Centre of Excellence in Exascale CFD

Topology Optimization Using Modern HPC Software

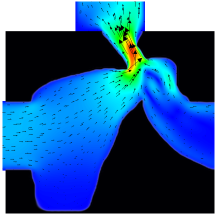
Harrison Nobis, Tim Felle, Abbas Mousavi,
Philipp Schlatter, Eddie Wadbro, Dan Henningson, Casper Andreasen, Niels Aage and
Martin Berggren

Topology Optimization Using Modern HPC Software

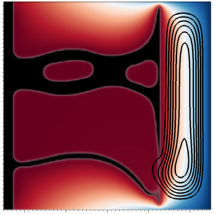
Structures

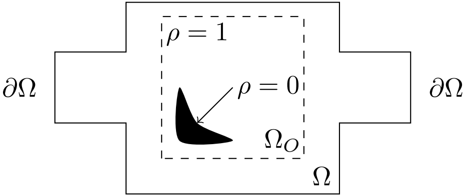
Heat transfer

Fluids



Multi physics





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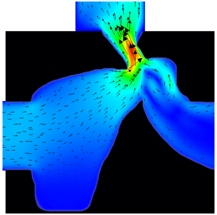

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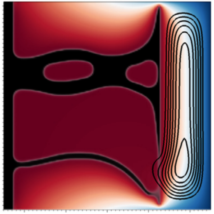

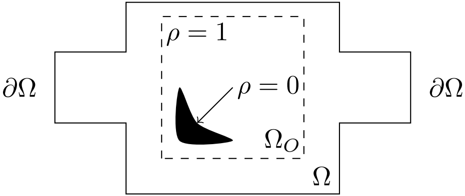

$$\begin{aligned} \underset{\rho \in \mathcal{A}}{\text{minimize}} \quad & \mathcal{F} & \\ \text{subject to} \quad & \text{Governing eqns} & \text{in } \Omega, \\ & \text{B.C's} & \text{on } \partial\Omega, \\ & \text{Constraints } C_i \leq 0 & \text{for } i = 1...M \end{aligned}$$

$$\mathcal{A} = \{\rho \in L^\infty(\Omega_O) \mid 0 \leq \rho \leq 1\}, \Omega_O \subseteq \Omega$$

FLOW

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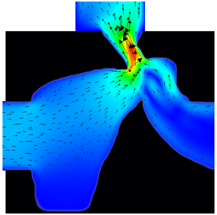
Structures

Heat transfer

Fluids



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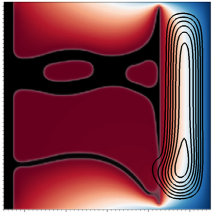

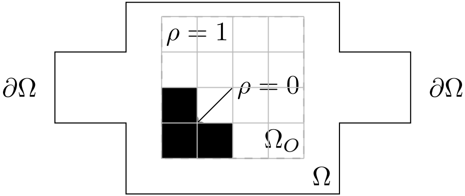

$$
\begin{aligned}
&\underset{\rho\in\mathcal{A}}{\text{minimize}} && \mathcal{F} && \\
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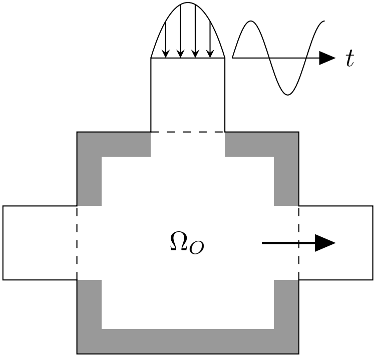
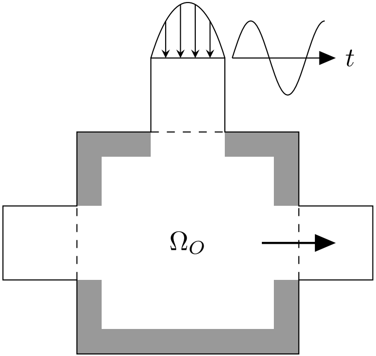
$$\mathcal{A} = \{\rho \in L^\infty(\Omega_O) \mid 0 \leq \rho \leq 1\}, \Omega_O \subseteq \Omega$$



Can we design a structure that converts the oscillatory inflow (top) into a flow from the left reservoir to the right reservoir?



