Numerical Methods for Multi-phase flow

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This lecture will focus on a few numerical methods for multi-phase flows, focusing on direct numerical simulation (DNS). The first part of the lecture will discuss predicting the behaviour of interfaces in fluid-fluid flows, such as a rising bubble, and the second part will focus on predicting the interface in a resolved fluid-particle flow.

Fluid/Fluid Interfaces

When resolving the behaviour of fluid-fluid interfaces, there are a few popular numerical techniques, which will be discussed in this lecture. These include level-set (Sussman et al., 1994), volume-of-fluid (Hirt and Nichols, 1981), and front-tracking (Unverdi and Tryggvason, 1992). Each of the methods will be derived from first principles, and a strength/weakness analysis of each of the methods will be presented. Moreover, some popular implementations of each of the methods will be discussed, and the methods will be compared to each other.

In most interfacial flows, surface tension plays an important role, and some methods for determining the surface tension and the curvature (Brackbill et al., 1992) will be discussed.

A number of example calculations will be given throughout the lecture.

DNS for particles

A method which has an increasing attention and popularity is the immersed boundary method (IBM), originally developed by Peskin (1972). The IBM is a well-suited tool for the direct numerical simulation of flows with dense particle suspensions, which feature strong and complex flow-particle and particle-particle interactions.

IBMs can generally be classified into two types: smooth- and sharp-interface types. The IBM was originally proposed as a smooth-interface type by Peskin (1972), and this is still the most generally applied implementation. In contrast to the smooth-interface IBMs, a sharp-interface formulation imposes a velocity that corresponds to the no-slip condition precisely at Lagrangian points, by constraining the nodal variables of only the nearest neighbouring fluid cells with an interpolation function, typically a linear one, such as in Mark and van Wachem (2008). This results in the sharp representation of the particle interface, which can show second-order spatial convergence behaviour on some specific features of the flow. Both of these implementations will be discussed during the lecture.

The recent results obtained from high fidelity simulations, where the flow is fully resolved at length scales smaller than the particles, have been increasingly useful to better understand the complex behaviour of such flows. Direct numerical simulation data, reliable results, and the ability to represent arbitrary physical situations, are useful to improve closure models for coarse-grained large scale simulations of dense particulate flows. Examples of these will be discussed.

References

Brackbill, J., Kothe, D., and Zemach, C. (1992). Continuum Method for Modeling Surface Tension. *Journal of Computational Physics*, 100:335–354.

Hirt, C. and Nichols, B. (1981). Volume of fluid (VOF) method for the dynamics of free boundaries. *Journal of Computational Physics*, 39(1):201–225.

Mark, A. and van Wachem, B. (2008). Derivation and validation of a novel implicit second-order accurate immersed boundary method. *Journal of Computational Physics*, 227:6660–6680.

Peskin, C. S. (1972). Flow patterns around heart valves: A numerical method. *Journal of Computational Physics*, 10(2):252–271.

Sussman, M., Smereka, P., and Osher, S. (1994). A Level Set Approach for Computing Solutions to Incompressible Two-Phase Flow. *Journal of Computational Physics*, 114(1):146–159.

Unverdi, S. and Tryggvason, G. (1992). A Front-Tracking Method for Viscous, Incompressible, Multi-fluid Flows. *Journal of Computational Physics*, 100:25–37.