Lubrication Film Friction Model for non-Symetrically Profiled Annular Seals

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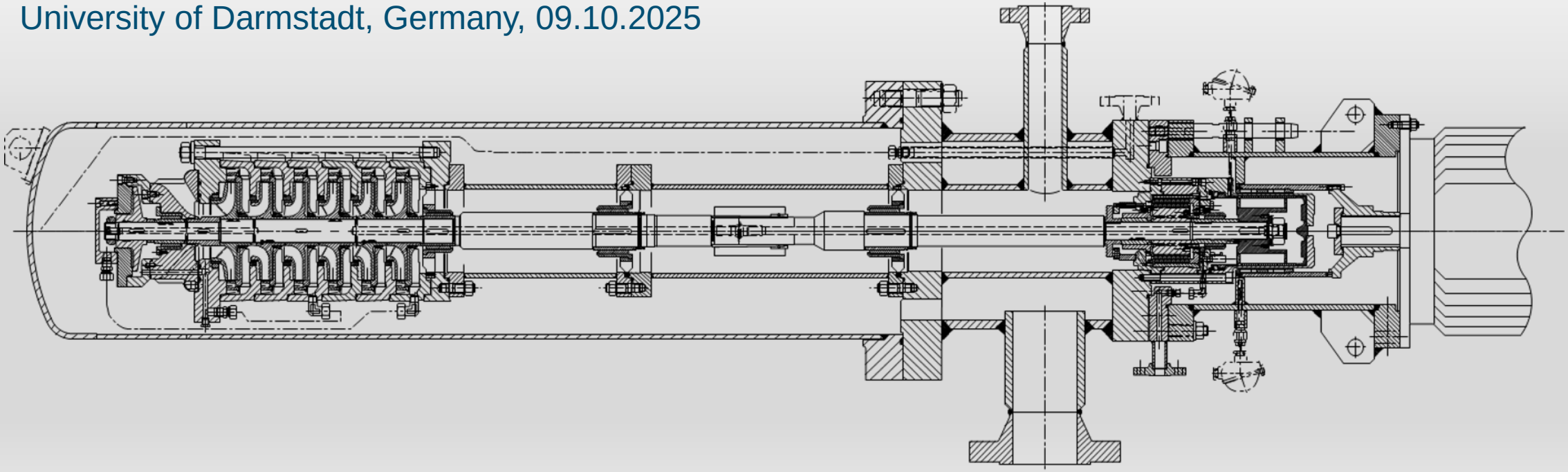
N. Billur Mohan, Dr.-Ing. R. M. Robrecht, Dr.-Ing. M. M.G. Kuhr, Prof. Dr. Ing. P. F. Pelz

ERCOFTAC Autumn Festival 2025 and 20th Da Vinci Competition,
University of Darmstadt, Germany, 09.10.2025



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Annular seals in pumps

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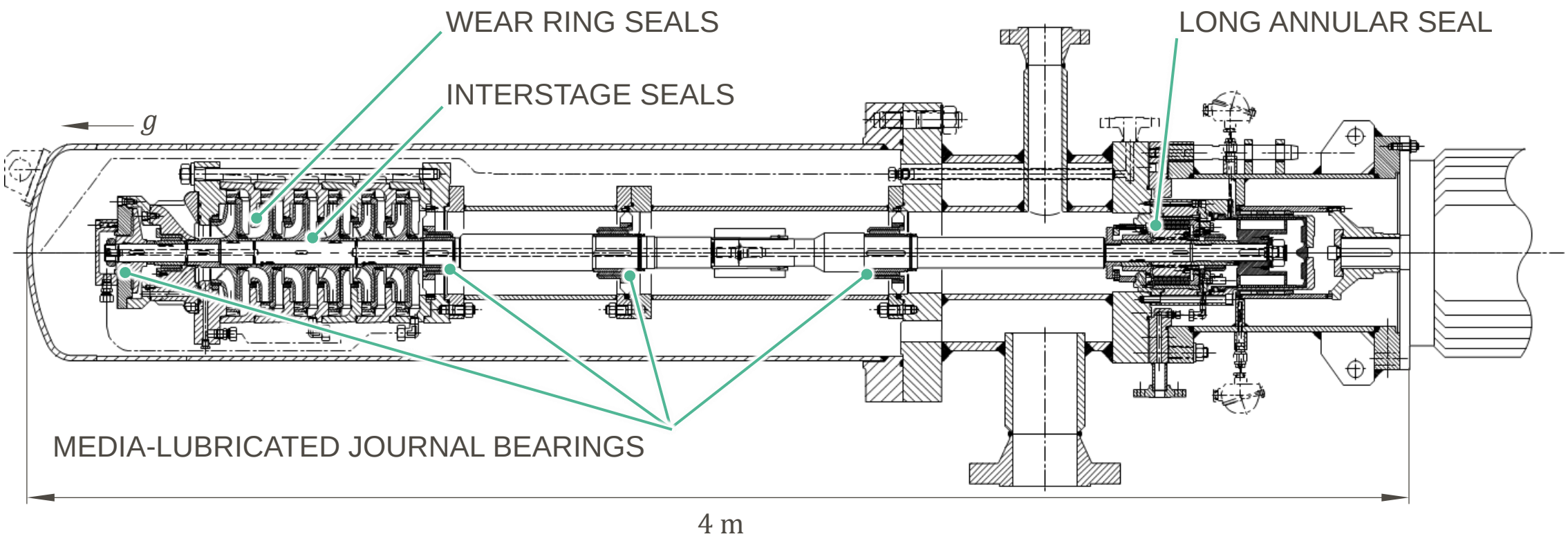
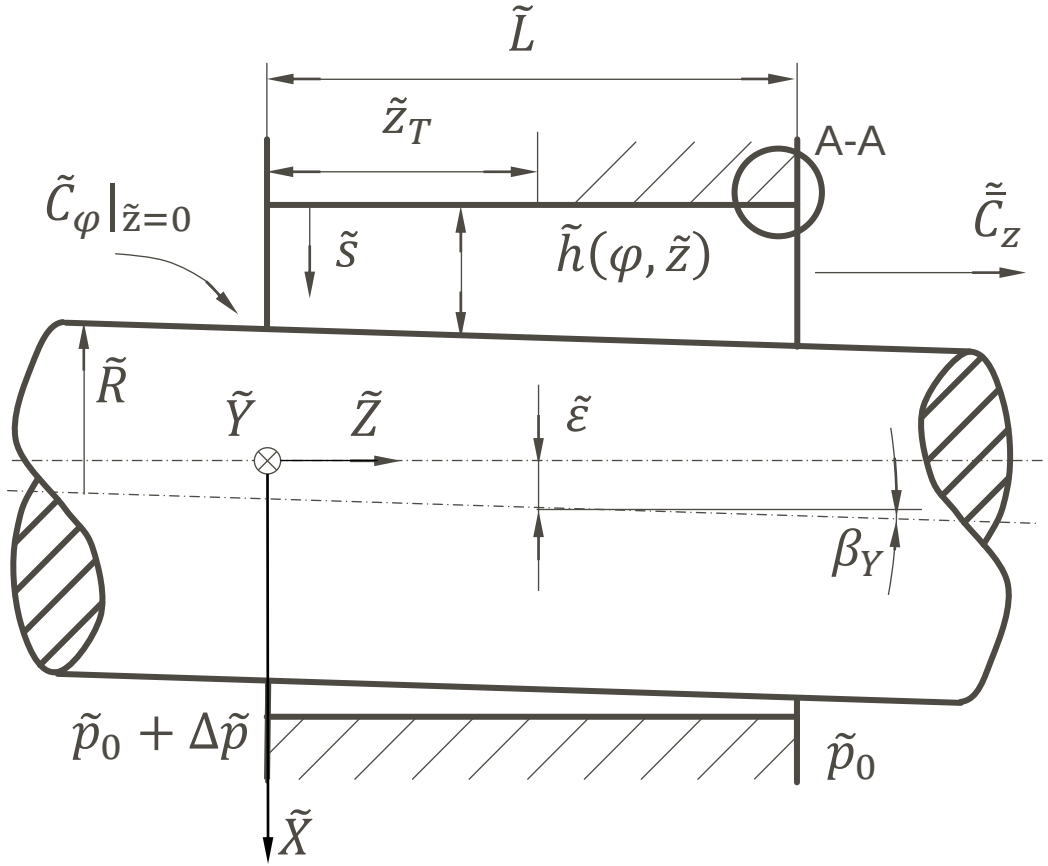
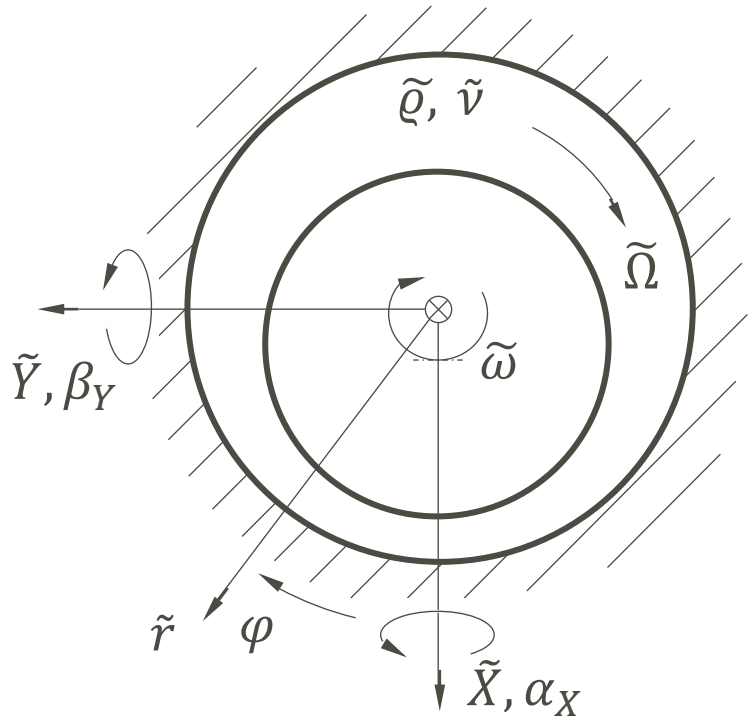
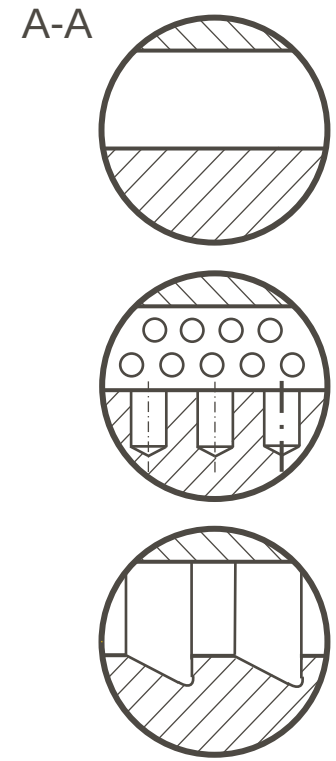
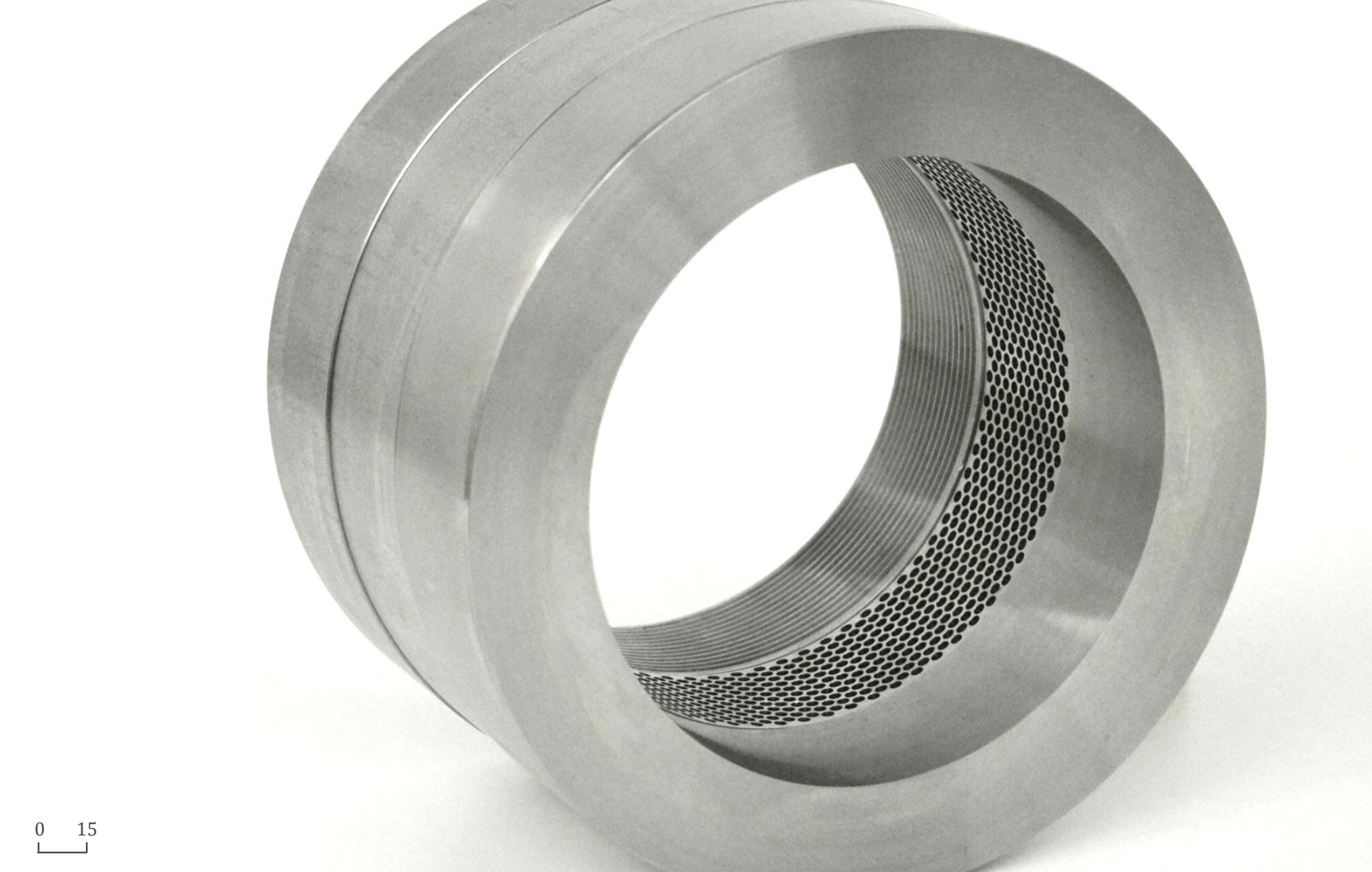