Can we design a structure that converts the oscillatory inflow (top) into a flow from the left reservoir to the right reservoir?

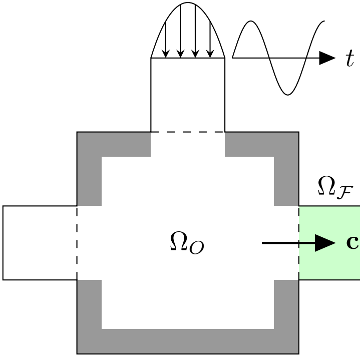
Governing equations:

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\cdot\nabla)\mathbf{u} - \frac{1}{Re}\nabla^2\mathbf{u} + \nabla p + \chi(\rho)\,\mathbf{u} = 0,$$
$$\nabla\cdot\mathbf{u} = 0,$$

where $\mathbf{u}$ and p denote the velocity and pressure respectively. Made non-dimensional with the Reynolds number Re and where

$$\chi(\rho) = \begin{cases} 0 & \text{in the fluid region,} \\ \text{BIG number} & \text{in the solid region,} \end{cases}$$

denotes a "Brinkman penalization" term.



Can we design a structure that converts the oscillatory inflow (top) into a flow from the left reservoir to the right reservoir?

Objective function:

Maximize:

$$\mathcal{F} = \frac{1}{T} \int_0^T \int_{\Omega_{\mathcal{F}}} \mathbf{c} \cdot \mathbf{u} \, d\Omega \, dt,$$

where $\mathbf{c}$ is a vector pointing to the right and T is integration time.

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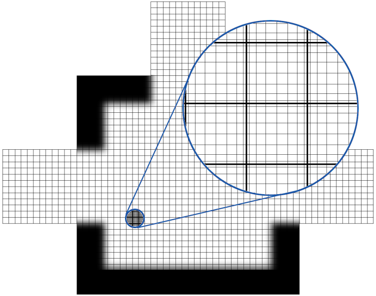
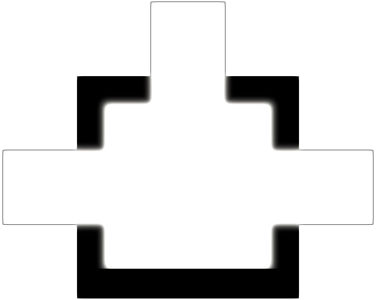


Can we design a structure that converts the oscillatory inflow (top) into a flow from the left reservoir to the right reservoir?

Discretization:
All computations are performed using the Spectral Element Method (SEM).

- Accuracy
- Diffusive and dispersive properties
- Highly parallelizable and scalable



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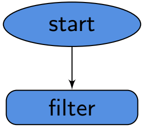
Optimization algorithm



FLOW

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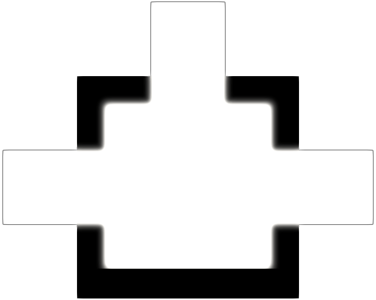
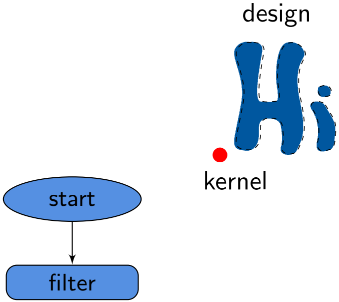


Optimization algorithm:



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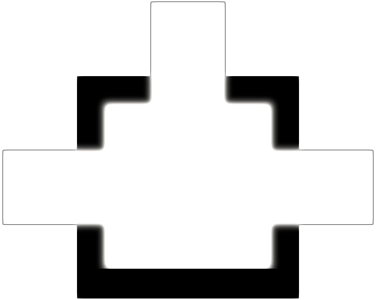

