Spatiotemporal tidal prediction and analysis using physics-informed ML

Thomas Monahan
Department of Engineering Science
University of Oxford
Oxford, UK
thomas.monahan eng.ox.ac.uk
+1-720-530-9658

Tianning Tang, Stephen Roberts, Thomas A.A.Adcock
Supervisors

Keywords Tides · Storm Surge · Satellite Altimetry · Tidal Energy · Scientific Machine Learning

1 Plain Language Summary

Coastal flooding is among the most devastating natural disasters, with over 1.8 billion people living in vulnerable coastal regions. Accurate forecasting is therefore imperative for these communities. These floods are compound events driven by tidal processes – phenomena partially caused by astronomical tides (Earth, Moon, Sun interactions) but also influenced by fluvial and meteorological forcing. While astronomical tides have been predictable for nearly a century via tidal harmonic analysis, the simplifying assumptions within this framework limit its application to many critical compound processes such as storm surges, tidal rivers, and tidal currents. Yet it is precisely these processes that govern flood risk and infrastructure impacts, making their prediction essential. Prior to this work, the principal approach to studying and forecasting such flows has been direct numerical simulation. Although physically constrained by the shallow water equations, the accuracy of these models is fundamentally limited by boundary conditions, most notably the bathymetry – the depth profile of the ocean. In the majority of low- and middle-income countries, which account for 80% of the vulnerable coastal population, bathymetric data is sparse or non-existent, rendering conventional numerical simulation intractable. This leaves coastal populations without robust flood forecasts and creates major engineering challenges.

This thesis introduces a new paradigm for studying and predicting tidal processes free from bathymetric dependency. It comprises two breakthroughs: (i) a novel physical theory of tidal processes, and (ii) the development of a time-invariant neural operator which retains an exact equivalence to the underlying physics. Combined, these innovations enable the automatic estimation of nonlinear response functions directly from observational data without bathymetric input, and allow for predictions at any time given knowledge of external forcing. The former was infeasible with numerical models, and the latter unattainable with conventional machine learning approaches. For the first time, this enables the analysis and forecasting of arbitrary tidal processes, with demonstrated applications to tidal rivers, tidal currents, and storm surge. The approach is operationally deployed within the Dutch storm surge forecast system and is being introduced into UK national operational forecasts, protecting hundreds of billions in assets and millions of lives. Critically, it lays the foundation for global storm surge forecasting, particularly in regions lacking conventional modeling infrastructure. The success of this work lies not in replacing physics with machine learning, but in using machine learning to exploit a new physical framework, eliminating reliance on data we lack and instead leveraging data we have in abundance.