Geometry of an annular seal





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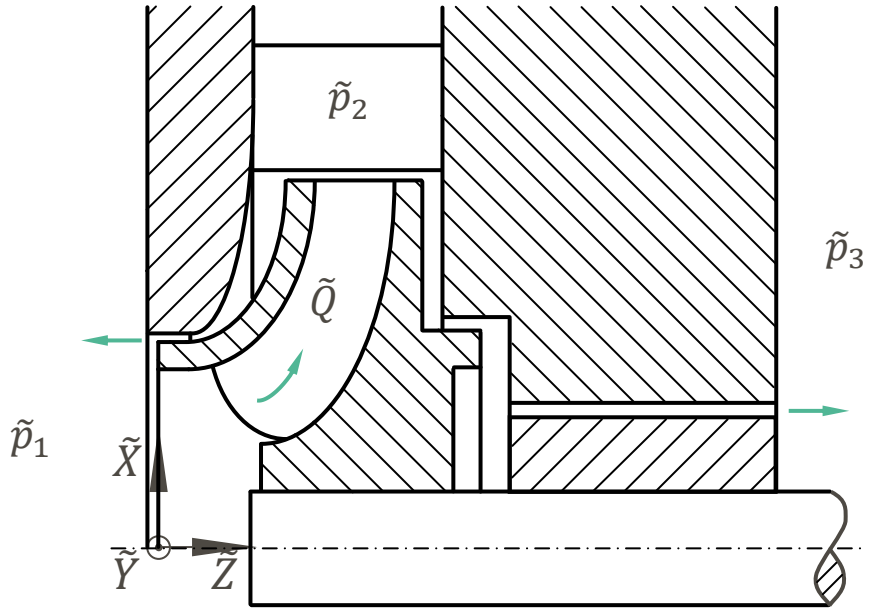
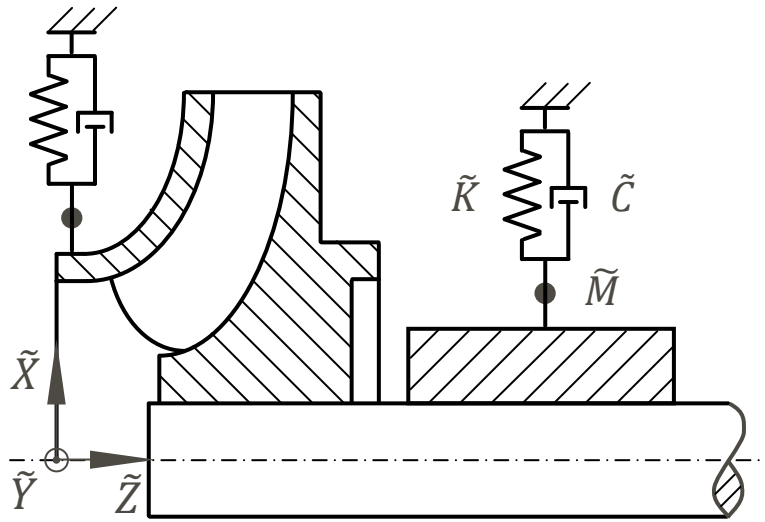
Static and dynamic influence of annular seals





Static characteristics:

$$\tilde{Q}_{\mathrm{L}} = \iint \tilde{c}_z \, \tilde{r} \, \mathrm{d}\tilde{r}\mathrm{d}\varphi$$

Dynamic characteristics:

$$-\begin{bmatrix}\tilde{F}_x\\ \tilde{F}_y\end{bmatrix} = \begin{bmatrix}\tilde{K} & \tilde{k}\\ -\tilde{k} & \tilde{K}\end{bmatrix}\begin{bmatrix}\tilde{x}\\ \tilde{y}\end{bmatrix} + \begin{bmatrix}\tilde{C} & \tilde{c}\\ -\tilde{c} & \tilde{C}\end{bmatrix}\begin{bmatrix}\dot{\tilde{x}}\\ \dot{\tilde{y}}\end{bmatrix} + \begin{bmatrix}\tilde{M} & \tilde{m}\\ -\tilde{m} & \tilde{M}\end{bmatrix}\begin{bmatrix}\ddot{\tilde{x}}\\ \ddot{\tilde{y}}\end{bmatrix}$$

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How to efficiently predict the dynamic characteristics of non symmetrically profiled annular seals?