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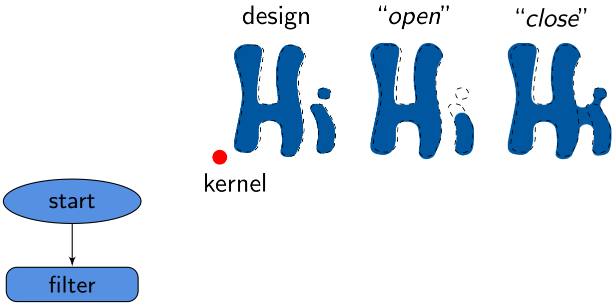

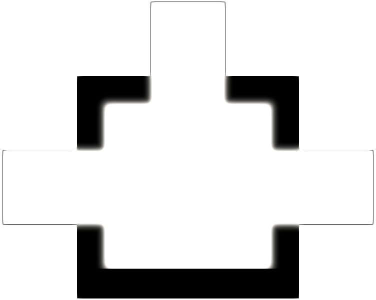

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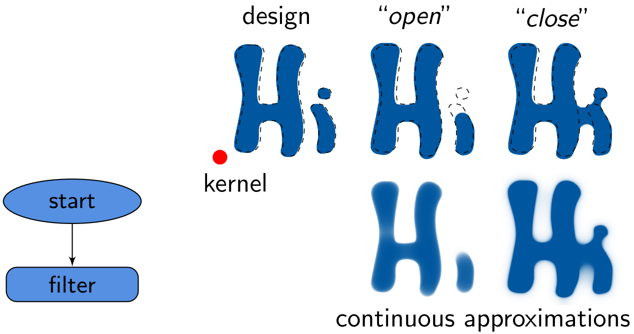

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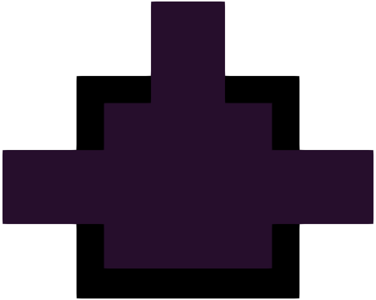

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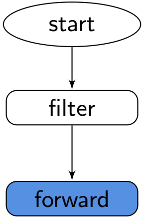
Optimization algorithm:



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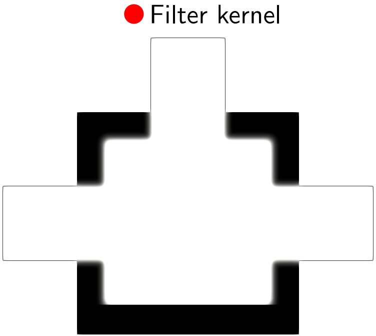
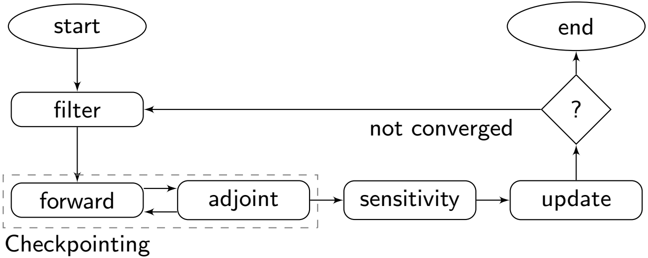


Can we design a structure that converts the oscillatory inflow (top) into a flow from the left reservoir to the right reservoir?

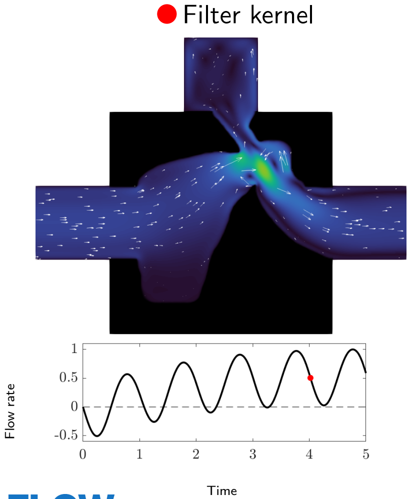
Optimization algorithm:

