University of Bremen

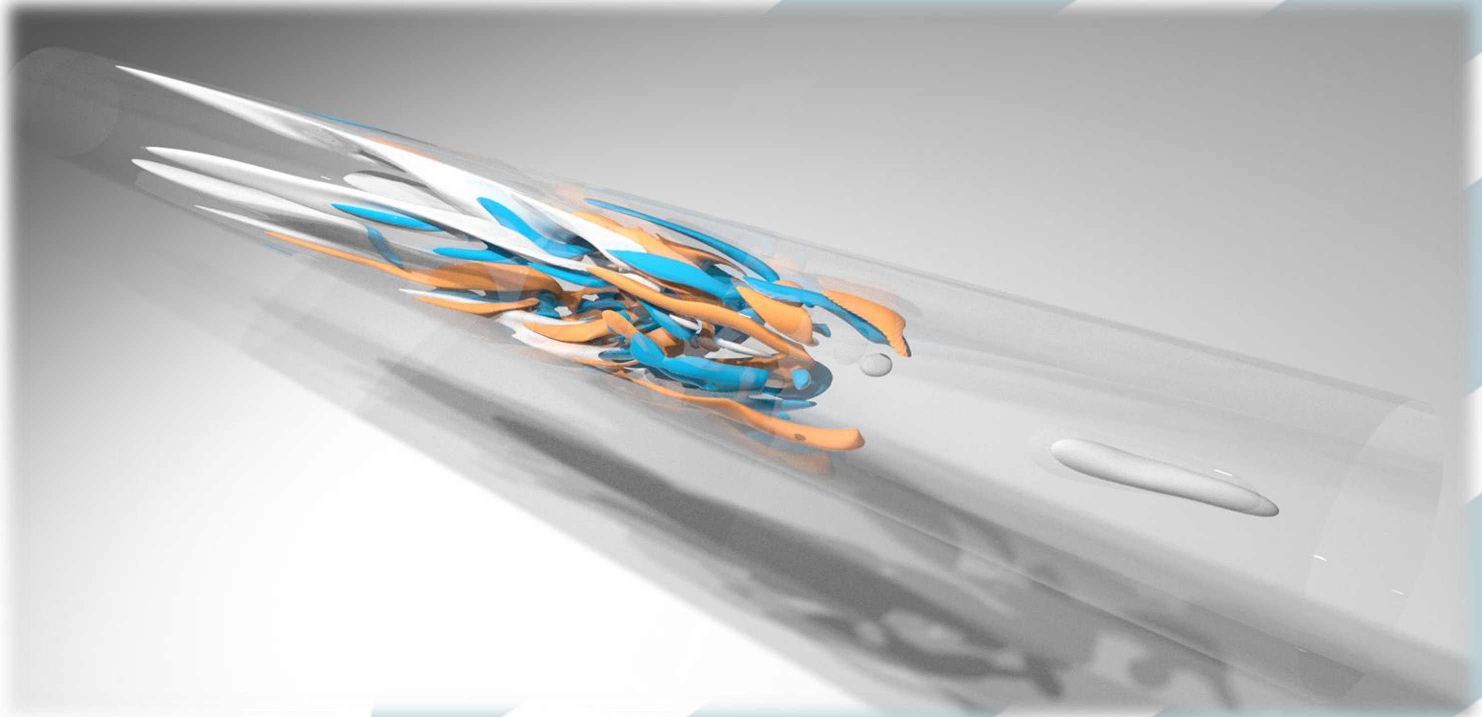
CENTER OF
APPLIED SPACE TECHNOLOGY
AND MICROGRAVITY

ZARM

THE TRANSITIONAL REGIME OF PULSATILE PIPE FLOW

DANIEL MORÓN MONTESDEOCA

ERCOFTAC Autumn Festival 2025



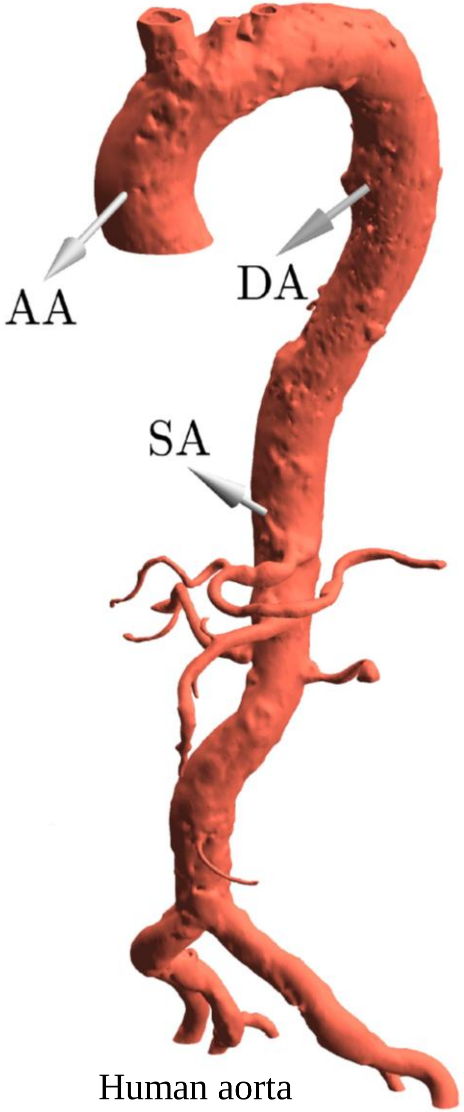
Turbulence in the cardiovascular system

The presence of irregular blood flow patterns or **turbulence** in the aorta has been linked with **cardiovascular diseases**.

<u>Still not clear under which conditions the flow may be turbulent!</u>

1. Re in the transitional regime!

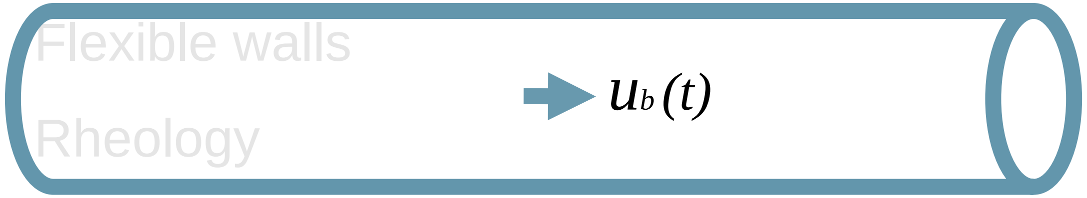
2. Difficult to measure/model



Human aorta

Modelling cardiovascular flows

1. Unsteady driven (pulsatile)
2. Complex geometry
3. Flexible walls
4. Rheology
5. …

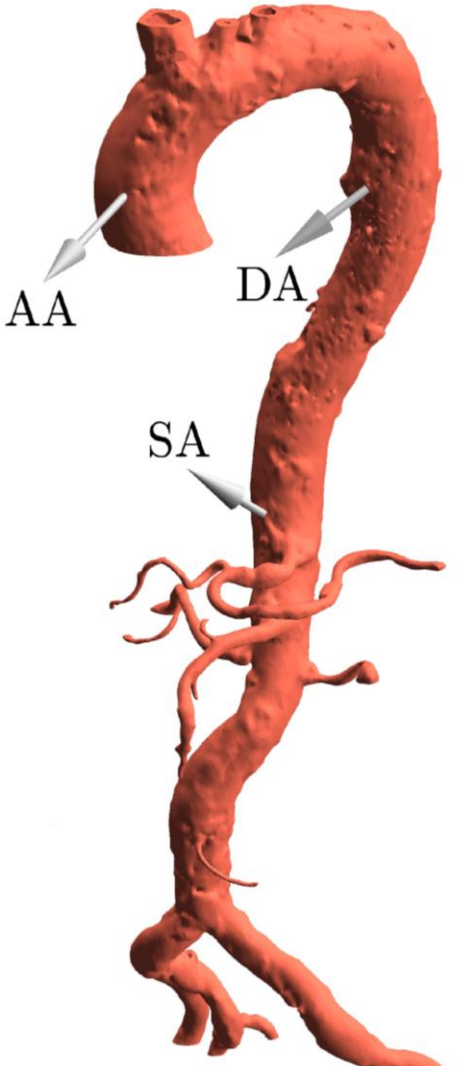


The Model:

- ✓ Cylindrical
- ✓ Rigid pipe
- ✓ Newtonian Fluid

What is the effect of pulsatile driving on
1. turbulence transition?
2. turbulence behavior?

I use DNS
Github with the
open source full GPU pipe flow code!



Human aorta

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Contents

1. Flow regimes in steady pipe flow

Time-Averaged bulk velocity

Pipe diameter

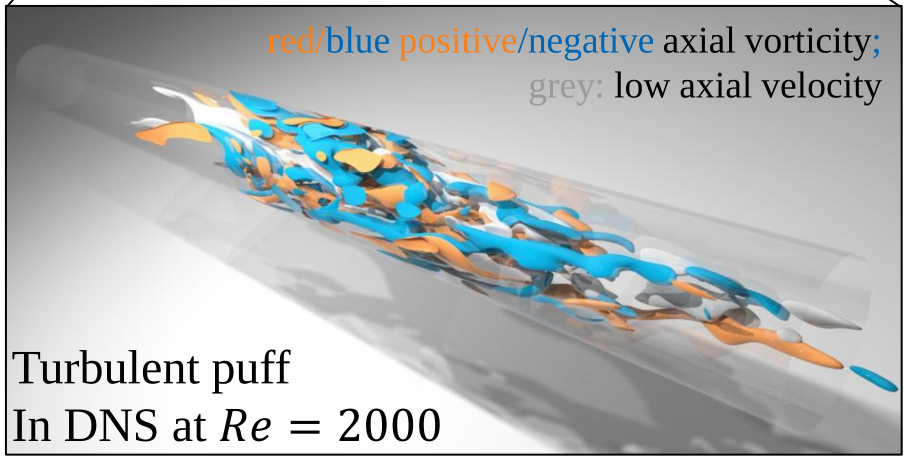
$$Re = \frac{UD}{\nu}$$

Kinematic viscosity

- **Laminar** flow: $Re \leq 1800$
- Transitional flow: $1800 \leq Re \leq 3000$
- Fully **turbulent** flow: $Re \geq 3000$

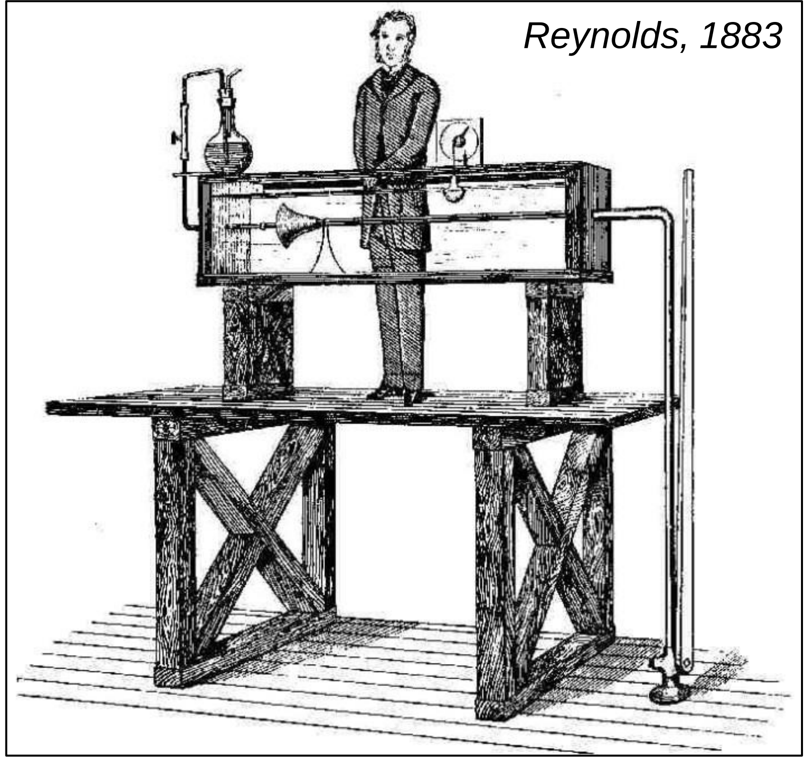
For a sufficiently perturbed pipe!