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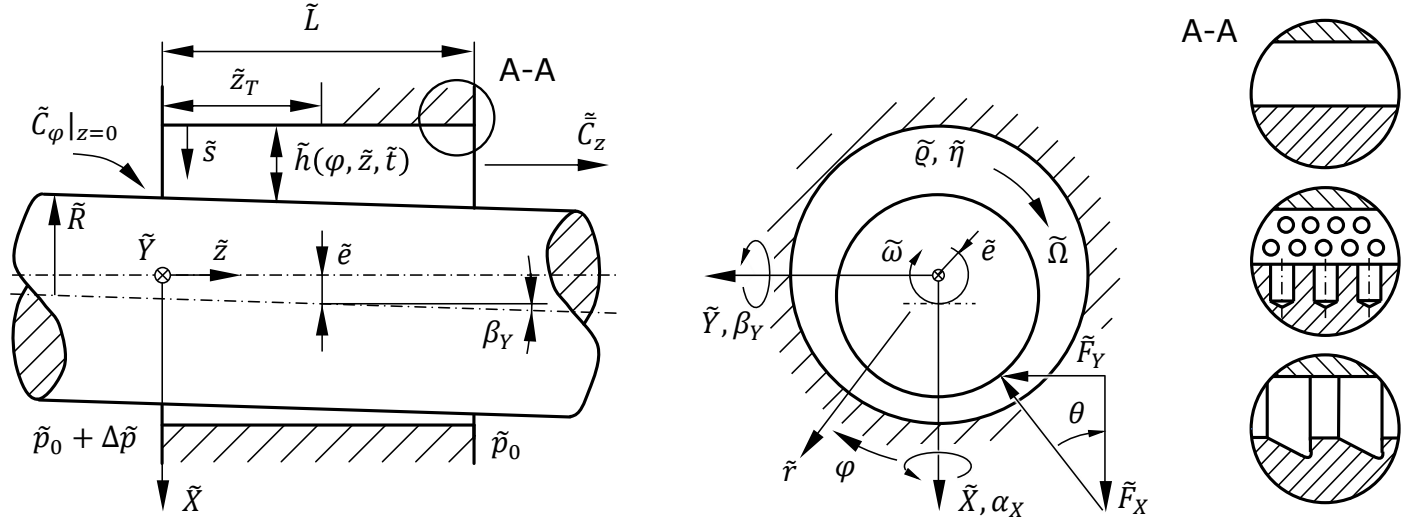
Clearance-Averaged Pressure Model (CAPM)

integro-differential lubrication film model



$$\frac{\partial}{\partial\varphi}\left(h\int_0^1 w_\varphi\,\mathrm{d}s\right)+\frac{\phi}{L}\frac{\partial}{\partial z}\left(h\int_0^1 c_z\,\mathrm{d}s\right)=0$$

$$\frac{\partial}{\partial\varphi}\left(h\int_0^1 w_\varphi^2\,\mathrm{d}s\right)+\frac{\phi}{L}\frac{\partial}{\partial z}\left(h\int_0^1 w_\varphi c_z\,\mathrm{d}s\right)=-\frac{h}{2}\frac{\partial p}{\partial\varphi}+\frac{1}{2\psi}\tau_{s\varphi}\Big|_0^1$$

$$\phi\frac{\partial}{\partial\varphi}\left(h\int_0^1 w_\varphi c_z\,\mathrm{d}s\right)+\frac{\phi^2}{L}\frac{\partial}{\partial z}\left(h\int_0^1 c_z^2\,\mathrm{d}s\right)=-\frac{h}{2L}\frac{\partial p}{\partial z}+\frac{1}{2\psi}\tau_{sz}\Big|_0^1$$

additional closure:

- velocity ansatz functions
- wall shear stress model

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Smearing shear stresses to model profiled walls using efficient 2D calculation tool

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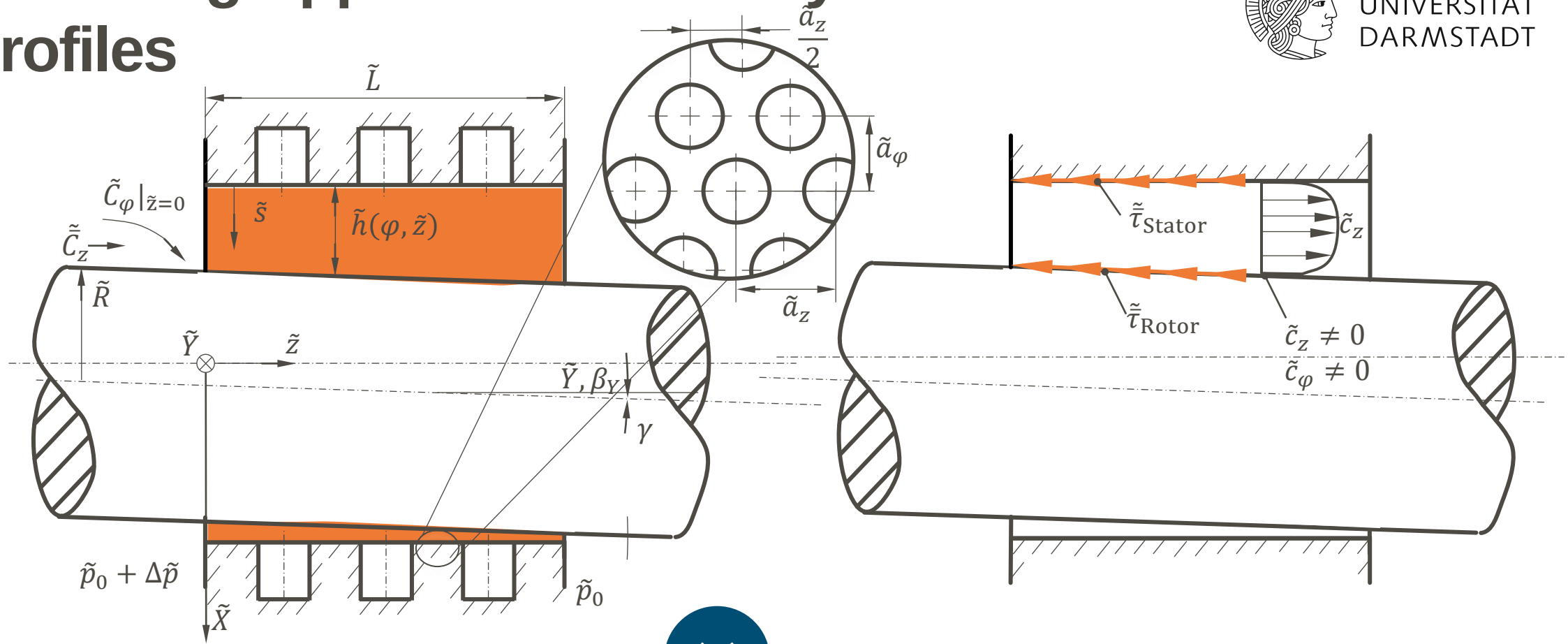
Modelling Approach for non symmetrical profiles