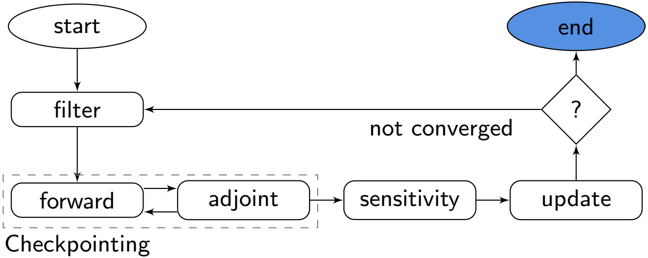
- Checkpointing with the `revolve` algorithm [Griewank and Walther, 2000]
- Design is updated with the Method of Moving Asymptotes (MMA) [Svanberg, 2002]



Can we design a structure that converts the oscillatory inflow (top) into a flow from the left reservoir to the right reservoir?

YES!!





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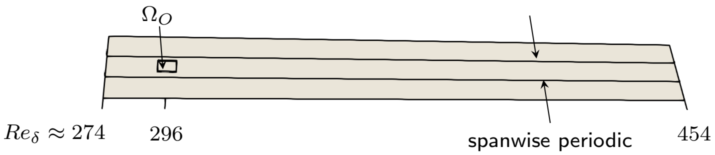

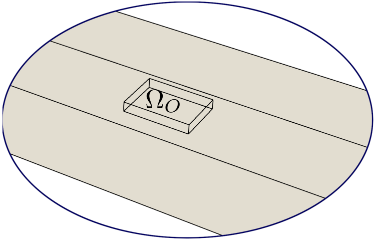

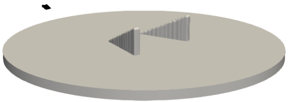

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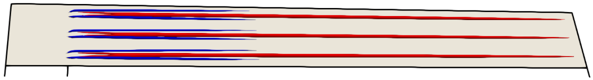
Streaks



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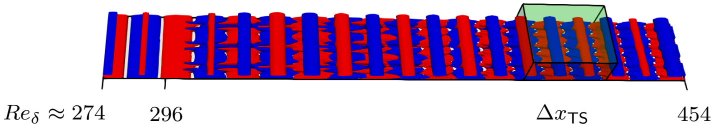
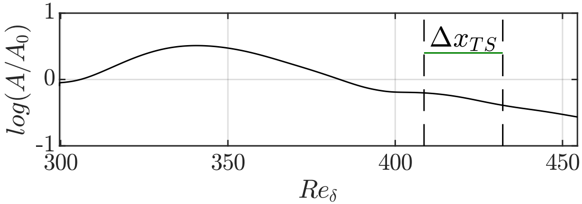
TS wave



Streamwise evolution of TS wave amplitude.



Objective function: Minimize perturbation kinetic energy in Δx_{TS}

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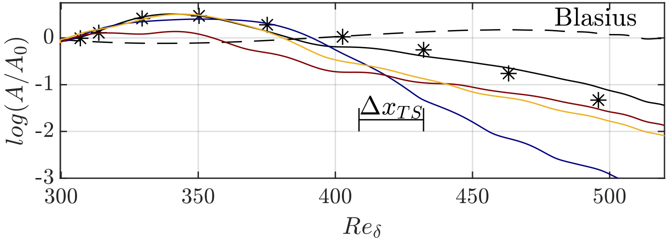