016)



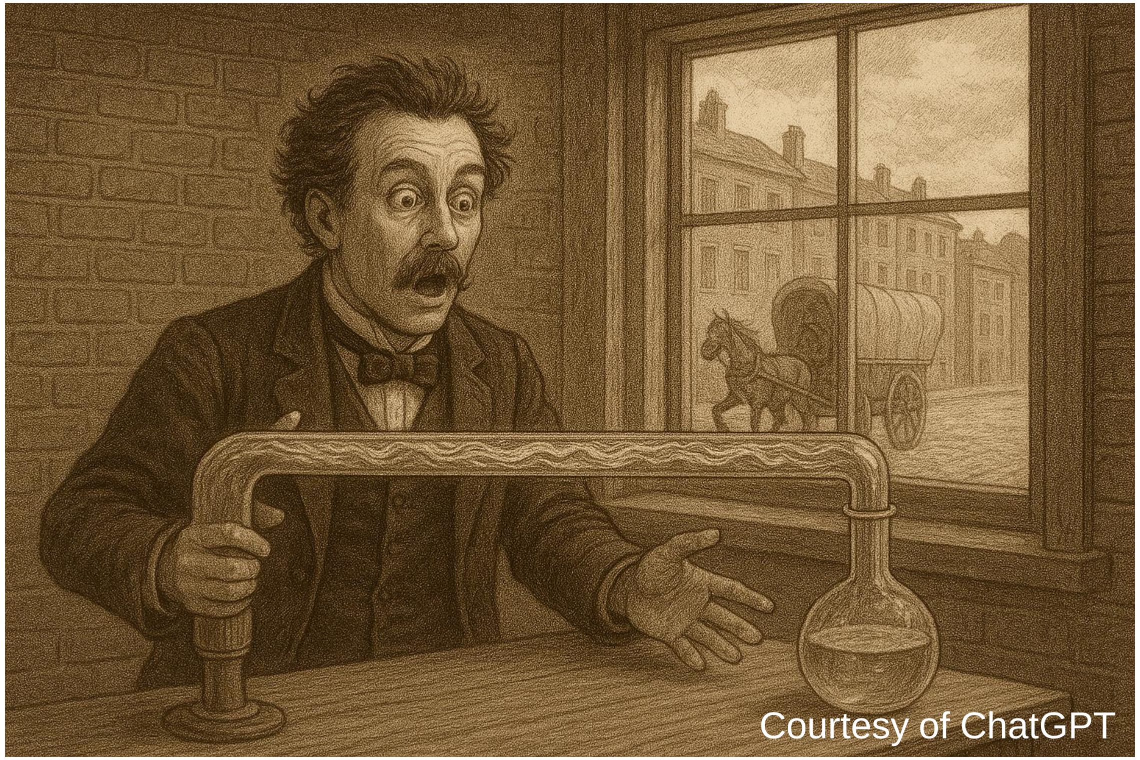
1. Transition to turbulence in steady pipe flow

By improving his experimental set-up Reynolds managed to get laminar flow up to $Re \approx 12000$



Reynolds, 1883

However, any **sufficiently big perturbation**, such as the vibrations caused by a wagon, could trigger turbulence at $Re < 12000$

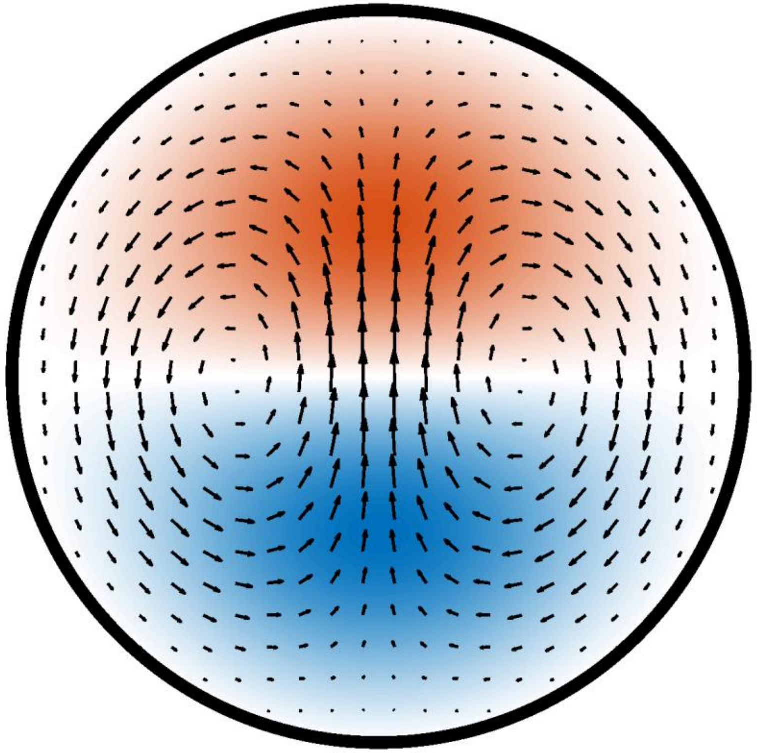


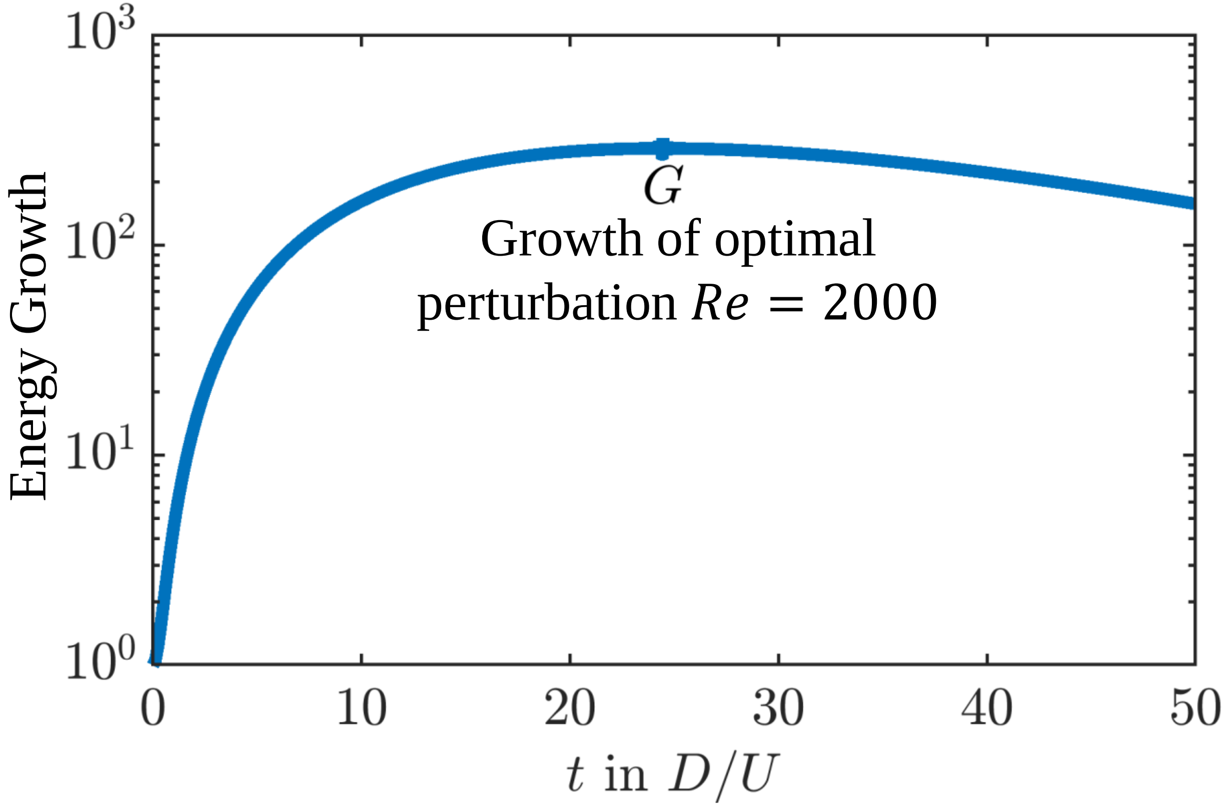
Courtesy of ChatGPT

1. Linear analysis

Pipe flow is linearly stable for all $Re \leq 1e7$ (Meseguer 2003)

Perturbations can grow (linearly) transiently! Total growth depends on its initial shape



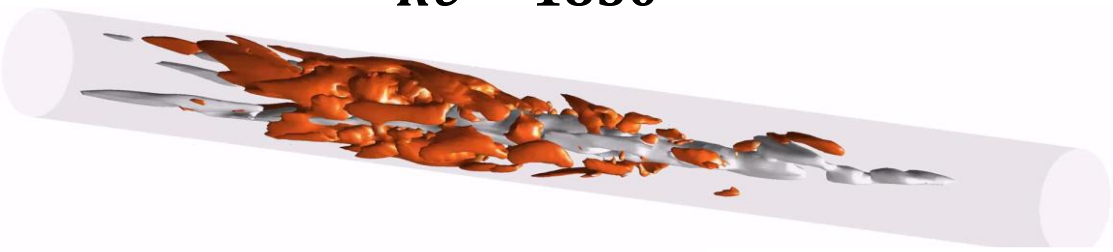


1. Transitional regime in steady pipe flow

Puffs **decay** at low $Re \leq 2040$, or **split/elongate** at $Re > 2040$

$\boldsymbol{Re = 1850}$

red: turbulence indicator
grey: low axial velocity



Co-moving reference frame

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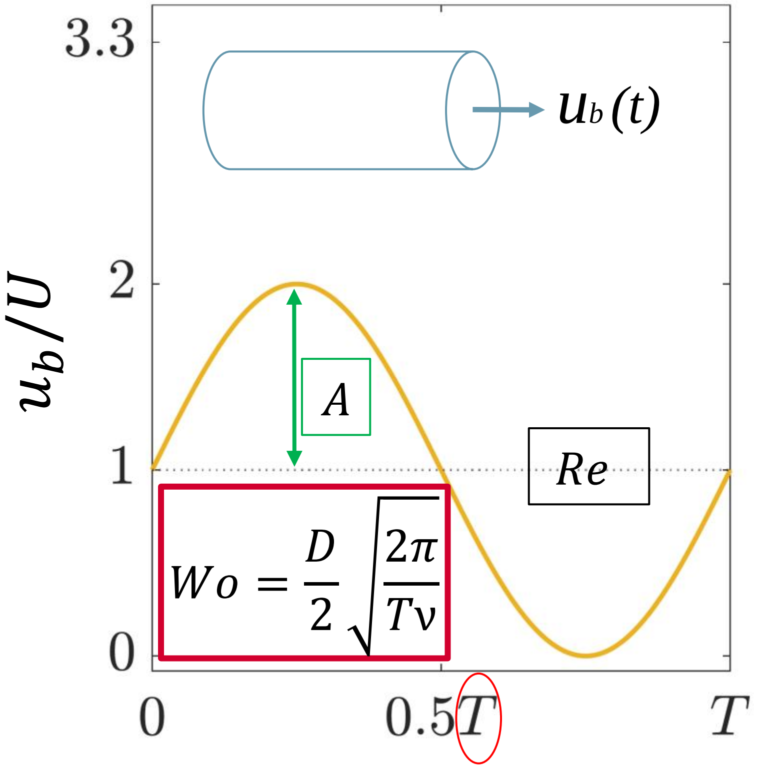
2. Pulsatile Pipe Flow

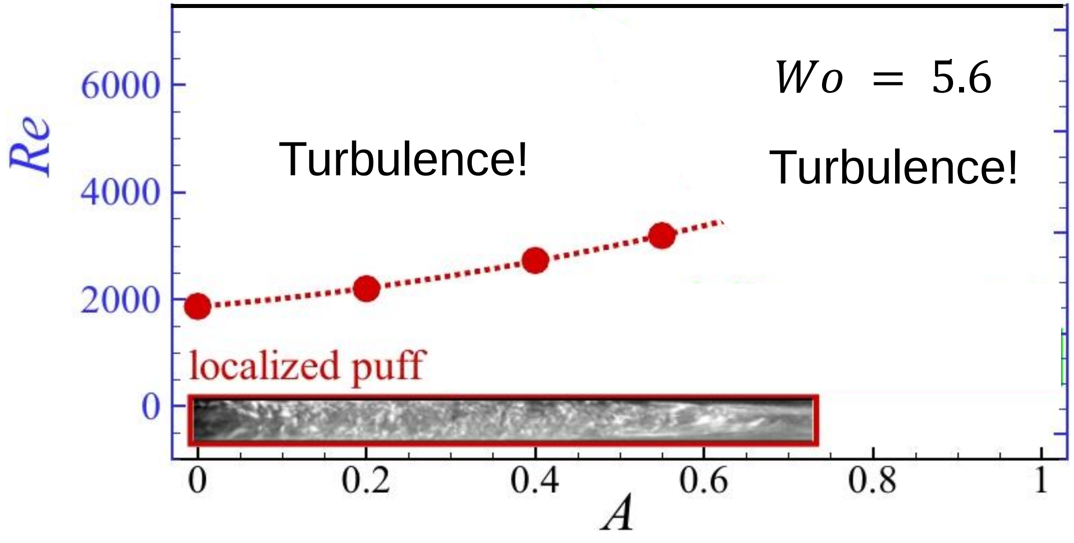
The flow depends on three parameters: $\boldsymbol{Re}, \boldsymbol{Wo}$ and waveform

I only consider $\mathbf{1000 < \boldsymbol{Re} < 3000}$ | $\mathbf{5 < \boldsymbol{Wo} < 22}$ | $\mathbf{0.5 \leq \boldsymbol{A} < 3}$

At these parameters transition occurs at low $\boldsymbol{Re}$.