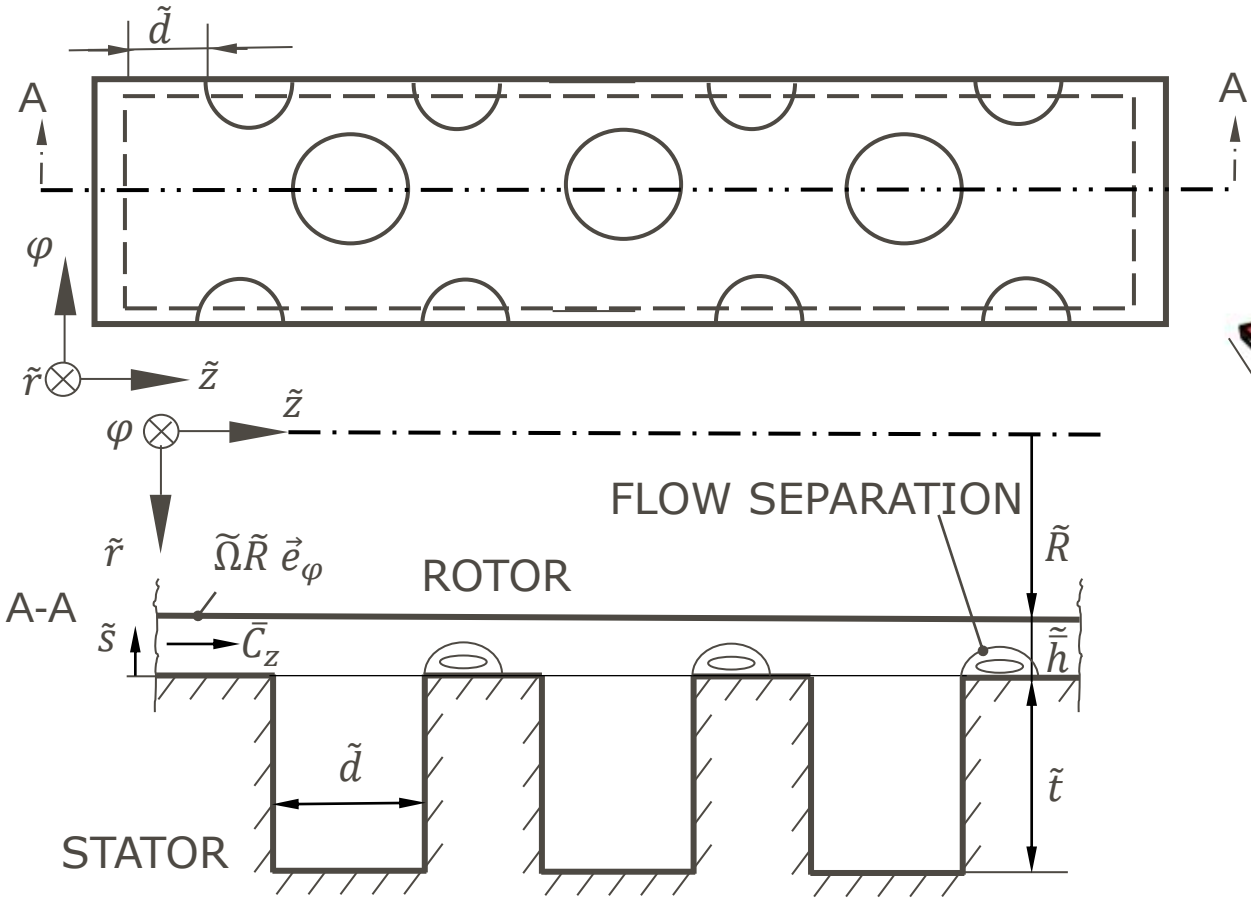
Non symmetrically profiled seal

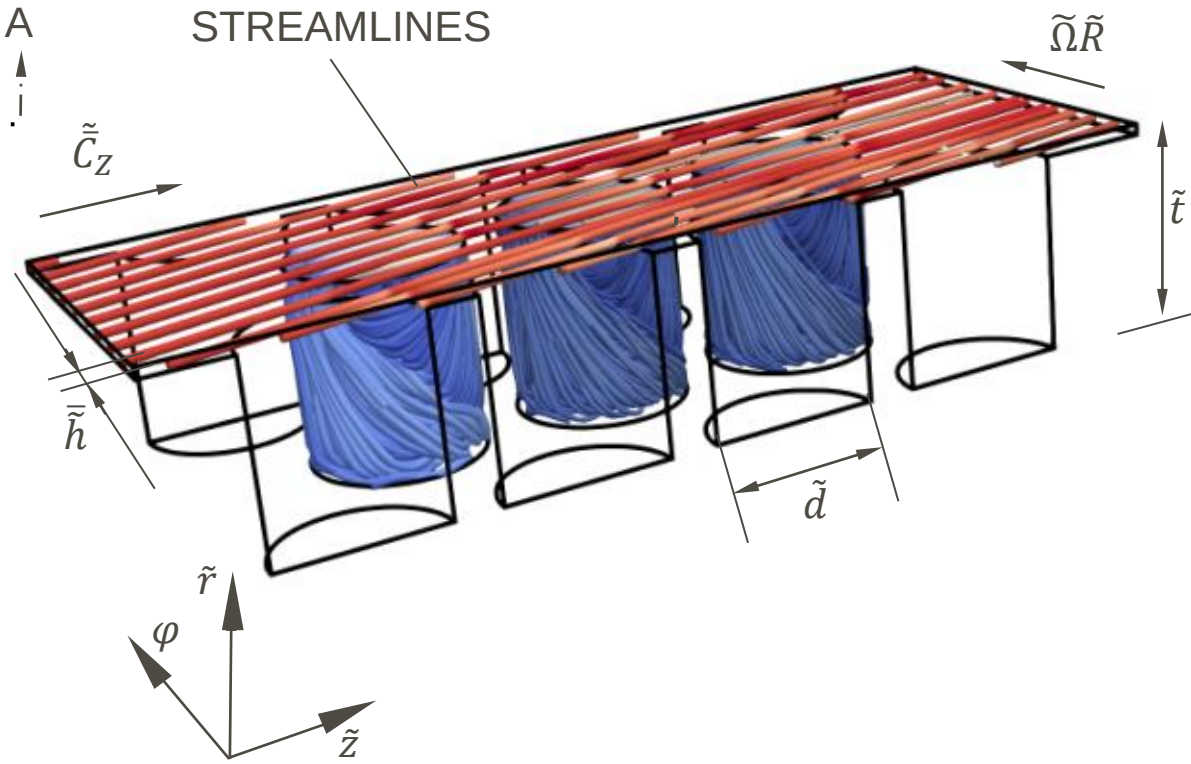
⇔

Plain seal with wall shear stress and velocity profiles

Analysis of a bore hole profile

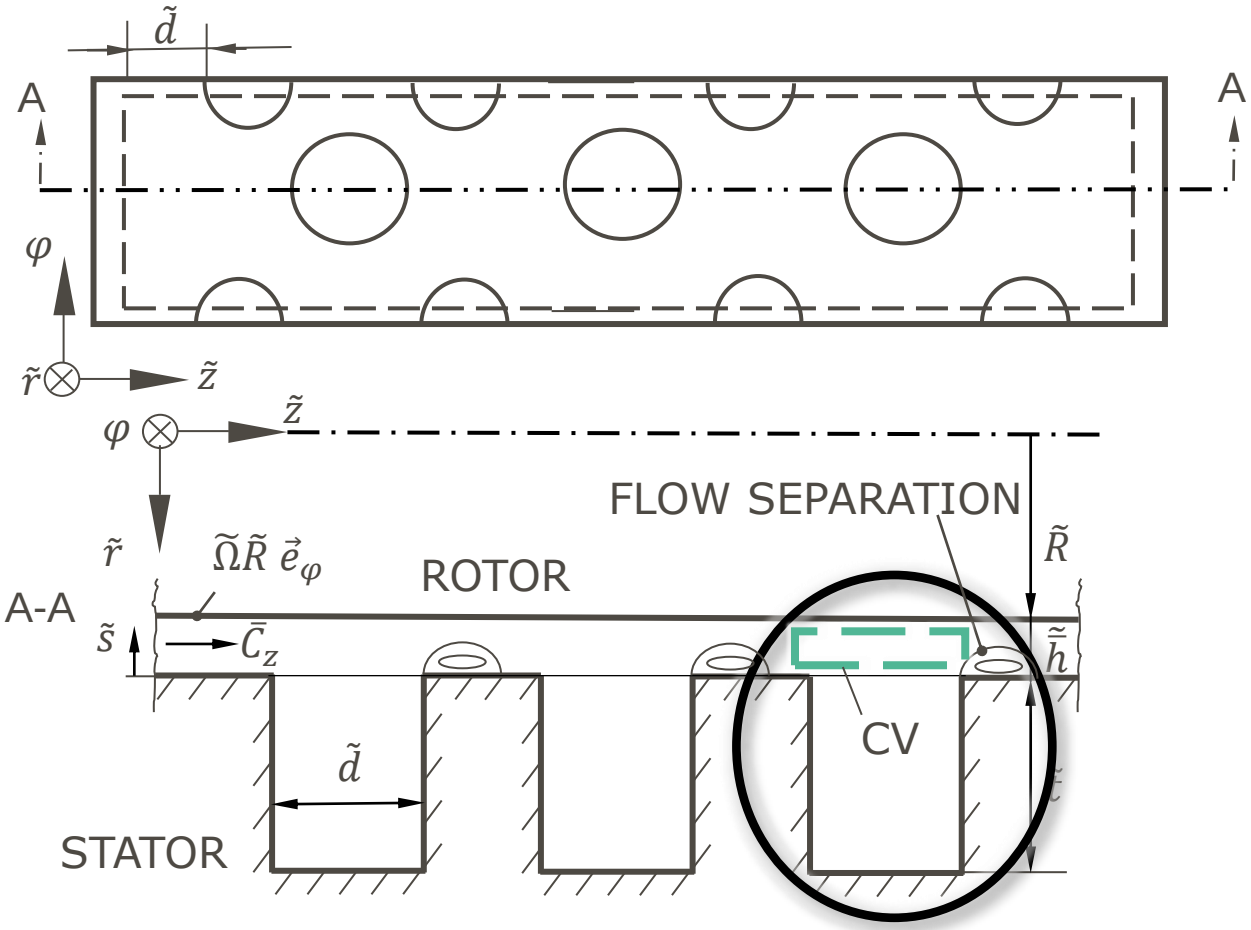
Concentric Rotor

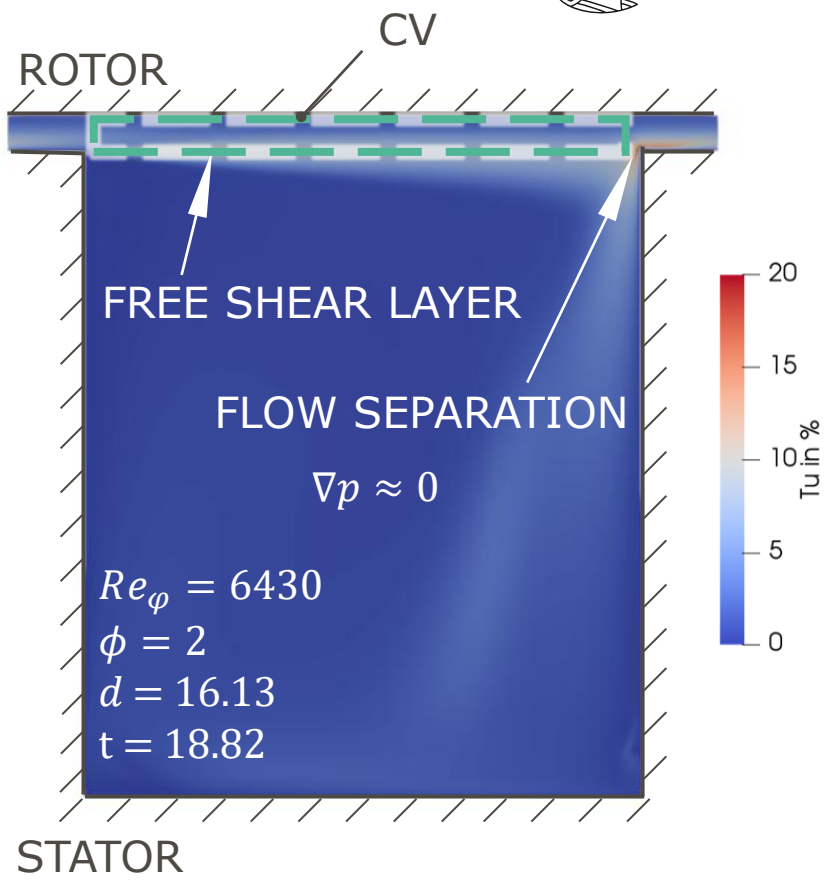




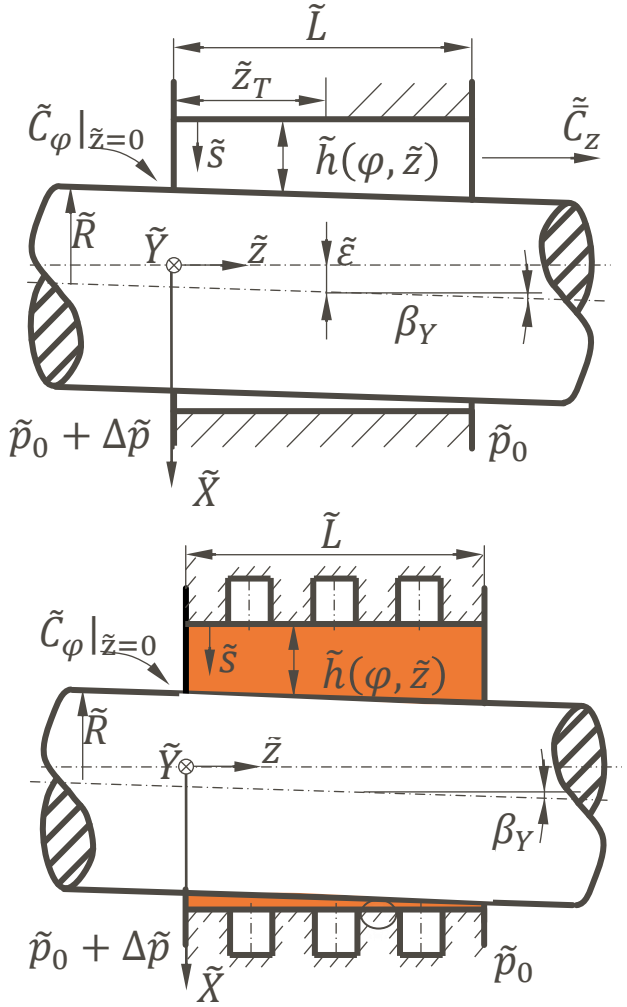
Analysis of a bore hole profile

Concentric Rotor





Friction Factor Model



$\Leftrightarrow$

$\Leftrightarrow$

$$\text{Wall} \in \{\text{Rotor; Stator}\}$$

$$f_{\text{Wall}}(\varphi,z) := \frac{2\,\tilde{\tau}_{\text{Wall}}}{\tilde{\varrho}\tilde{\bar{w}}^2_{\text{Wall}}} = \frac{\tau_{\text{Wall}}}{\bar{w}^2_{\text{Wall}}}$$

$$f_{\text{Wall}}(\varphi,z) = m_{\text{Wall}}\,(Re_{\text{Wall}})^{n_{\text{Wall}}}$$

?

Friction Factor Results

$$f_{\mathrm{Wall}} = m_{\mathrm{Wall}} \left(Re_{\mathrm{Wall}}\right)^{n_{\mathrm{Wall}}} = \frac{\tau_{\mathrm{Wall}}}{\bar{w}^2_{\mathrm{Wall}}}$$