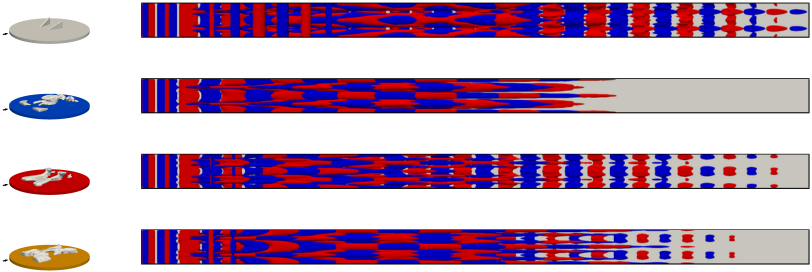

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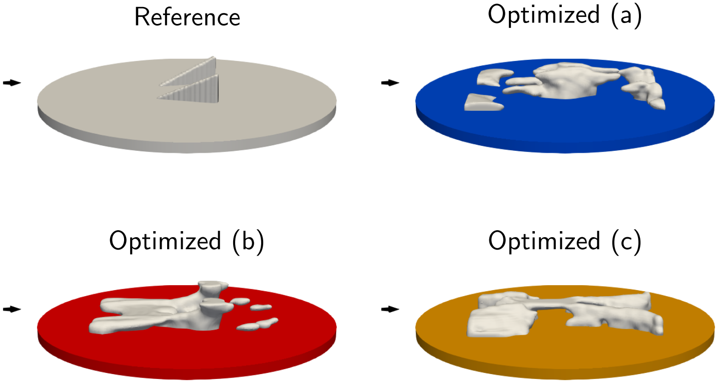

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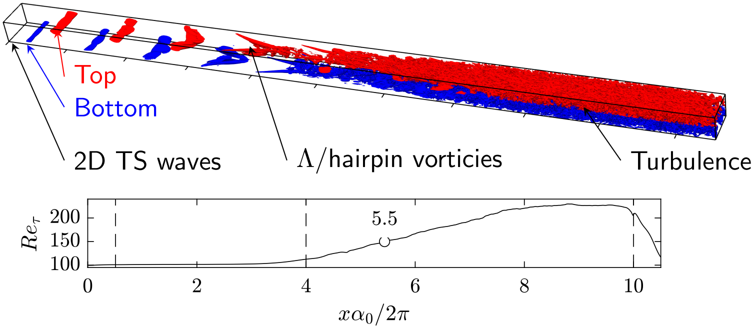

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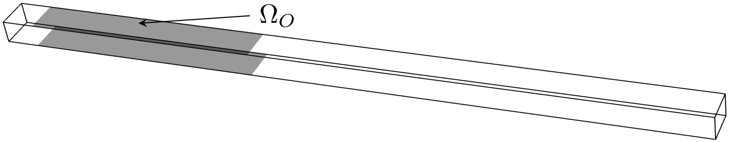

Can we paint a pattern using superhydrophobic surfaces which will delay this transition scenario?

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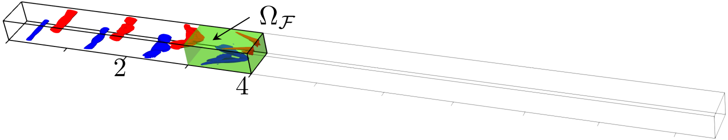

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Can we paint a pattern using superhydrophobic surfaces which will delay this transition scenario?

We minimize the pseudo-dissipation:

$$\mathcal{F} = \frac{\omega}{2\pi|\Omega_{\mathcal{F}}|}\int_{T-\frac{2\pi}{\omega}}^{T}\int_{\Omega_{\mathcal{F}}}\frac{1}{2}|\nabla \mathbf{u}|^2\,d\Omega\,dt.$$

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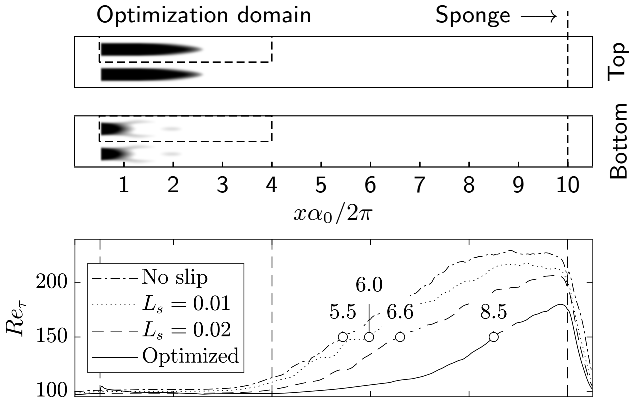

Spatial evolution of friction Reynolds number, with circles indicating where $Re_\tau = 150$.