2 Scientific Summary

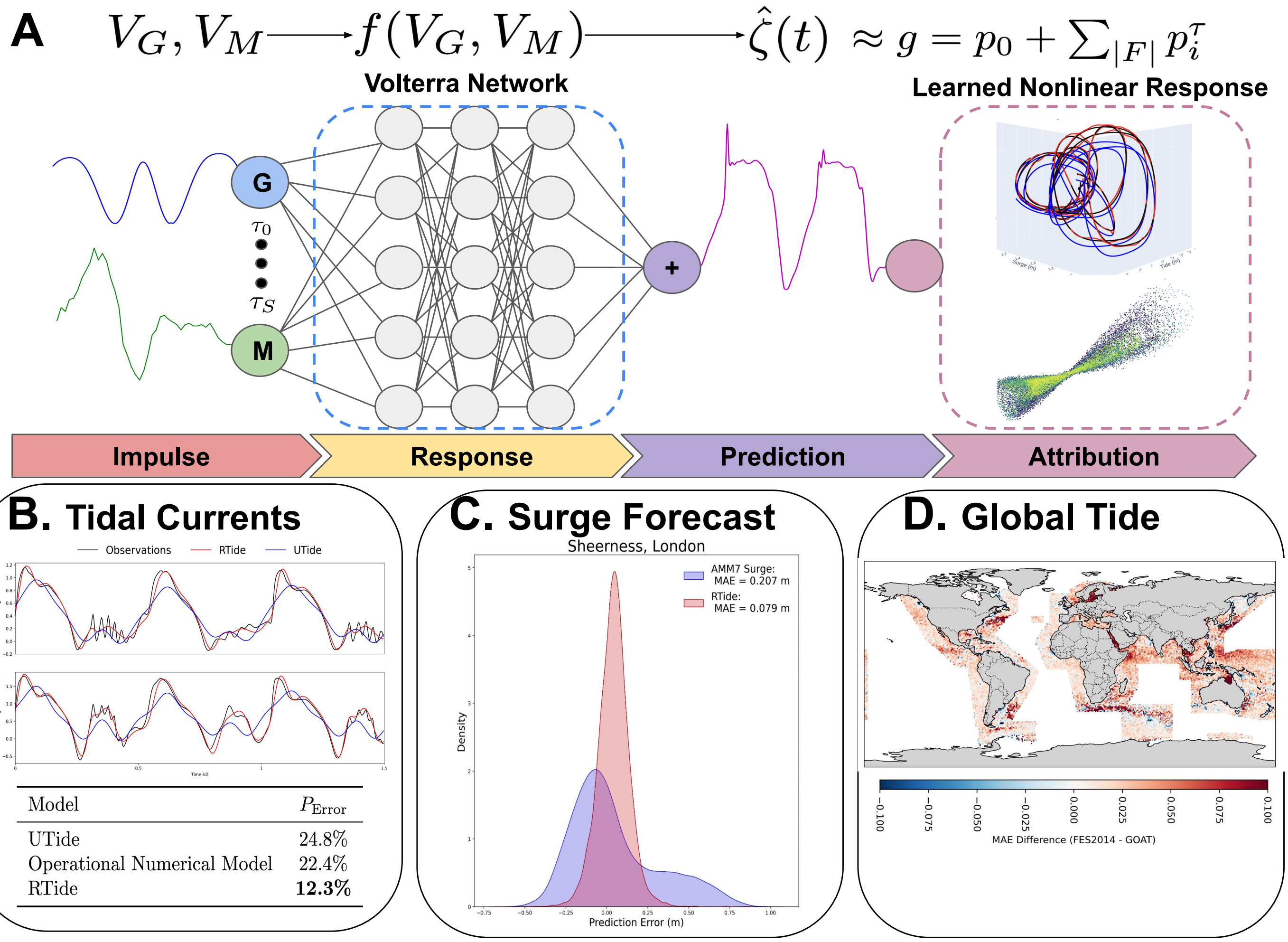
While numerical simulation is not typically regarded as data-driven, data is fundamental to the success of these models in providing a set of initial conditions, constraints, and for model calibration. It is well known that the propagation of tidal processes and shallow-water flows is depth-dependent, as can be seen in the shallow-water equations which approximate these flows. As described above, in many regions this data is either of low quality or is unavailable. While observational data exists, it may be temporally sparse as with satellite observations, or if measurement devices are damaged or moved. Both limitations present challenges for conventional machine learning. This work answers the question of what to do under such conditions.

RTide Framework. The approach I develop stems from the recognition that the ocean's response to time-varying forcing – which drives tidal processes – is both time-invariant and weakly nonlinear. This means that if we can learn the underlying time-invariant response function, we can predict sea-level changes at any time, provided the forcing is known. For astronomical tides, predicting the forcing is straightforward, as it derives directly from Keplerian mechanics. With modern meteorological and hydrological models, predicting other types of forcing has also become routine. However, despite the theoretical foundations laid in the 1960s by Walter Munk and David Cartwright, attempts to exploit this relationship have historically failed to account for nonlinearities in the oceanic response. This is because the nonlinear response is described by a Volterra series, whose terms increase exponentially with the order of interaction, making classical estimation intractable:

$$\hat{\zeta}(t) = \underbrace{\sum_{s} w(s)c(t-\tau_s)}_{\text{Linear response}} + \underbrace{\sum_{s}\sum_{s'} w(s,s')c(t-\tau_s)c(t-\tau_{s'})}_{\text{Bilinear response}} + \dots \tag{1}$$