D. Xu, A. Varshney, X. Ma, B. Song, M.Riedl, M. Avila, B.Hof , PNAS 2020



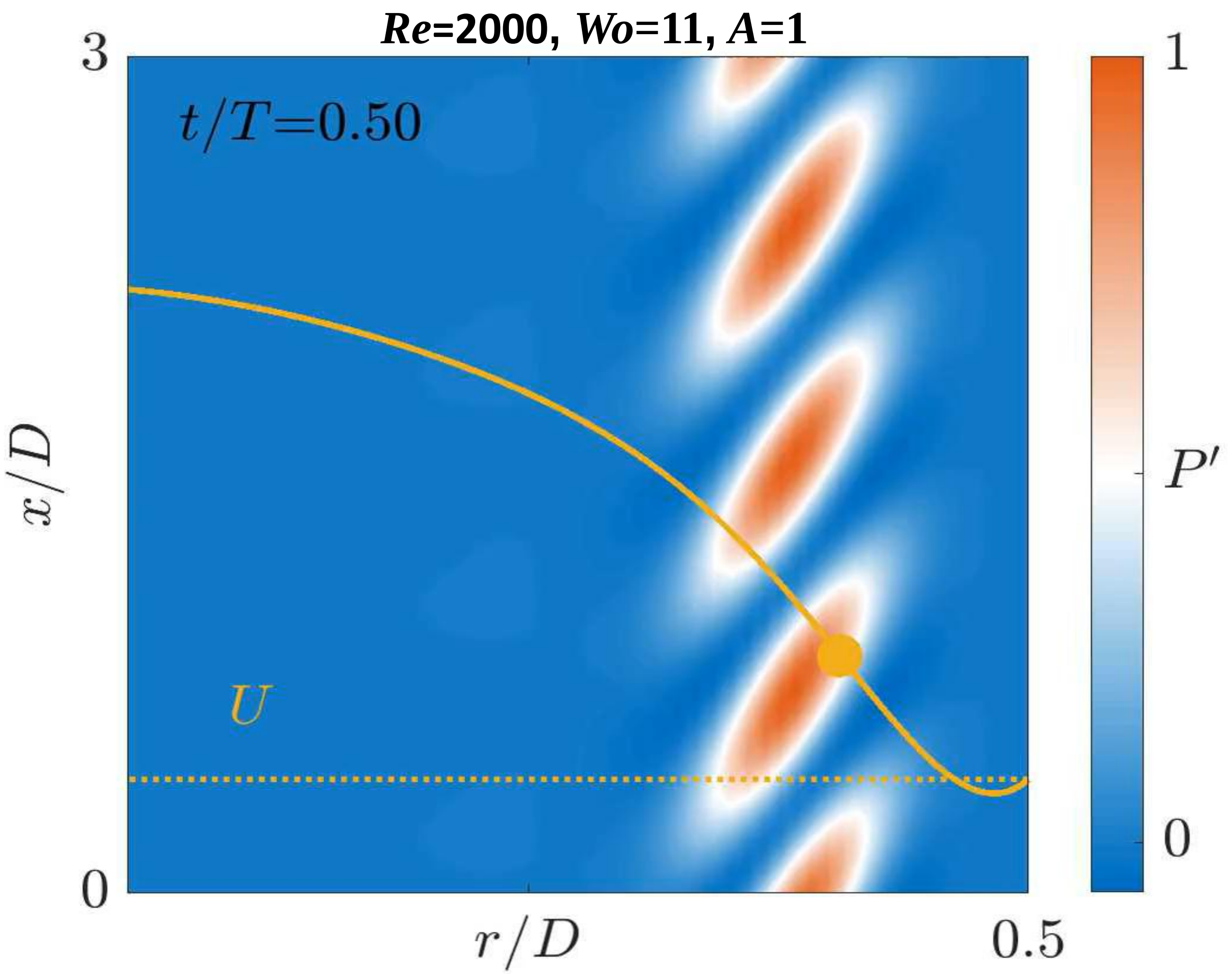
2. Transient growth of perturbations

We look for the **initial condition** that can **grow** (G) the most on top of the flow.
The optimal perturbation is helical. G (max. growth) depends on the parameters.

Re**=2000,** Wo**=11,** A**=1**

2. Transient growth of perturbations

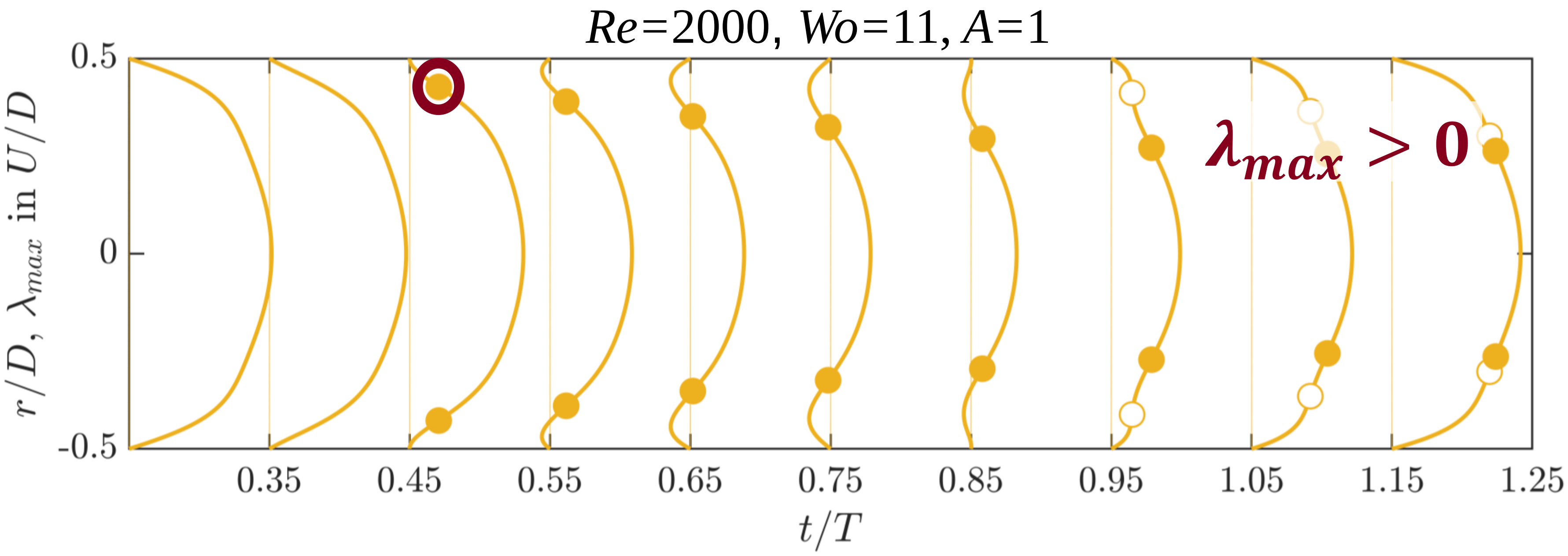
Turbulent **production sticks** to the radial location of **inflection points in the laminar profile**



2. Origin of the helical perturbation

Inflectional profiles are **linked** with **inviscid instabilities (Rayleigh, Fjørtoft)**

At sufficiently high $Re, Wo > 5, A > 0.5$: $\boldsymbol{\lambda_{max}} > 0$!! (Relevant for cardio. Flows!)



The key is the difference on time scales between the (slow) laminar profile and (fast) perturbations.

Effect of waveform on turbulence transition in pulsatile pipe flow, Morón et.al, JFM (2022)

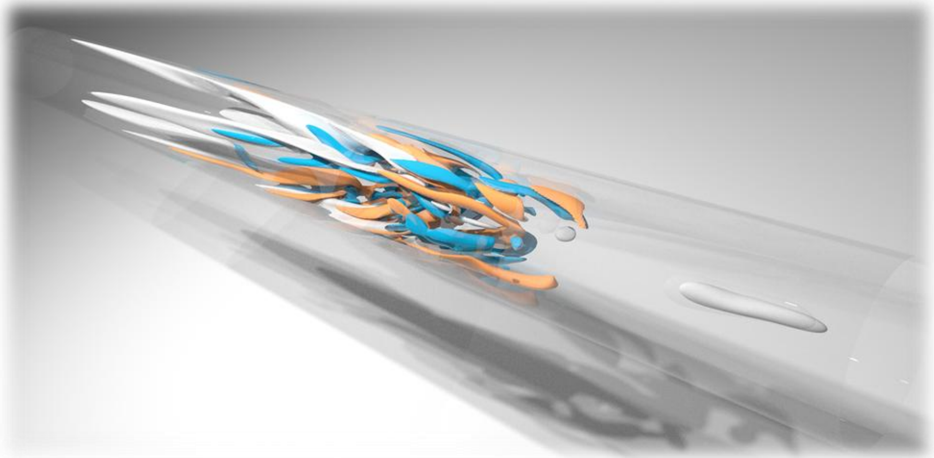
Contents

3. Triggering turbulence in pulsatile pipe flow

Direct Numerical Simulation (DNS) at Re=2000, Wo=11, A=1

3. Puffs in pulsatile pipe flow

The first long lived structures are
localized turbulent puffs



Puff at $Re = 2400, Wo = 11, A = 1.4$

Behavior depends on the flow parameters:

3. Are inflection points important for turbulence?

I perfomed master-slave pairs of DNS:
1 with 1 without inflection points
in the mean profile

At $A \geq 0.5$ and $5 \leq Wo \leq 14$ **puffs** make use of **inflection points** to **survive!**

Thus turbulence also takes **advantage of instabilities** to survive low Re phases of the period!

3. Reduced-order Model

I **extended the model** by Barkley (2015) at $Wo > 5, 0.5 \leq A \leq 1$

I added two physical mechanisms:
1) **instantaneous instability**
2) a **time lag** between **turbulence** and **driving**!

The model **reproduces** all the **behaviors** in pulsatile pipe flow in a broad parametric regime

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4. Conclusions

1) **Transition** depends on **instantaneous instabilities**

2) **Transition** is more likely to happen during flow **deceleration**

Instability → **Growth** → **Turbulence**

3) Turbulence appears as **localized puffs**

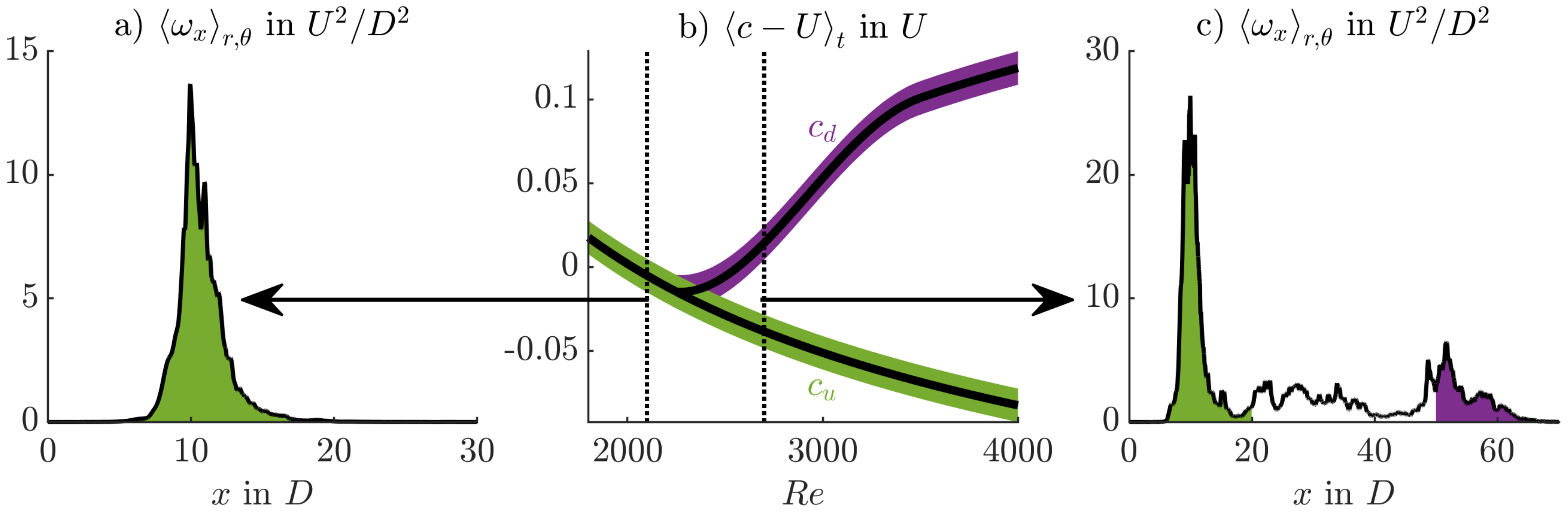
4) A simple **reduced-order model** reproduces their dynamics

Open access codes and databases can be found in pangaea.de

Check my PhD thesis!

Github with the GPU pipe flow code!

