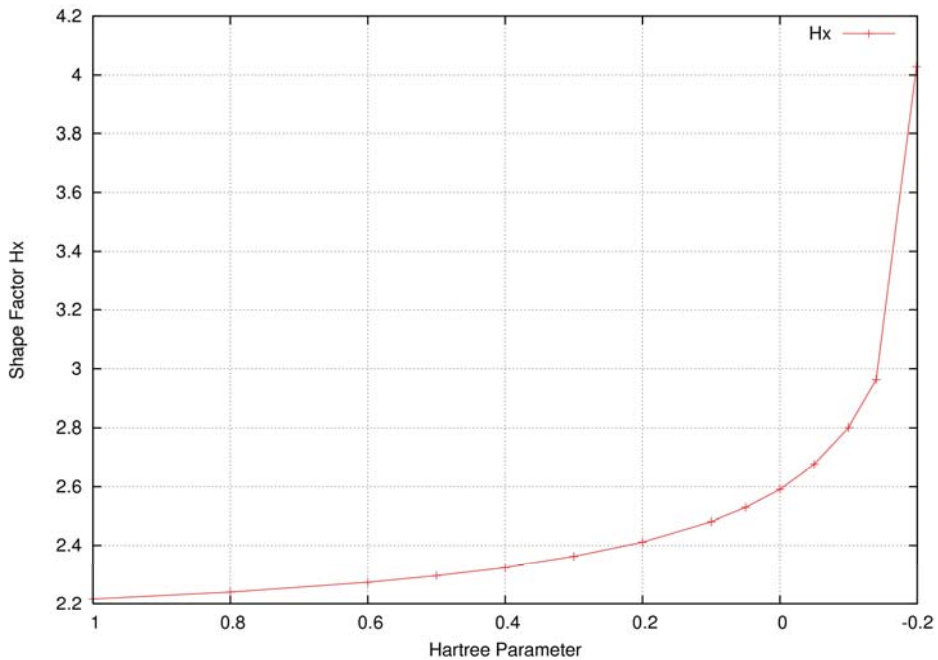




Spatial and temporal stability curves A.R. Wazzan, T.T. Okamura and A.M.O. Smith Rep No. DAC-67086


(−αᵢ δ*/R_δ*) x 10³
0
.560
.492
.3315
.100
β=−.1988
ω_r
R_δ*
10⁻²
10⁻³
10⁻⁴
10⁵
10⁻⁶
10⁻⁷
10¹
10²
10³
10⁴
10⁵


Rcr
Critical Reynolds number
Hartree Parameter
14000
12000
10000
8000
6000
4000
2000
0
1
0.8
0.6
0.4
0.2
0
-0.2


Hx
Shape Factor Hx
Hartree Parameter
4.2
4
3.8
3.6
3.4
3.2
3
2.8
2.6
2.4
2.2
1
0.8
0.6
0.4
0.2
0
-0.2

What about the cross flow, how to handle the problem ?

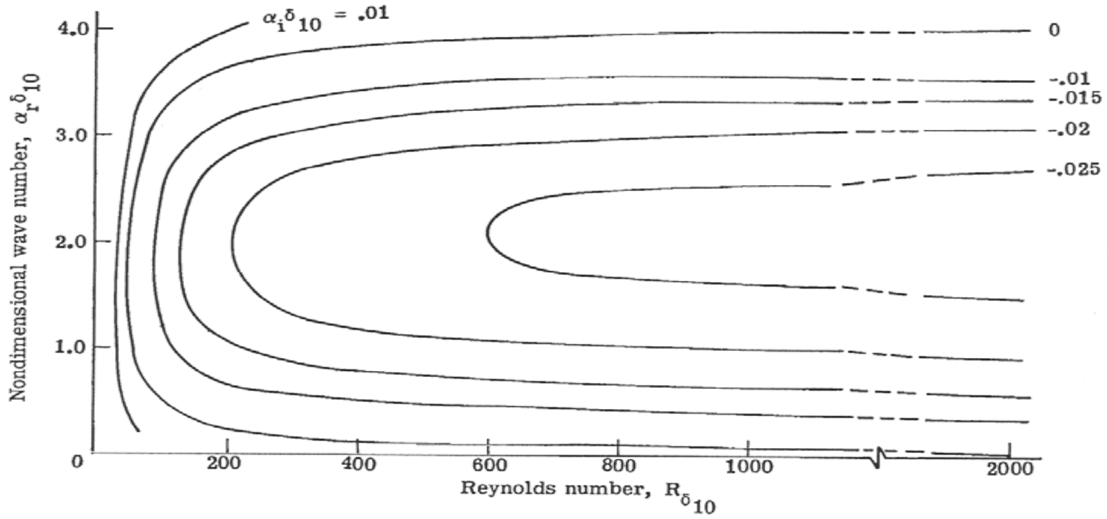
Proposed approach to solve for the Cross flow Database:

- One very important conclusion from the analysis of the velocity profiles is that the tangential flow is not affected by the cross flow velocity profile. Therefore is possible to consider the stability of the two velocity profiles independly.