Here, $w(s)$ are the weights for time-lag s, and $c(t)$ is the forcing. The rapid growth in terms with increasing nonlinearity, and when considering the interactions between input forcing, renders manual specification of these interactions *a priori* infeasible, particularly when studying new phenomena. My work overcomes this by exploiting the equivalence between Volterra series and a class of neural networks known as Volterra Networks. By embedding these networks within an impulse-response framework (Figure 2), the model learns both the weights and the structure of the Volterra kernels directly from observational data via gradient descent, without requiring explicit specification of physical interactions beforehand [4].

Once trained, the model weights can be utilized to predict tidal processes purely from input forcing, never seeing sea-level observations directly during prediction. This contrasts with conventional machine learning approaches, which map inputs to outputs empirically rather than learning a time-invariant physical operator. Moreover, the causal structure of the impulse-response framework allows the learned physical interactions to be interpreted: in fact, the exact Volterra kernels can be recovered analytically from the network weights via a Taylor expansion of the activation functions. For example, when trained on meteorological data, the model accurately recovers the localized inverse barometric pressure effect – something unattainable through direct regression of barometric pressure against sea-level due to the presence of multiple correlated inputs. This generalized framework can be applied to any tidal process where appropriate forcing functions are specified. We demonstrate novel applications to



Model	P_{Error}
UTide	24.8%
Operational Numerical Model	22.4%
RTide	**12.3%**

Figure 1: **Schematic of the proposed ML Response framework [4].** The input consists of the lagged Gravitational (V_G) and Multivariate (V_M) input functions. The oceanic response function $f(V_G, V_M)$, is learned by a three-layer neural network (Volterra network) and used to predict the tidal process $\hat{\zeta}$ at any time instance t. Learned models can then be taken apart and studied by transforming into an equivalent linear model g using SHapley Additive feature exPlainers which quantify the contribution of each feature p_i^{τ} to the final prediction. **B** Example 1.5 day tidal current predictions between UTide (classic separable) and RTide (new coupled) models. Table shows comparisons of predicted power error P_{Error}. **C** Forecast residuals from 2021-2022 for AMM7 Surge (UK Operationally forecast model), and RTide (using AMM7 as input function). **D** Comparisons of average coastal tide error between FES2014 (assimilative tide model), and GOAT (my model). Red = GOAT better, blue = FES better.

storm surge, tidal rivers, and interactions with changing mean sea-level, marking a significant advance in both operational oceanography and in our ability to study the dynamics of these processes.

Extension to Tidal Currents. Tidal currents are a special case of tidal processes owing to the fact that they are multidimensional. Classical tidal analysis treats the orthogonal tidal velocity components as independent. However, these components are in fact coupled, with the strength of this coupling increasing in more nonlinear regimes (e.g. fast moving coastal flows). I developed the first coupled model for tidal currents based on the response framework described above [3], with the predicted tidal current response $\hat{B}(t)$ to input forcing $c(t)$ given by

$$\hat{\beta}(t) = \begin{bmatrix} \hat{u}(t) \\ \hat{v}(t) \end{bmatrix} = \sum_s W(s)\, c(t - \tau_s), \text{ with } W(s) = \begin{bmatrix} x_{\vec{x}}(s) + y_{\vec{x}}(s) & x_{\vec{x}\to\vec{y}}(s) + y_{\vec{x}\to\vec{y}}(s) \\ x_{\vec{y}\to\vec{x}}(s) + y_{\vec{y}\to\vec{x}}(s) & x_{\vec{y}}(s) + y_{\vec{y}}(s) \end{bmatrix} \tag{2}$$