Appendix: Puffs front speeds



1.Model of puffs in SSPF

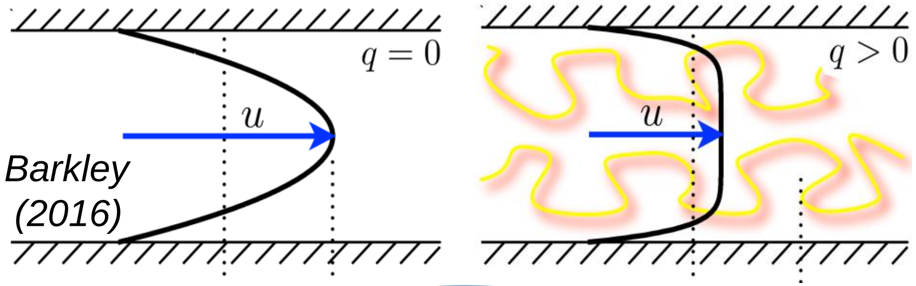
Barkley (2015, *et al.*)

Only **axial direction** and **time** (x,t)

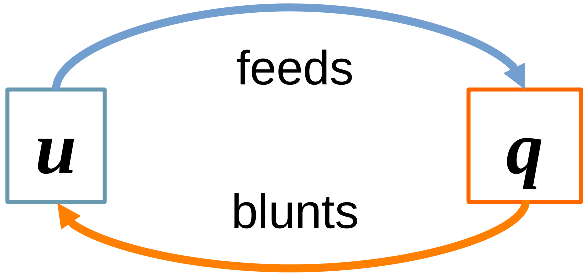
$\boldsymbol{q} \rightarrow$ local turbulent intensity

$\boldsymbol{u} \rightarrow$ local mean shear

Fits the **front speed** of puffs



Barkley (2016)



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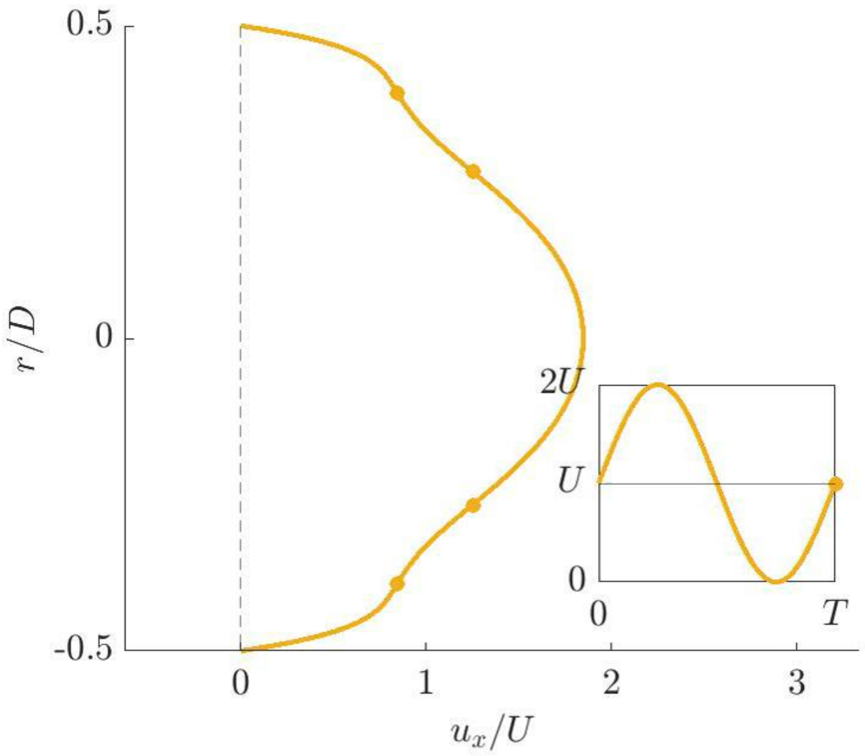
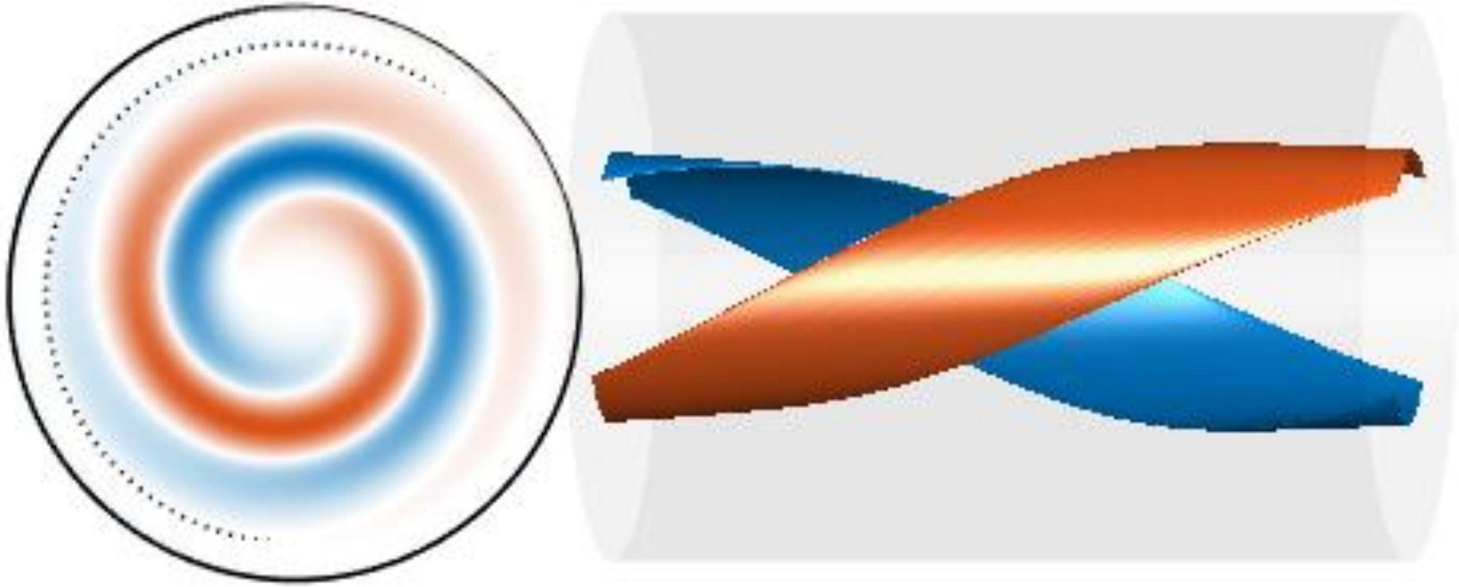
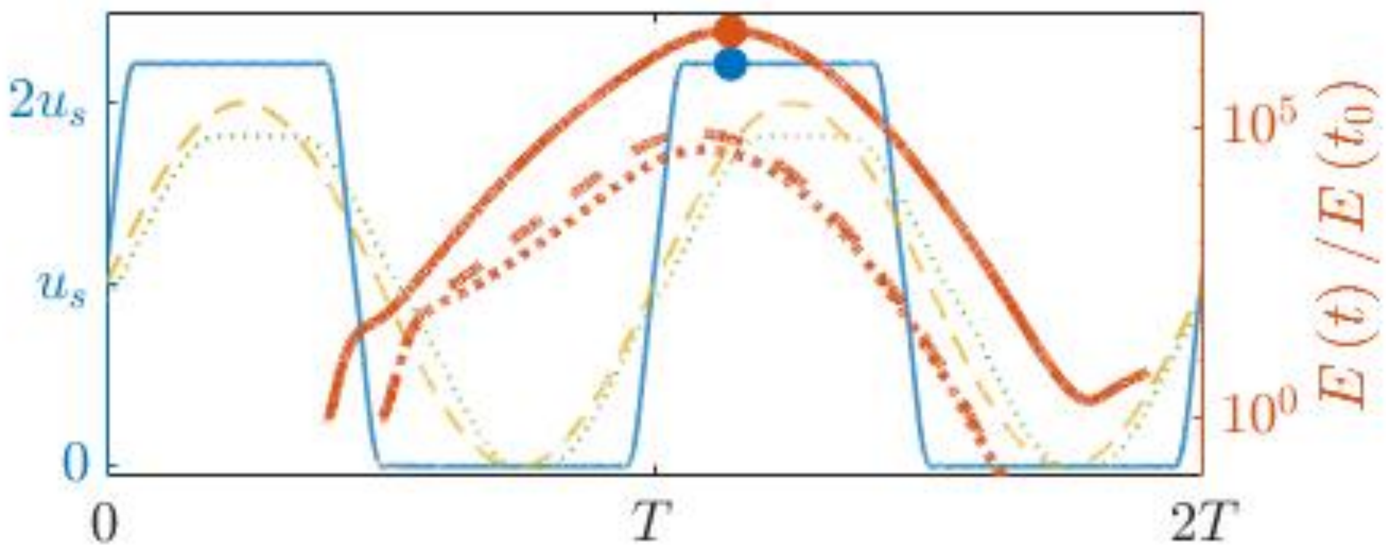
***Re*=2000, *Wo*=11, *A*=1**



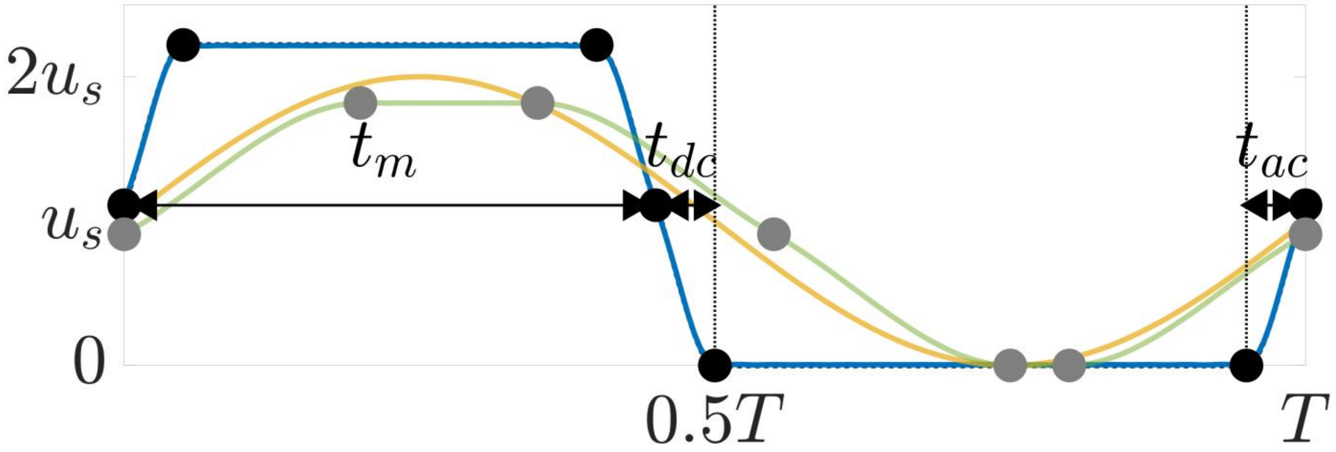
Appendix:





Appendix: effect of waveform on growth

Huge parametric analysis tells us:



How to increase perturbation growth (*G*):

Increase ***Re***

Maximal at ***Wo*≈7**

Decrease t_{dc}, t_{ac}, t_m

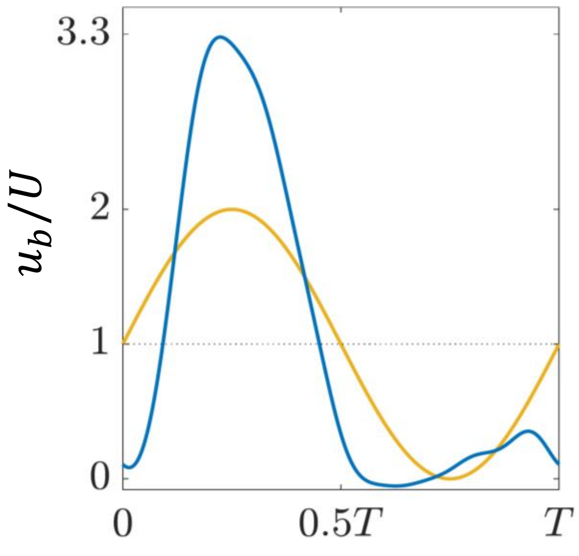
2.Effect of the waveform on transition

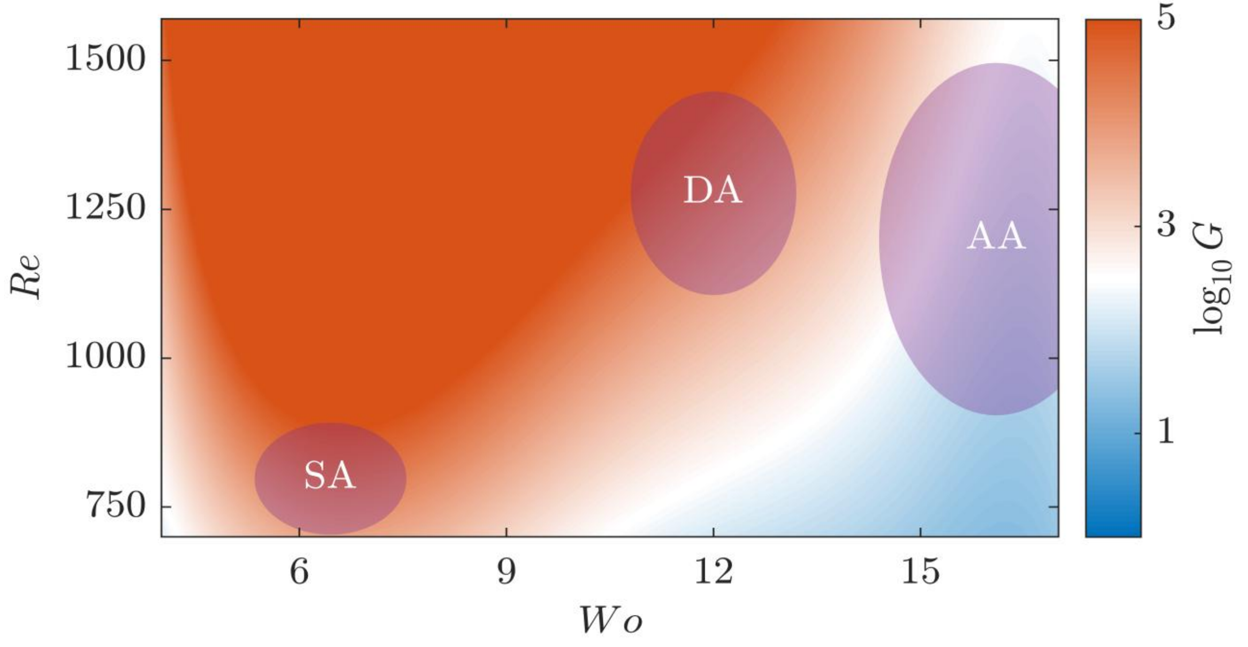
Transition depends on **the instantaneous instability**