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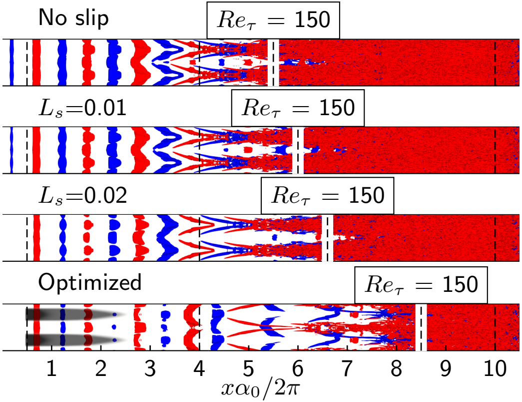

Colours indicate the top and bottom surfaces. View is from above.

FLOW

Neko:

- Spectral element method
- Modern HPC architectures
  - CPUs & GPUs
- Nominated for ACM Gordon Bell Prize

  Jansson et al. (2023).
  Exploring the Ultimate Regime of Turbulent Rayleigh-Bénard Convection Through Unprecedented Spectral-Element Simulations
  SC23.
- 12 million spectral elements $\approx$ 6 billion points

  Massaro et al. (2024).
  Direct numerical simulation of the turbulent flow around a Flettner rotor
  Sci Rep 14, 3004.

Jansson et al. (2024).
Neko: A modern, portable, and scalable framework for high-fidelity computational fluid dynamics.
Computers & Fluids, 275:106243.

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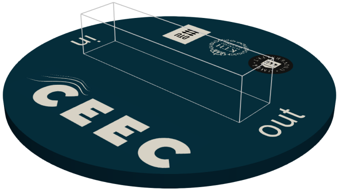

Can we design an internal structure that passively enhances mixing?

<u>Governing equations:</u>

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} - \frac{1}{Re} \nabla^2 \mathbf{u} + \nabla p + \chi(\rho)\,\mathbf{u} = 0,$$

$$\nabla \cdot \mathbf{u} = 0,$$

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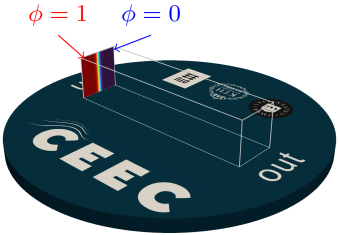

Can we design an internal structure that passively enhances mixing?

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\begin{aligned}
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\nabla \cdot \mathbf{u} &= 0, \\
\frac{\partial \phi}{\partial t} + (\mathbf{u} \cdot \nabla)\phi - \frac{1}{Pe}\nabla^2 \phi &= 0.
\end{aligned}
$$

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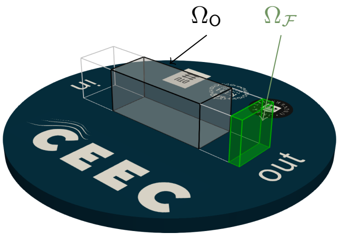

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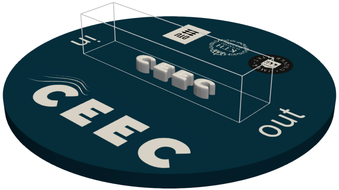
\nabla \cdot \mathbf{u} &= 0, \\
\frac{\partial \phi}{\partial t} + (\mathbf{u} \cdot \nabla)\phi - \frac{1}{Pe}\nabla^2 \phi &= 0.
\end{aligned}$$

Objective function:

$$\mathcal{F} = \frac{1}{|\Omega_{\mathcal{F}}|} \int_{\Omega_{\mathcal{F}}} \frac{1}{2}\left(\phi - \phi_{\text{ref}}\right)^2 d\Omega.$$

FLOW



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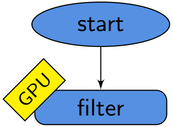
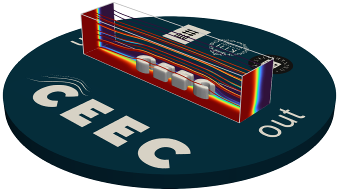
Optimization algorithm:

- Filtering based on a PDE [Lazarov & Sigmund, 2010]



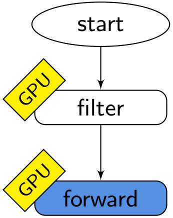
Topology Optimization Using Modern HPC Software



Can we design an internal structure that passively enhances mixing?