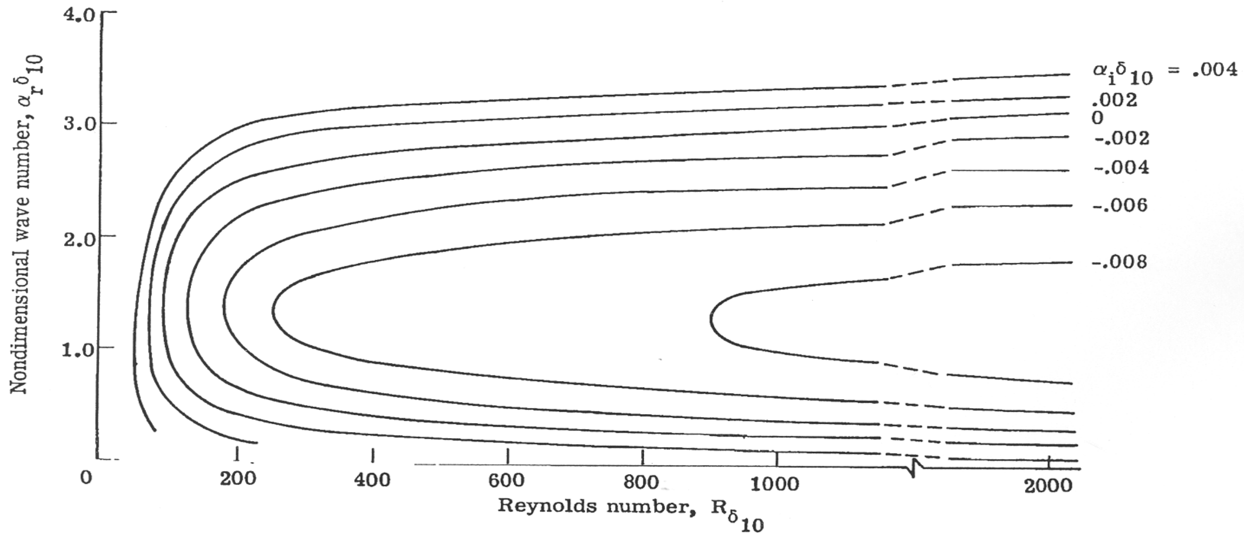
- Is important to notice that the method used to solve the eigenvalue problem is very close related to method used to define the transition criteria.

- From Annu. Rev. Fluid. Mech. 1996. 28, H Reed, W.S. Saric and D. Arnal, for the case of subsonics flows, recommend to separate or uncouple the problem of the cross flow instability from the TS instability, since the mechanism are different.

<u>Stability Diagram for the cross flow velocity profile:</u>



(c) H_c = 0.4299 and $W_M/U_{e,t}$ = −0.04624.



(d) H_c = 0.2444 and $W_M/U_{e,t}$ = 0.01678.

Figure 11.– Continued.

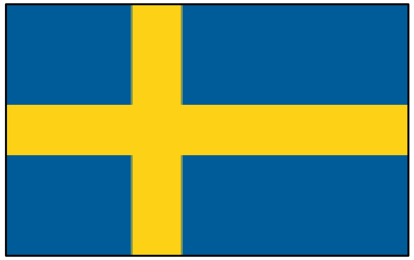
J Ray Dagenhart NASA TP-1902

The approach requires the following:

- Reformulation of the Orr-Sommerfeld equation and the parameters in the stability diagrams, (Reynolds number) and disturbance shape and boundary conditions.

- Two stability modes are considered: Traveling Cross flow waves and stationary cross flow instability.

- Alternative formulation of the problem for instance the envelope method, to verify the solution. A kind of Orr-Sommerfeld equation including rotational effects have been derived. (Still need to check the boundary conditions for solution)



Thank you :)