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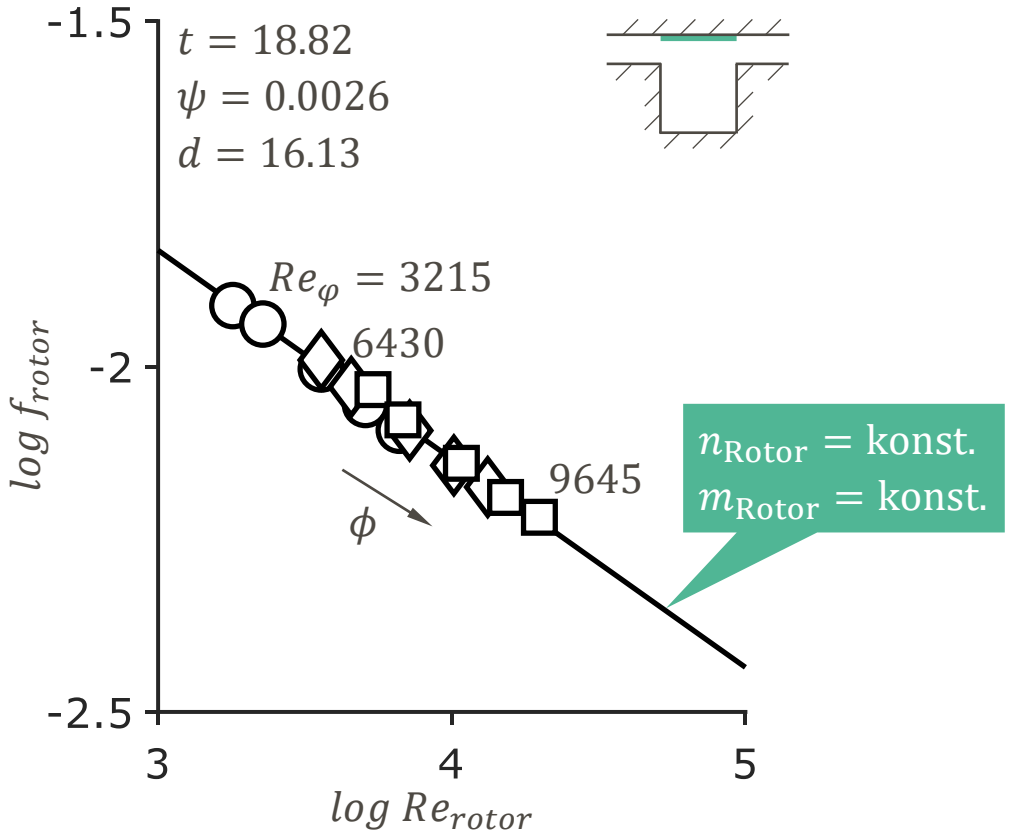
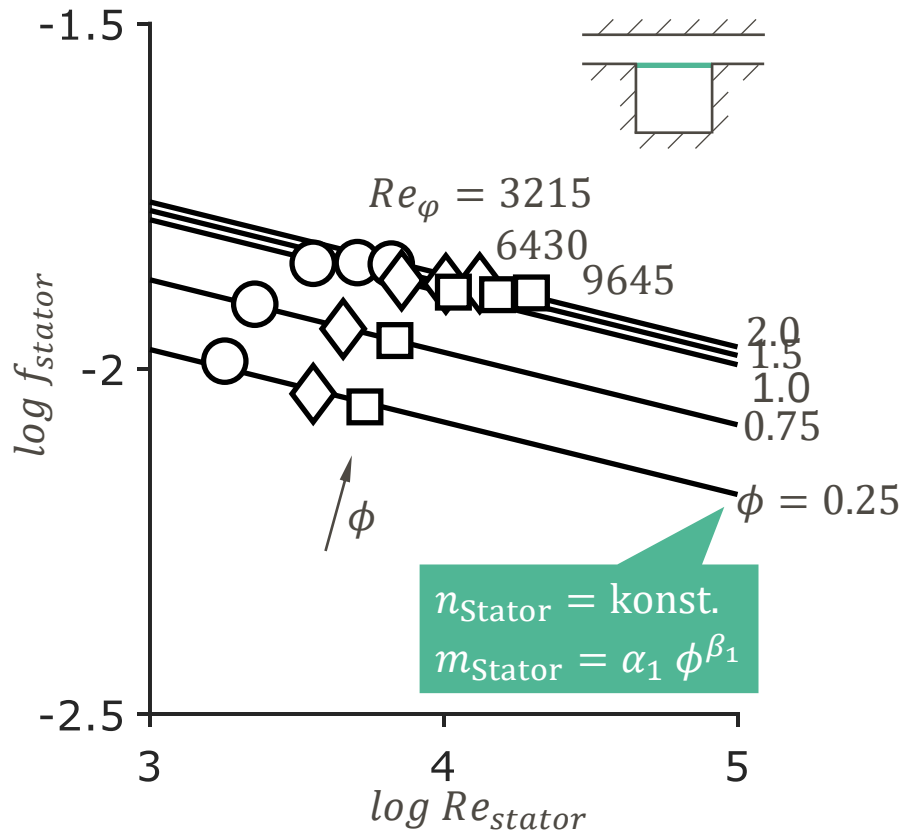
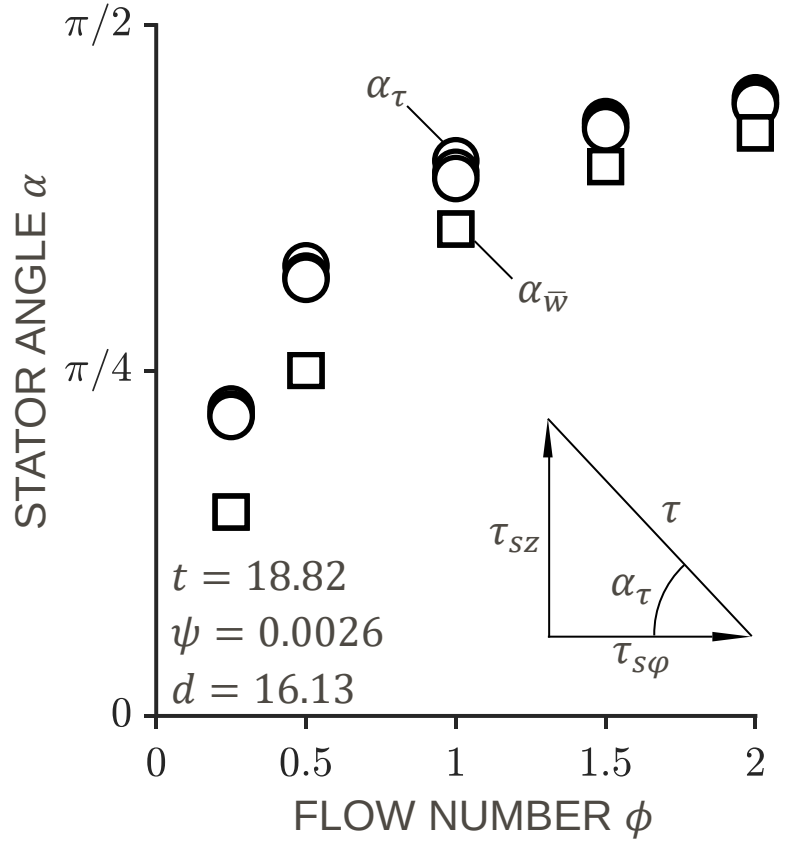
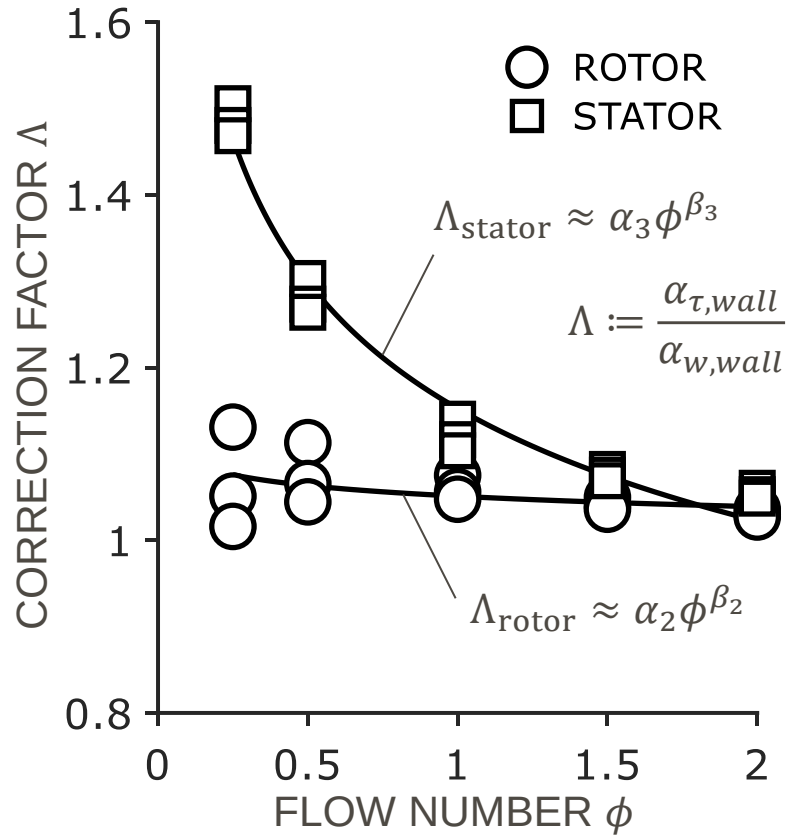




Model Extension:
Velocity Angle Correction





Entrance pressure loss coefficient
for flow from bore into gap

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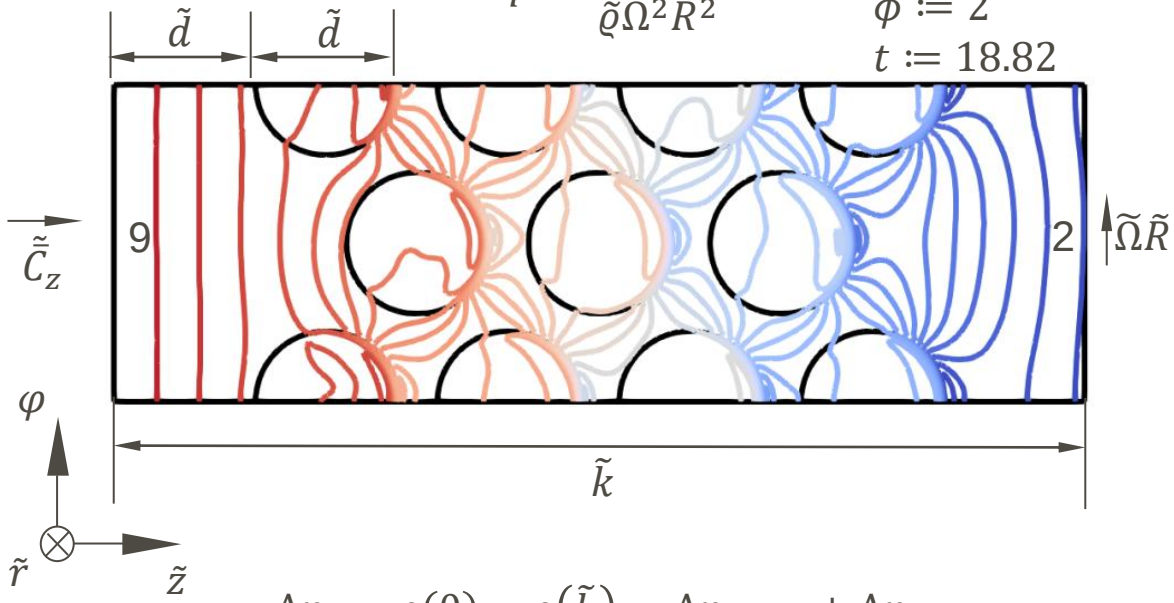
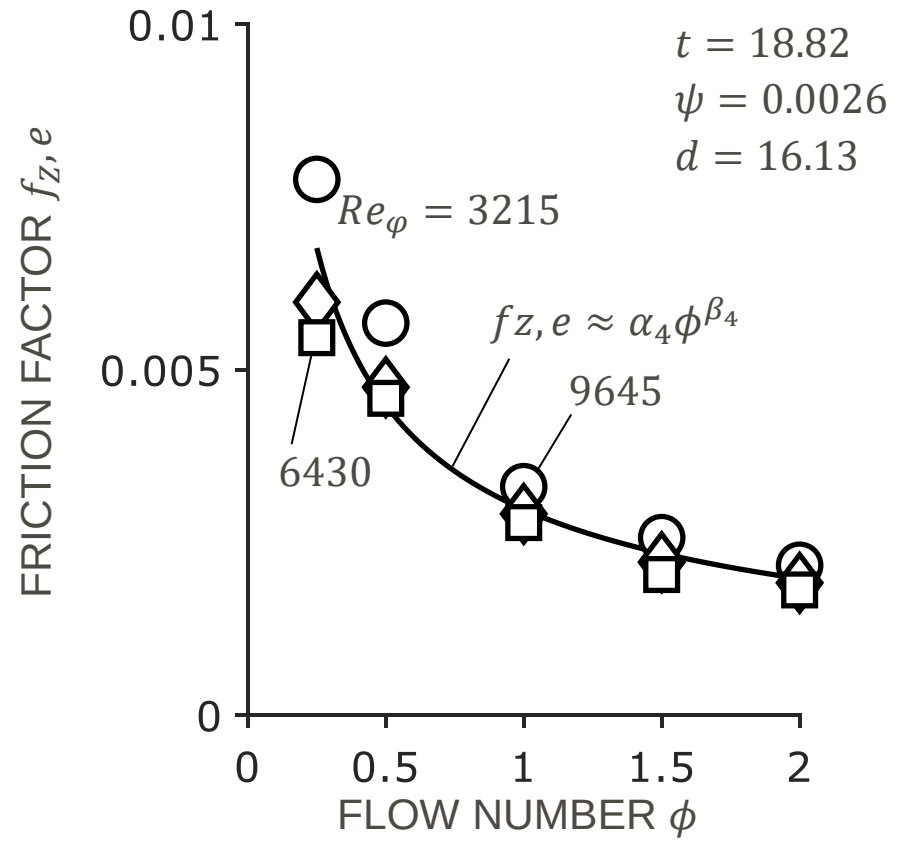
$$p := \frac{2\tilde{p}}{\tilde{\varrho}\tilde{\Omega}^2\tilde{R}^2}$$