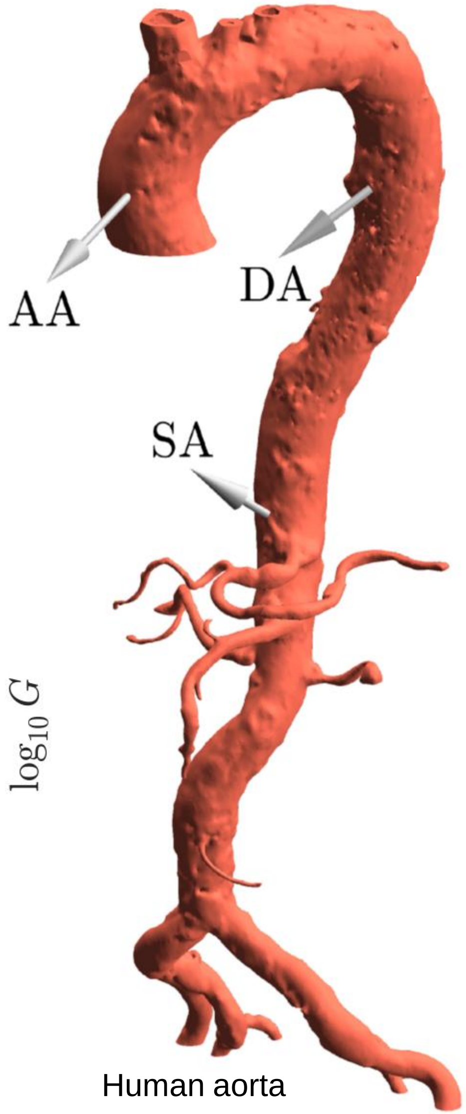
Instability **depends** on **parameters (*Re*, *Wo*, waveform)**

Instability → $G \approx C e^{2\lambda_i T}$ → **Turbulence**

Aorta waveform is highly susceptible to transition!

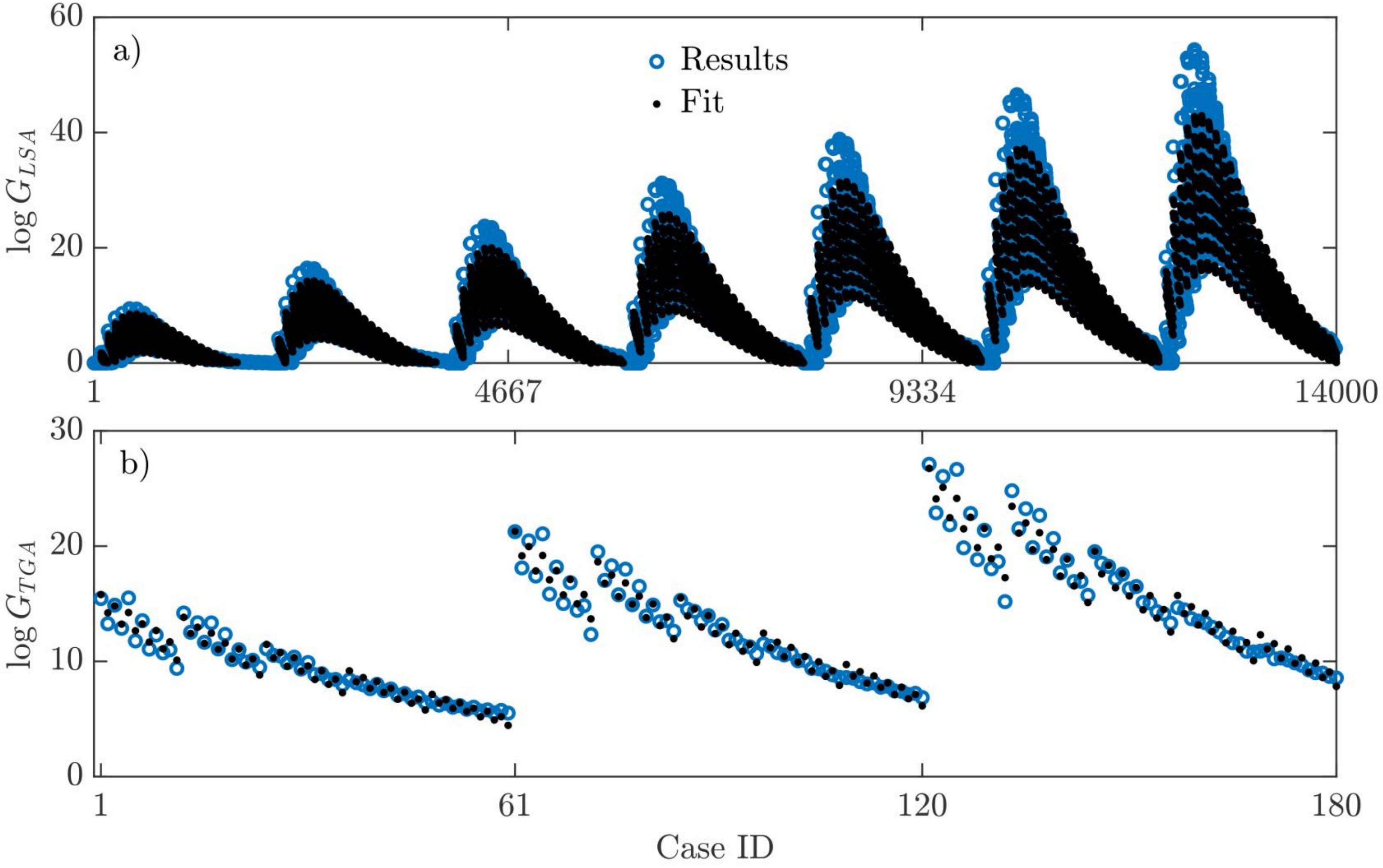




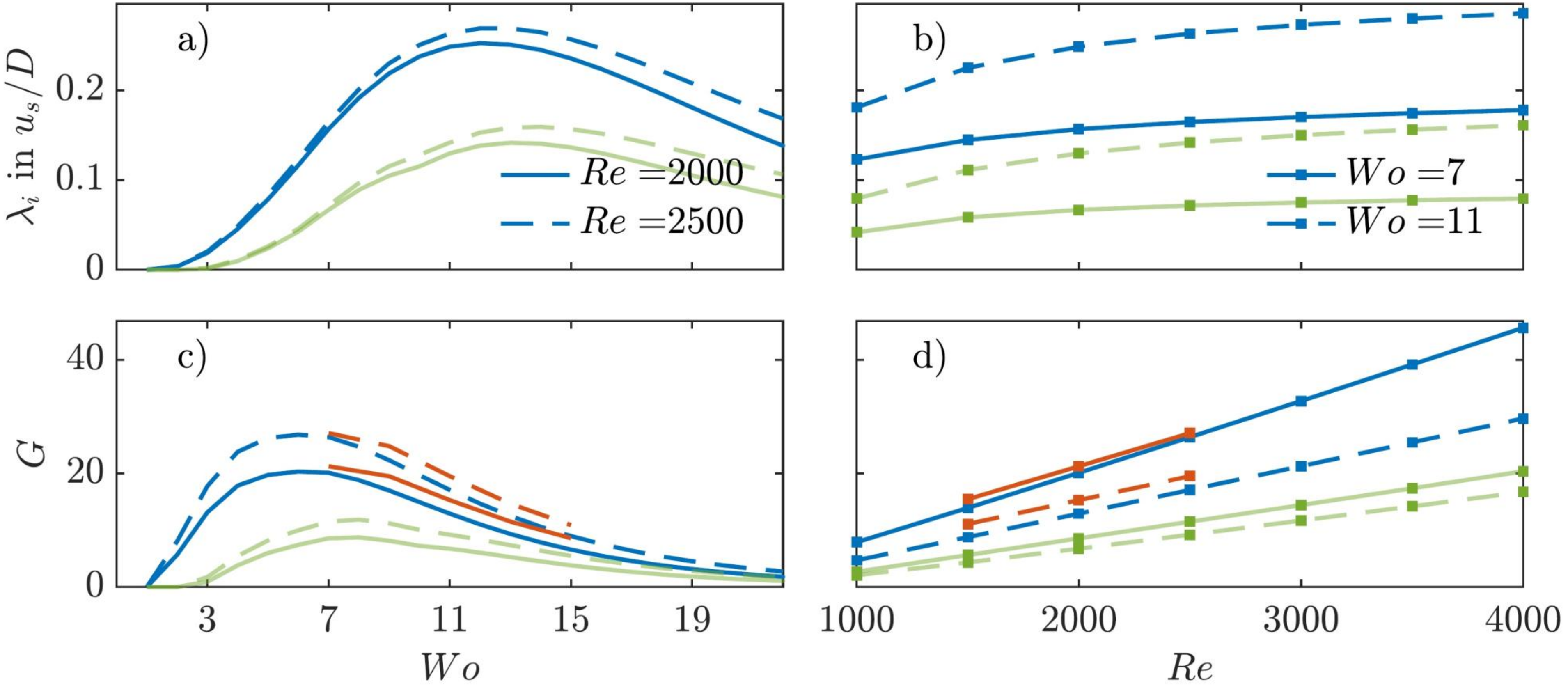


Human aorta

Appendix: LSA and waveform

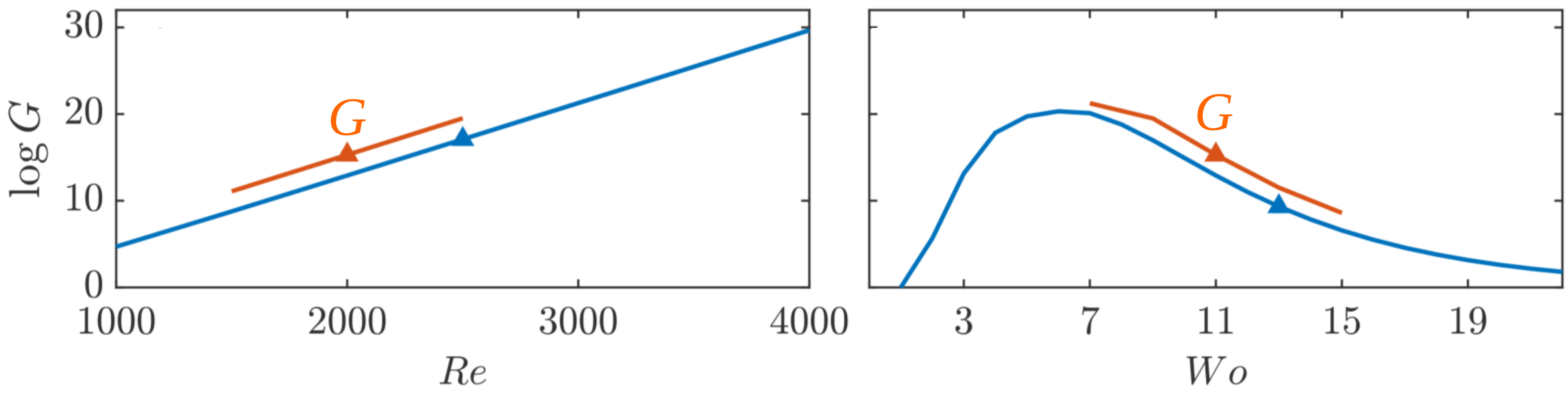


Appendix: growth of perturbations and waveforms



Appendix: TGA vs LSA growth

$Re = 2000, Wo = 11$



Inflection points render the laminar profile instantaneously **unstable**

Growth depends on:

- How **long** it is unstable
- How **much** unstable it is:

$$\lambda_i = \frac{1}{T}\int_{t_0}^{t_0+\Delta t_u} \lambda_{\max}(t)\,\mathrm{d}t$$

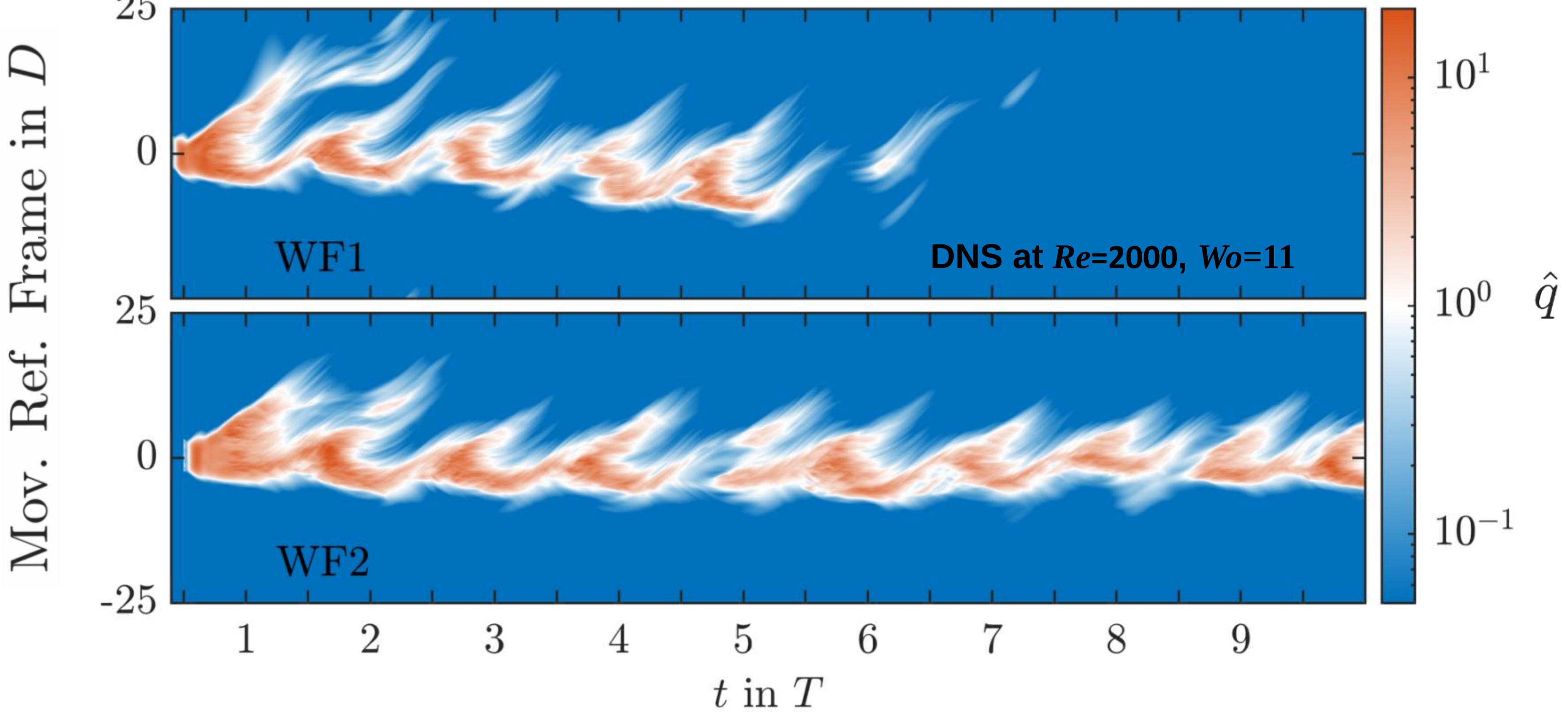
$$\frac{E_{\max}}{E_0} \propto \exp\left(2 \cdot \lambda_i \cdot T\right)$$

Good agreement with optimal perturbation transient growth

Appendix: Effect of waveform on puffs

Waveforms with rapid accelerations/decelerations (WF1) promote **transition** but also **turbulent decay**.

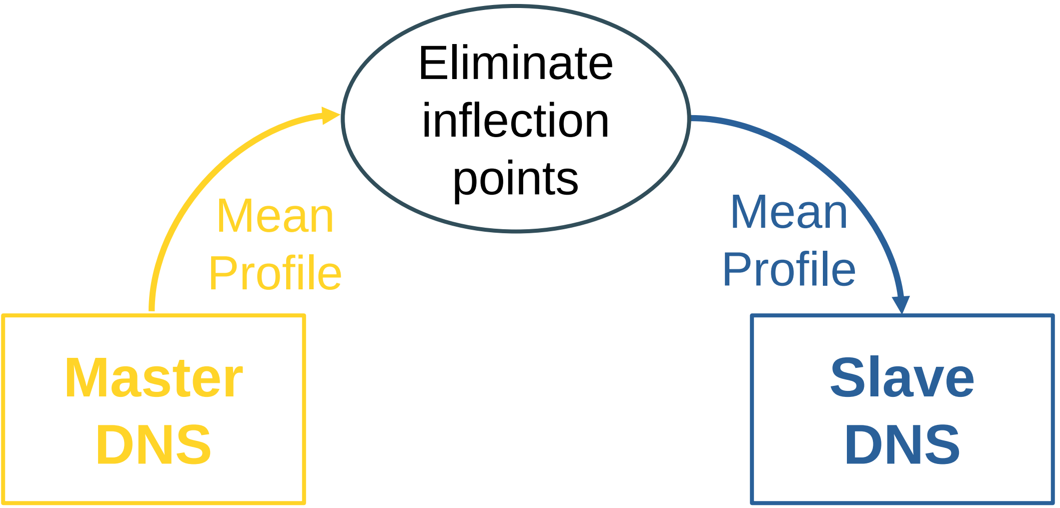
D. Morón, D. Feldmann, M. Avila, JFM 2022

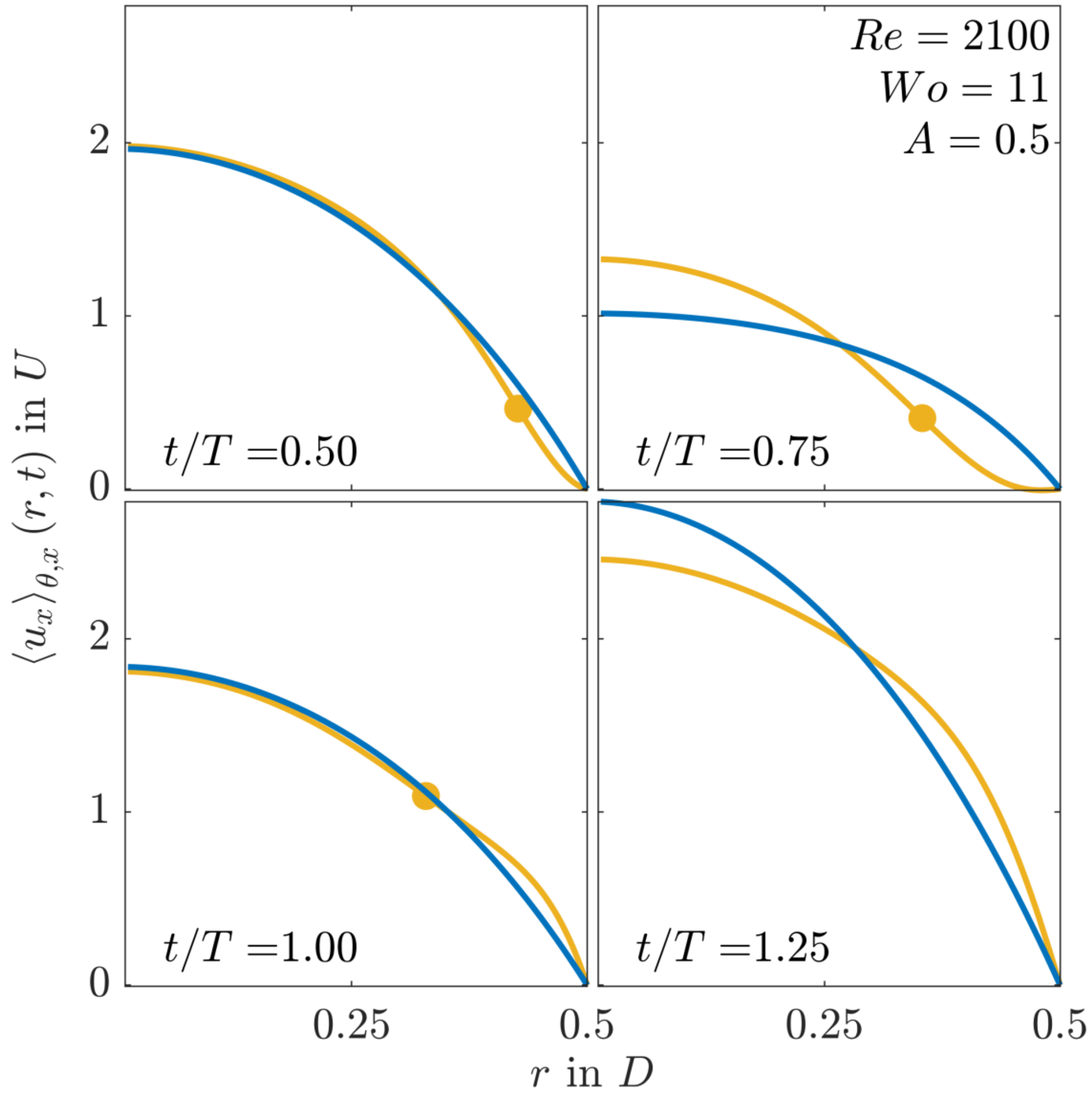


3.Effect of inflection points

I perform pairs of **DNS** with the same initial puff

- **1:** DNS of pulsatile pipe flow **(master)**
- **2:** DNS of pulsatile pipe flow with **artificial** mean profiles **without** inflection points **(slave)**





Appendix: Model of puffs in SSPF

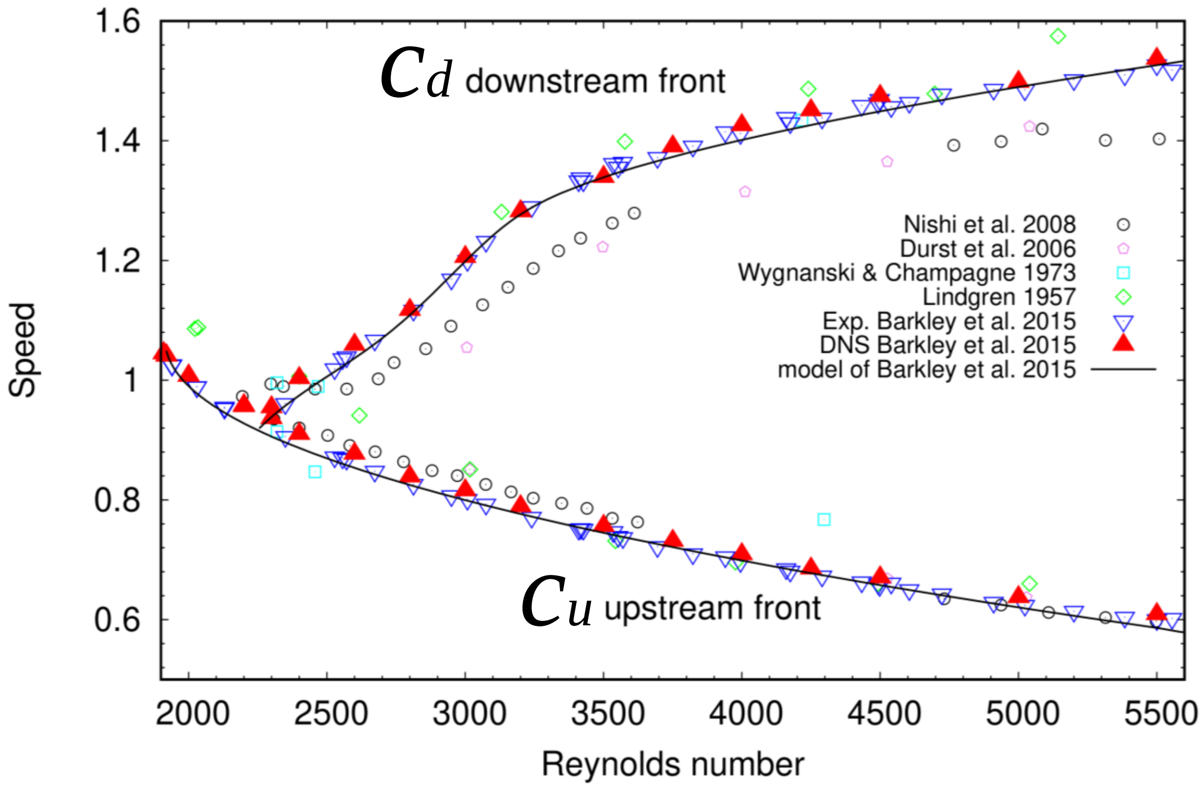
$$\frac{\partial q}{\partial t} + (u - \boxed{\zeta})\frac{\partial q}{\partial x} = f(q,u) + \boxed{D}\frac{\partial^2 q}{\partial x^2}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = g(q,u)$$

$$f(q,u) = q\left(\boxed{r} + u - U_0 - (\boxed{r} + \boxed{\delta})(q-1)^2\right)$$

$$g(q,u) = \boxed{\epsilon_1}(U_0 - u) + \boxed{\epsilon_2}(\bar{U} - u)q$$

$$\begin{tikzcd} \boxed{u} \arrow[r, bend left, "\text{feeds}"] & \boxed{q} \arrow[l, bend left, "\text{blunts}"] \end{tikzcd}$$



Appendix: Slave Profile

We minimize a functional at each time step

$$\mathcal{S} = 2\int_0^R \mathcal{L}\left(r, U_S, U_S'\right) \mathrm{d}r$$

$$\mathcal{L} = \frac{1}{2}{U'}_S^2 r + \lambda_L \left(U_S r - \frac{Q}{2R}\right) + \mu_L \left(\frac{1}{2}U_S^2 r - \frac{E_M}{2R}\right)$$

The first condition makes the shear monotonic

The second condition sets the bulk velocity.