Optimization algorithm:

- Filtering based on a PDE [Lazarov & Sigmund, 2010]



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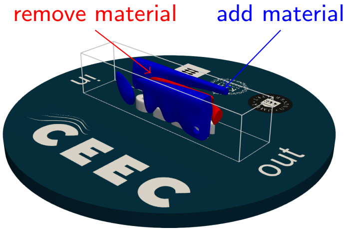

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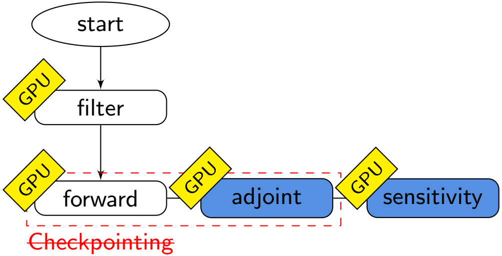

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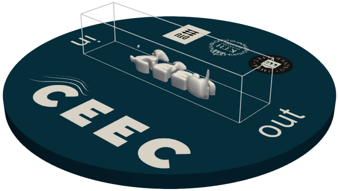

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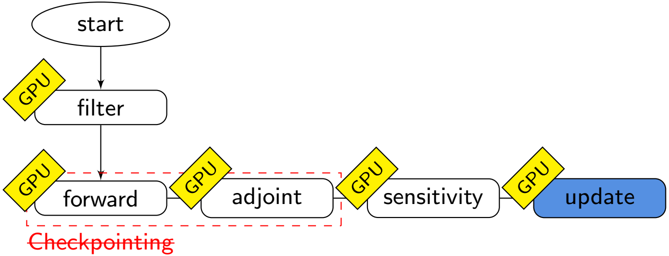

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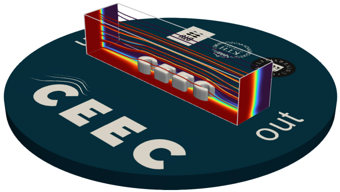

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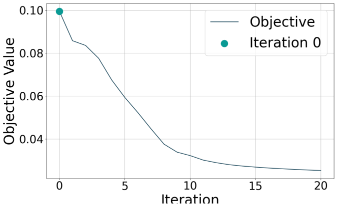

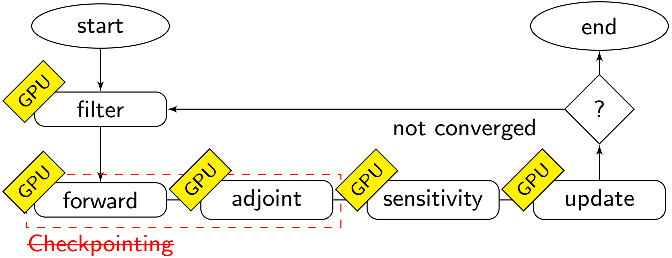

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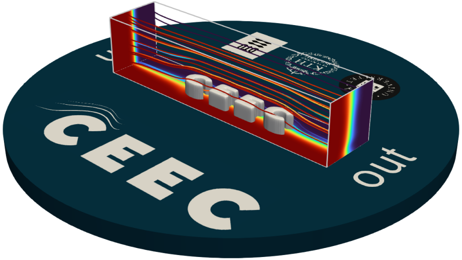
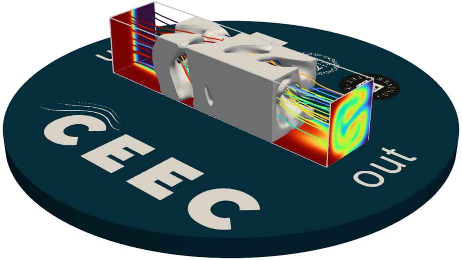
Thank you for your attention!





Neko-top

github.com/
ExtremeFLOW/neko-top

Neko

github.com/
ExtremeFLOW/neko

CEEC

ceec-coe.eu

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