Where the weights W at each time-lag s now given by the 2 x 2 matrix where the arrow denotes the interaction of the $\vec{x}$ or $\vec{y}$ component with its orthogonal counterpart. If the two components are independent then the classical approach can be recovered as a diagonal matrix, however, this approach allows the coupling between terms to be accounted for. The network based RTide approach can take on an equivalent form to this by using a shared set of network weights to predict both velocity components simultaneously, again retaining exact equivalence to the physical operator. The coupled framework is able to capture significantly more of the embodied energy – proving that the tidal current signal has significantly more predictable variance than previously thought, garnering a 30% improvement over the classical theory shown in Figure 2 Panel B. We also demonstrate the coupled approach yields superior power predictive accuracy for tidal energy sites to classical theory, and numerical models, using around an order of magnitude less data. As will be discussed in the engineering impact section, this has significant implications for tidal energy site operations.

Augmentation of numerical models A key contribution of this work has been in developing an approach which can *augment* existing numerical models rather than replace them. Traditional physics-based models, constrained by computational limits, exhibit systematic errors from both resolution limitations (missing small-scale processes) and neglected physics (e.g., waves, fluvial interactions). To address this, I developed a scheme that uses numerical model output as an additional forcing input for the RTide model. This allows RTide to learn a time-invariant, coupled response between the numerical model and true observations, compensating for neglected nonlinearities while the numerical model captures external processes. Unlike traditional post-processing, this physically grounded framework provides predictions without real-time observations. When applied to the

UK's AMM7 model, this hybrid approach reduces errors by an average of 47% and is operationally deployed in the Netherlands. Figure 2 Panel C compares forecast residuals over the year 2021-2022 at Sheerness, London, used to inform Thames Barrier operations.

Global Forecasting All of the preceding approaches are applicable for single locations where in-situ observation devices exist. However, in many regions globally, particularly in the low- middle-income countries for which numerical models are deficient, these observations are at their sparsest. To overcome this, I advance an approach which can make use of a rich, but underutilized resource for storm surge forecasting; satellite altimetry. While over 30 years of global data exists, the temporal sparsity of these data introduces significant challenges as repeat observations of a given location may be separated by weeks to months. I developed an extension to the response theory which treats the oceanic response function as a continuous spatial function. This function is learned by a type of neural operator which parameterizes the time-invariant response function using a second network. By imposing this spatial coherence, the approach can be trained on sparse altimetric observations without requiring measurement binning for the first time. Comparisons of the new Global Operator-based Altimetric Tide model (GOAT) with the current state of the art global assimilative tide model (FES2014) demonstrates a 14% reduction in average error even when trained on less than 1% of all available satellite data (Figure 2 Panel D). Notably, this is the first pure empirical model to exceed the performance of assimilative tide models.

2.1 Additional work

My thesis also makes major contributions to the study of astronomical tides by improving harmonic analysis, which models tides as a sum of fixed-frequency waves driven by astronomical forcing. Classic approaches assume Gaussian noise and often underestimate uncertainty, particularly in sparse, noisy satellite altimetry data. I developed a spatially coherent variational Bayesian harmonic analysis that reduces these inaccurate assumptions about both the tidal signal and the noise, enabling robust uncertainty quantification and outlier detection [5]. Two major applications include: (i) state-of-the-art tidal estimates in complex coastal and estuarine regions using only 90 days of SWOT data, supporting the next-generation global tide model with uncertainty estimates; and (ii) the discovery of the earth-shaking seiche in East Greenland after the 2023 mega-tsunami, the first direct observation of such a phenomenon from space [2]. See Figure 2 panel D for results. This work has identified novel fluid mechanics relating to the dissipation of seiches due to changing stratification which is being pursued in a new study.