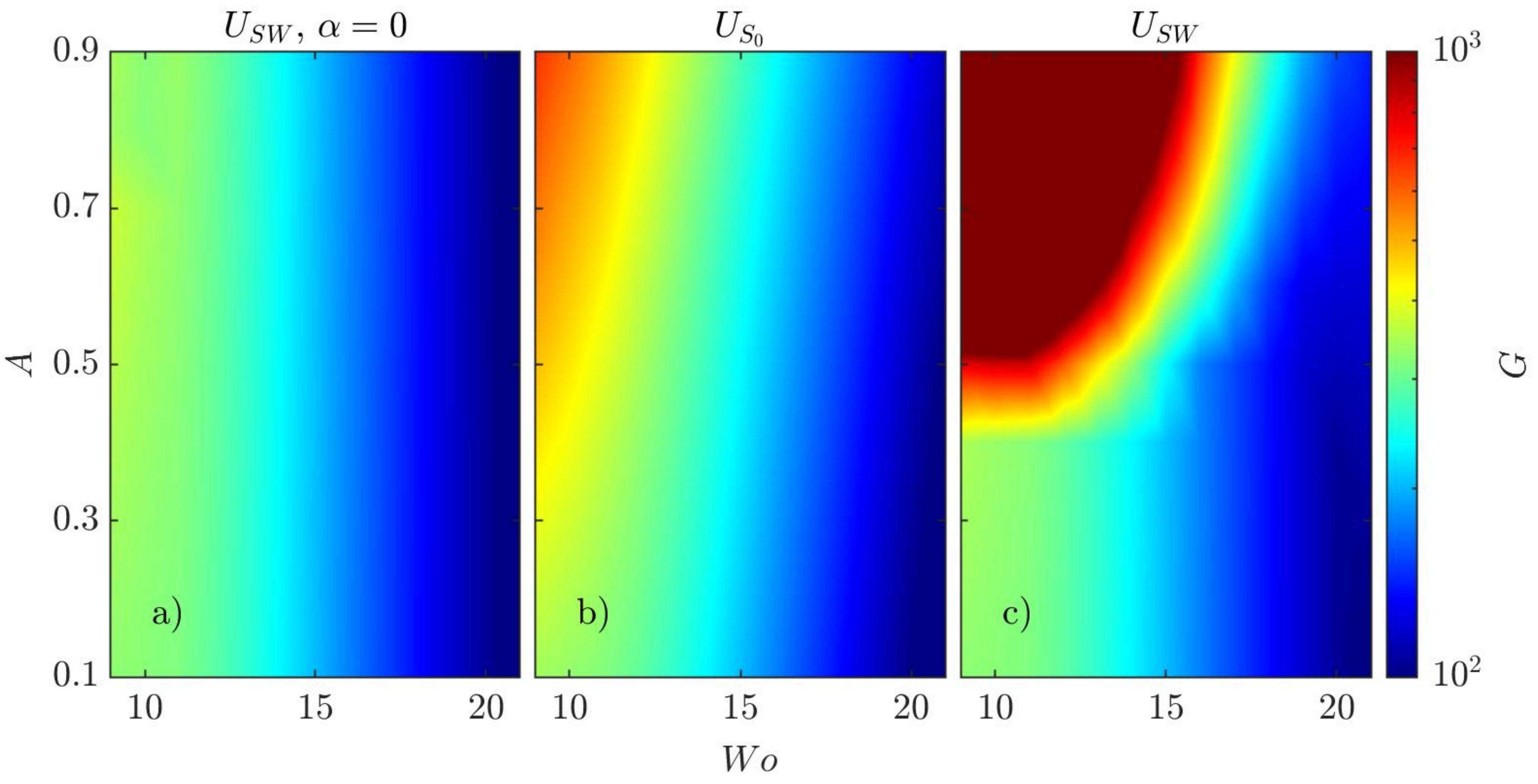
The third the energy of the profile

$$2\int_0^1 U_S r \mathrm{d}r\mathrm{d} = Q = \sqrt{\frac{3E_L}{2}}$$

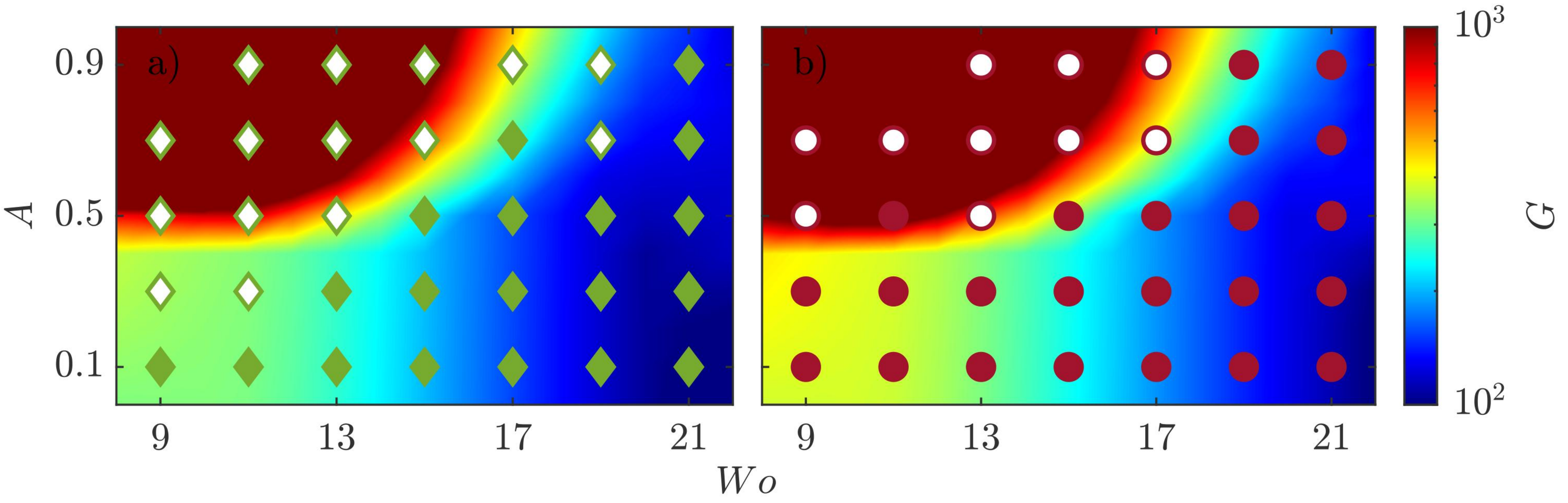
$$2\int_0^1 \frac{1}{2}U_S^2 r \mathrm{d}r\mathrm{d} = E_M$$

$$U_S = \frac{\lambda_L}{\mu_L}\left(\frac{I_0\left(\sqrt{\mu_L} r\right)}{I_0\left(\sqrt{\mu_L}\right)} - 1\right)$$

Appendix: Transient growth of master/slave profiles



Appendix: Puff survival in master/slave DNS



Appendix: Model for puffs in pulsatile pipe flow

$$\frac{\partial q}{\partial t} = -\left(u - \boxed{\zeta}\bar{U}(t)\right)\frac{\partial q}{\partial x} + f_{EBM}(q,u) + \boxed{D}\frac{\partial^2 q}{\partial x^2} + \sigma(Re)\,\tau(t,x)\,q$$

$$\frac{\partial u}{\partial t} = -u\frac{\partial u}{\partial x} + g_{EBM}(q,u)$$

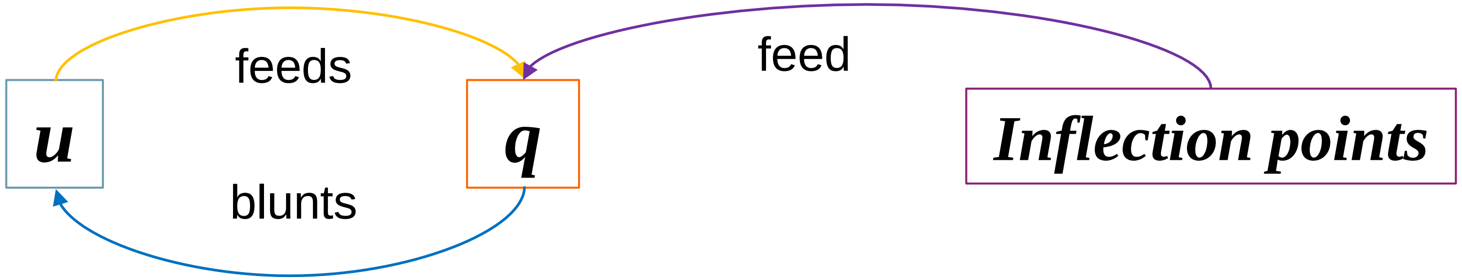
$$f_{EBM}(q,u) = q\left[\boxed{r}\bar{U}(t+\phi) + \boxed{\gamma\lambda} + u - U_c(t) - \left(\boxed{r}\bar{U}(t+\phi) + \boxed{\delta}\right)(q-1)^2\right]$$

$$g_{EBM}(q,u) = \boxed{\epsilon}\left(U_c(t) - u\right) + 2\boxed{\epsilon}\left(\bar{U}(t) - u\right)q + P_G(t) + F_{v_0}(t)$$

$$\boxed{1\frac{D}{U} \rightarrow 0.28\, t_c}$$

$$\boxed{\phi \propto \tan^{-1} Wo}$$

$$\boxed{\sigma \propto Re}$$



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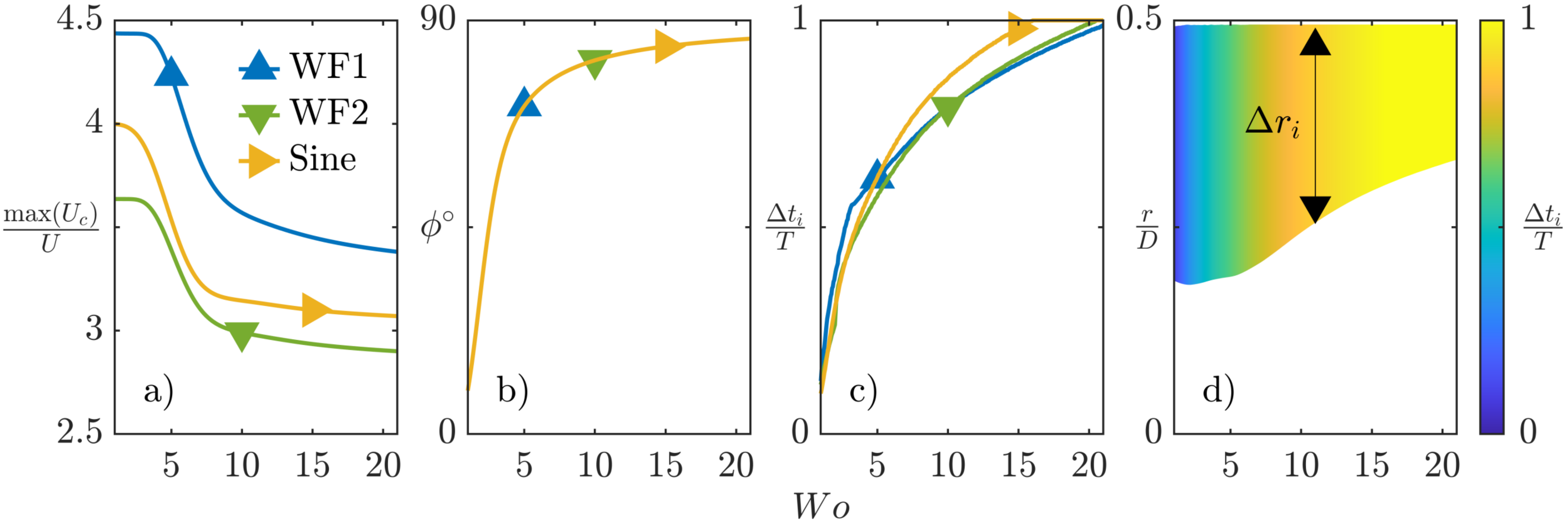
Appendix: Justification of phase lag in the model