$Re_\varphi := 9641$
$\phi := 2$
$t := 18.82$

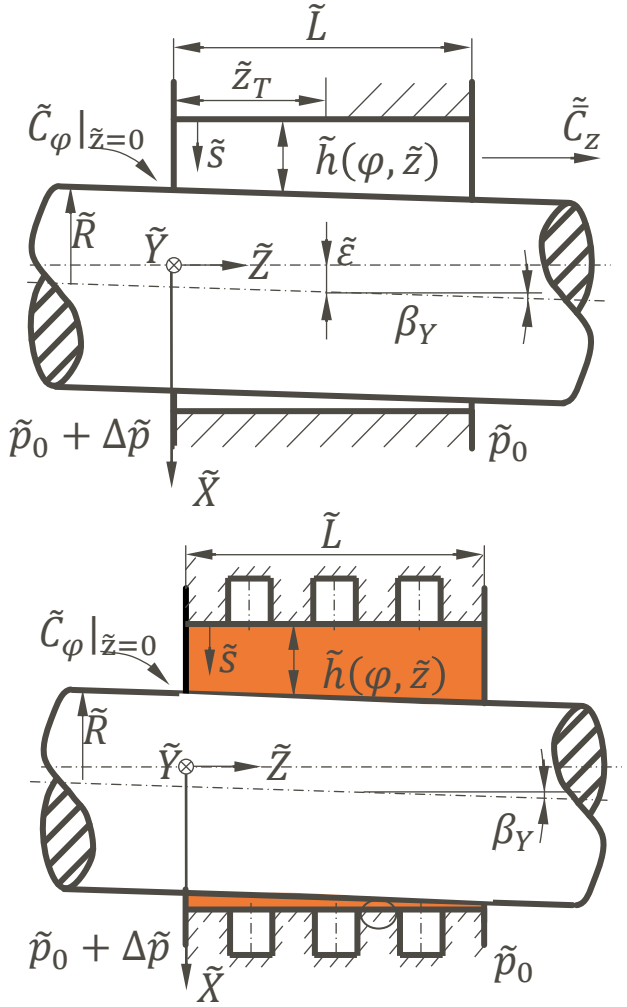


$$\Delta p_g = \mathrm{p}(0) - \mathrm{p}(\tilde{k}) = \Delta p_{\mathrm{shear}} + \Delta p_e$$

$$\tau_{z,e} = \frac{\Delta p_e}{\tilde{k}} = f_{z,e}\phi^2\bar{c}_z^{\,2}$$



Friction Factor Model



$\Leftrightarrow$

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$$f_{\text{Wall}}(\varphi,z) := \frac{2\,\tilde{\tau}_{\text{Wall}}}{\tilde{\varrho}\tilde{\bar{w}}^2_{\text{Wall}}} = \frac{\tau_{\text{Wall}}}{\bar{w}^2_{\text{Wall}}}$$

$$f_{\text{Wall}}(\varphi,z) = m_{\text{Wall}}\,(Re_{\text{Wall}})^{n_{\text{Wall}}}$$

$\Leftrightarrow$

$$m_{\text{Stator}} = \alpha_1\,\phi^{\beta_1} \checkmark$$
$$\Lambda_{\text{Rotor}} = \alpha_2\,\phi^{\beta_2} \checkmark$$
$$\Lambda_{\text{Stator}} = \alpha_3\,\phi^{\beta_3} \checkmark$$
$$fz,e\ = \alpha_4\,\phi^4 \checkmark$$
$$\tau_{s\varphi,\text{Wall}} = \pm f_{\text{Wall}}\bar{w}^2_{\text{Wall}}\cos\left(\Lambda_{\text{Wall}}\theta_{\bar{w},\text{Wall}}\right) \checkmark$$
$$\tau_{sz,\text{Wall}} = \pm f_{\text{Wall}}\bar{w}^2_{\text{Wall}}\sin\left(\Lambda_{\text{Wall}}\theta_{\bar{w},\text{Wall}}\right) \checkmark$$

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Evaluation of model performance

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Static and dynamic influence of annular seals

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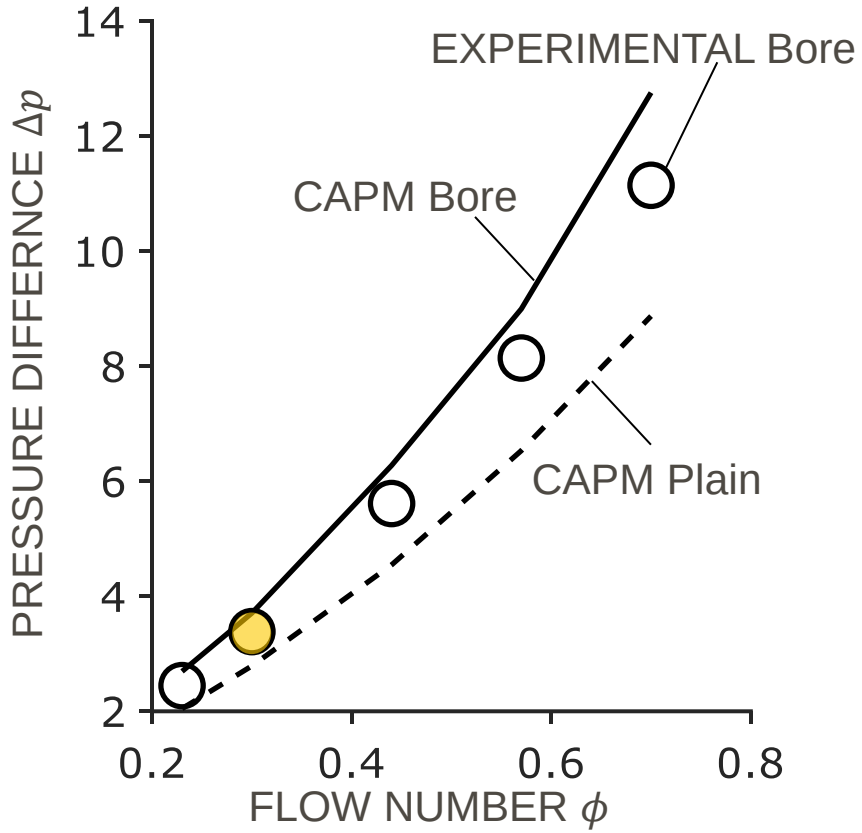