2.2 Scientific Contributions and Engineering Impact

This thesis hosts several scientific firsts. I developed the first automated framework for analysing and predicting arbitrary tidal processes, including the first coupled response theory of tidal currents, demonstrating that much more variance in tidal current signals is predictable than previously thought. I was also the first to show that the oceanic response function is spatially coherent, enabling the use of altimetric data for storm surge forecasting. Finally, my discovery of a seiche using variational Bayesian harmonic analysis constitutes the first observation of persistent fluid sloshing without an external driver lasting over a week. Beyond these contributions, the methods developed here have the potential to greatly accelerate climate studies and improve their accuracy, as well as enhance our understanding of compound flooding events.

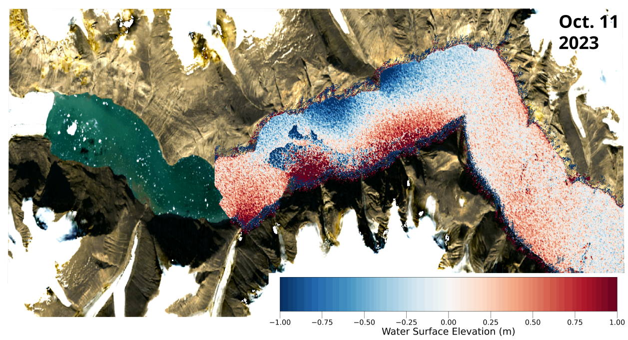


Figure 2: **Processed SWOT observations of the earth-shaking seiche in the Dickson Fjord, Eastern Greenland [2].** Data are corrected for tides using the spatially coherent variational Bayesian harmonic analysis procedure.

The approaches developed in this thesis have already had global engineering impact. The response method is implemented in the open-source RTide Python package, now used by agencies including NASA, the UK National Oceanography Centre, the Dutch Hydrographic Service, the Environment Agency, the Met Office, and others. RTide is operationally deployed in the Dutch storm surge forecast system and is being introduced into UK operational forecasts, protecting over £300 billion in assets and millions of lives. It is also under consideration to replace forecasts at more than 600 secondary ports in the UK. A pilot project with NASA GSFC and Google is underway to implement RTide for coastal flood forecasts in Chesapeake Bay, USA. Additionally, the improved accuracy and lower data requirements of the RTide coupled tidal currents model can significantly enhance tidal energy resource assessments. RTide has demonstrated state-of-the-art results relative to the operational model at Meygen, the world's largest tidal stream energy site, where efforts are ongoing to implement it operationally [1].

3 Selected Publications and Output

In total, 7 peer-reviewed publications have resulted from this thesis, 5 of which are already published, with the remaining 2 in second-round reviews. My discovery of the seiche that shook the world [2], published in Nature Communications, received global media coverage across 91 news outlets and holds the sixth-highest Altmetric score among Nature Communications papers in 2025. This work has been presented at nine international conferences, and I have been an invited speaker at the UK National Oceanography Centre, the Met Office, the Alan Turing Institute, and several universities.

References

[1] T. Monahan, T. Tang, S. Roberts, and T. Adcock. Using a machine learning enhanced implementation of the response method for the prediction of tidal currents. In *Proceedings of the European Wave and Tidal Energy Conference*, Sept. 2025. Accepted.

[2] T. Monahan, T. Tang, S. Roberts, and T. A. A. Adcock. Observations of the seiche that shook the world. *Nature Communications*, 16(1):4777, 2025.

[3] T. Monahan, T. Tang, S. Roberts, and T. A. A. Adcock. Response-based prediction of tidal currents. *Journal of Geophysical Research: Oceans*, 2025. 2nd round of reviews.

[4] T. Monahan, T. Tang, S. Roberts, and T. A. A. Adcock. Rtide: Automating the tidal response method. *Journal of Geophysical Research: Machine Learning and Computation*, 2(2):e2024JH000525, 2025.

[5] T. Monahan, T. Tang, S. Roberts, and T. A. A. Adcock. Tidal corrections from and for SWOT using a spatially coherent variational Bayesian harmonic analysis. *Journal of Geophysical Research: Oceans*, 130(3):e2024JC021533, 2025.