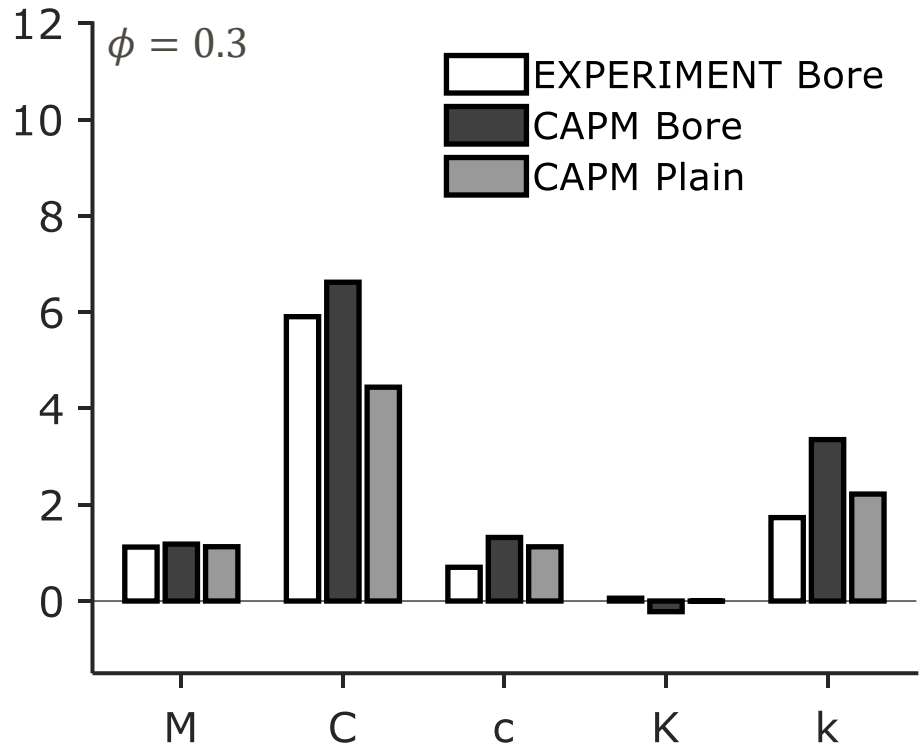

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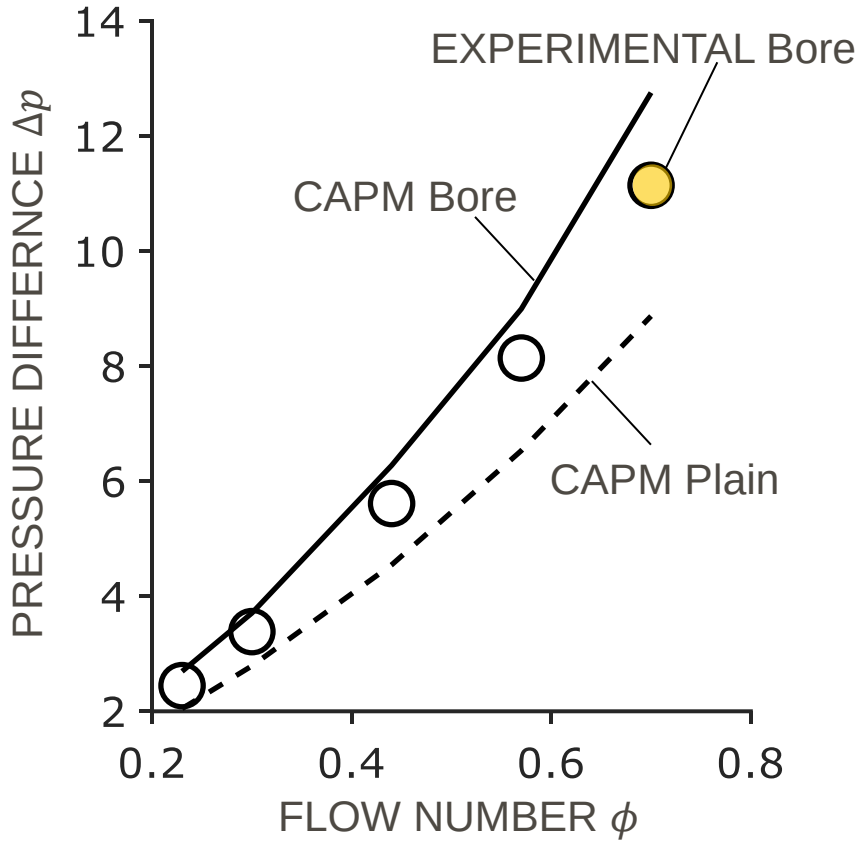

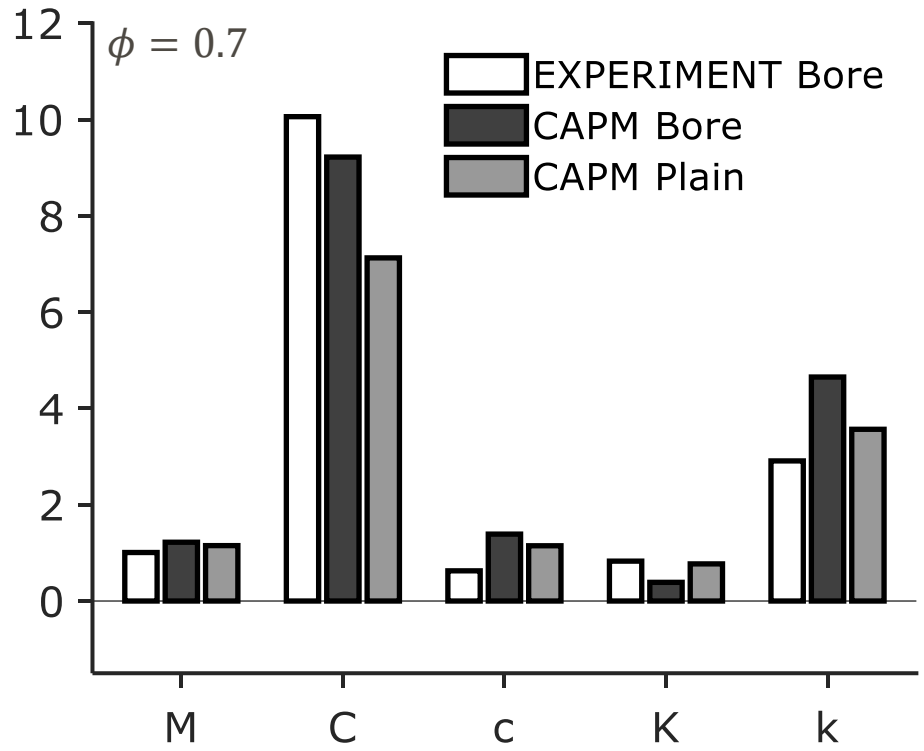

Take away

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1. The model successfully extends the range of seals that can be analysed both efficiently and with acceptable accuracy.
2. The model shows good agreement with experimental data when predicting static characteristics
3. It is a good tool which gives rough estimates for obtaining the dynamic characteristics for the non-symmetrically profiled seals
4. The 2D tool can generate results up to 10,000 times faster than 3D RANS simulations

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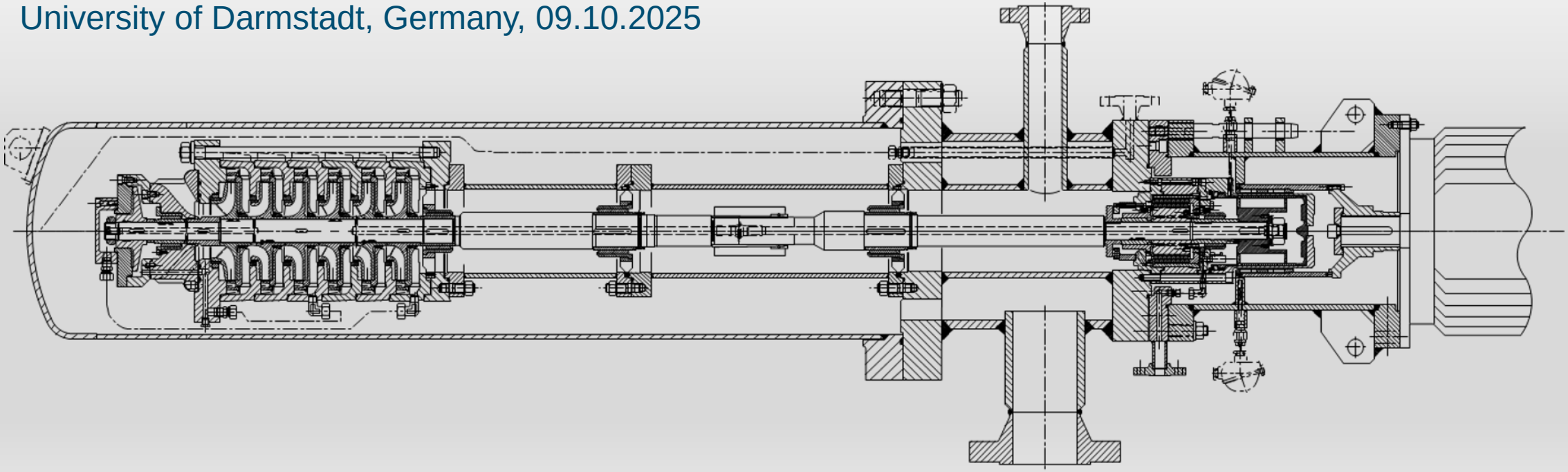

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