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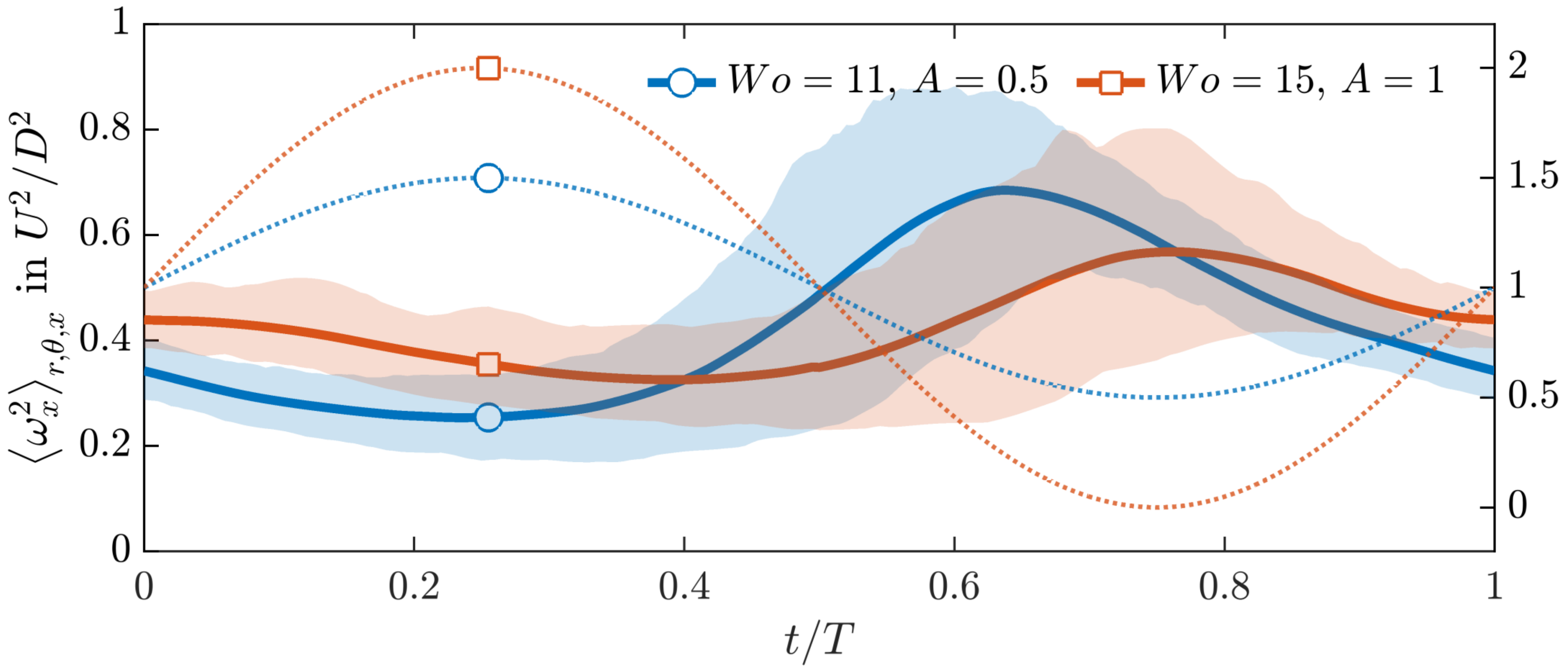

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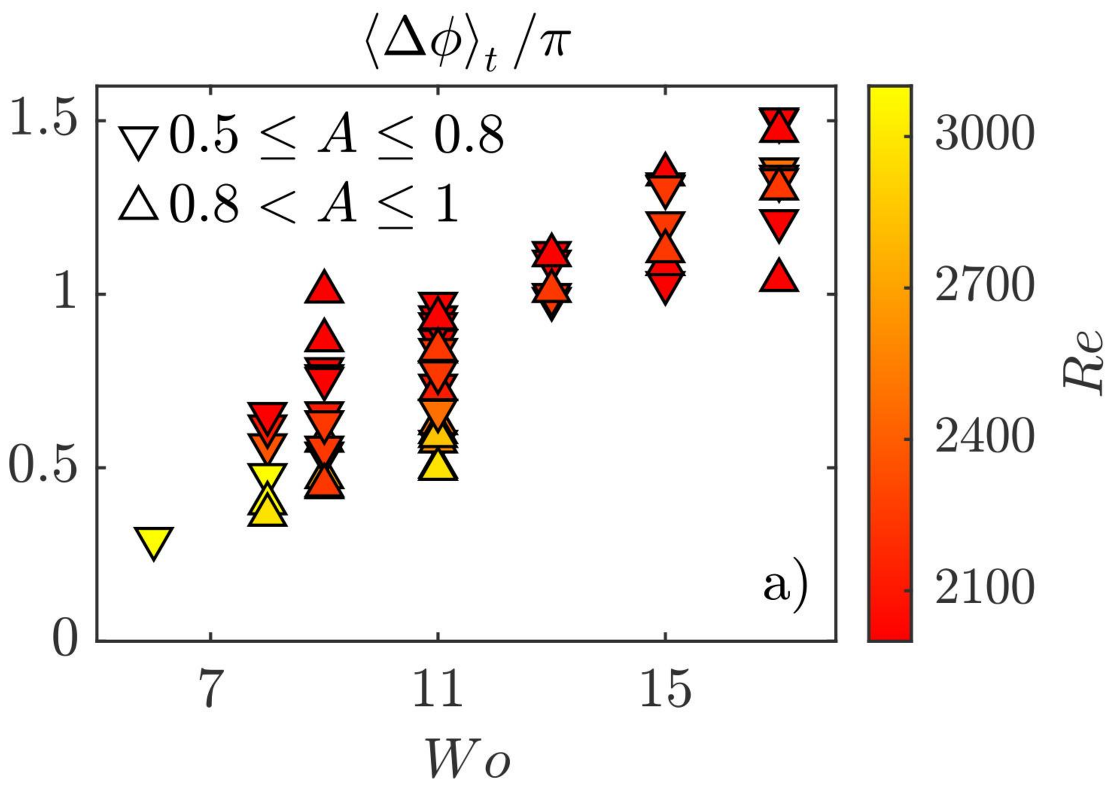
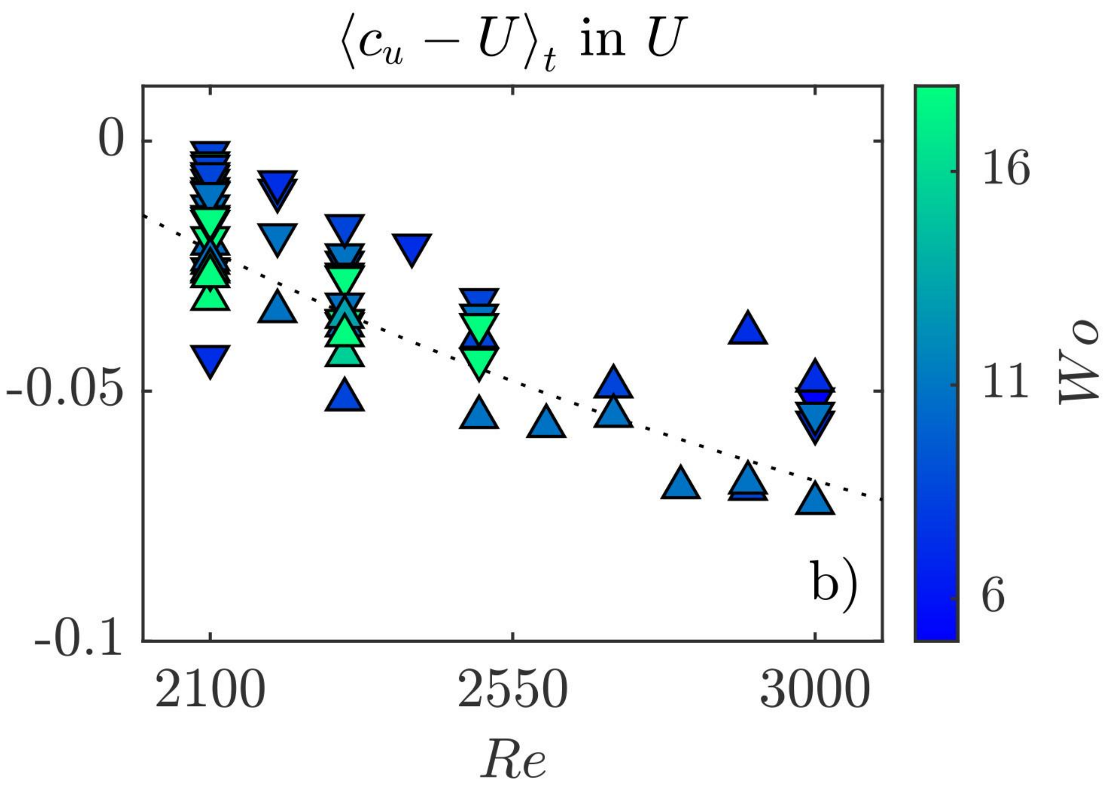
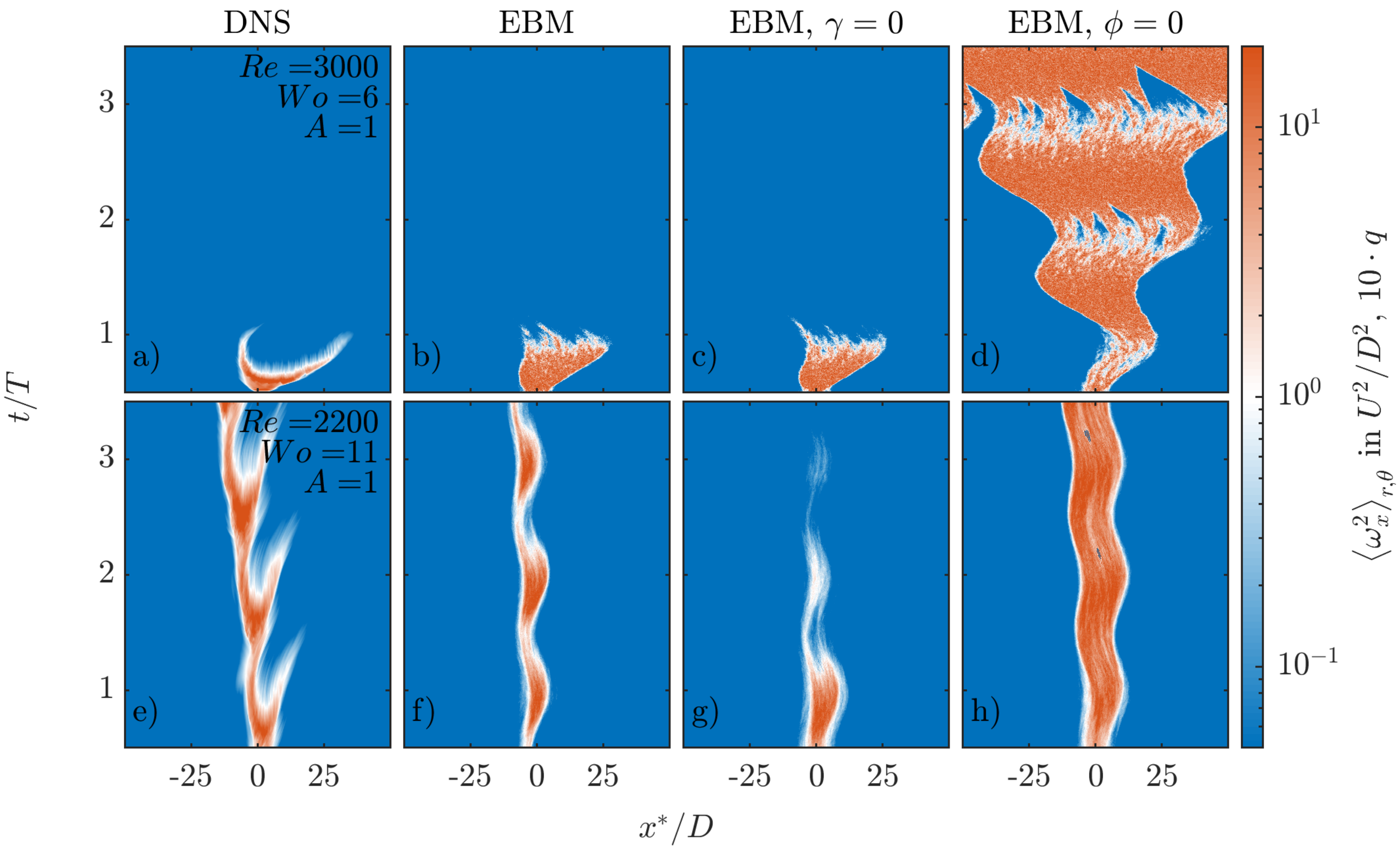




Appendix: Model for puffs in pulsatile pipe flow



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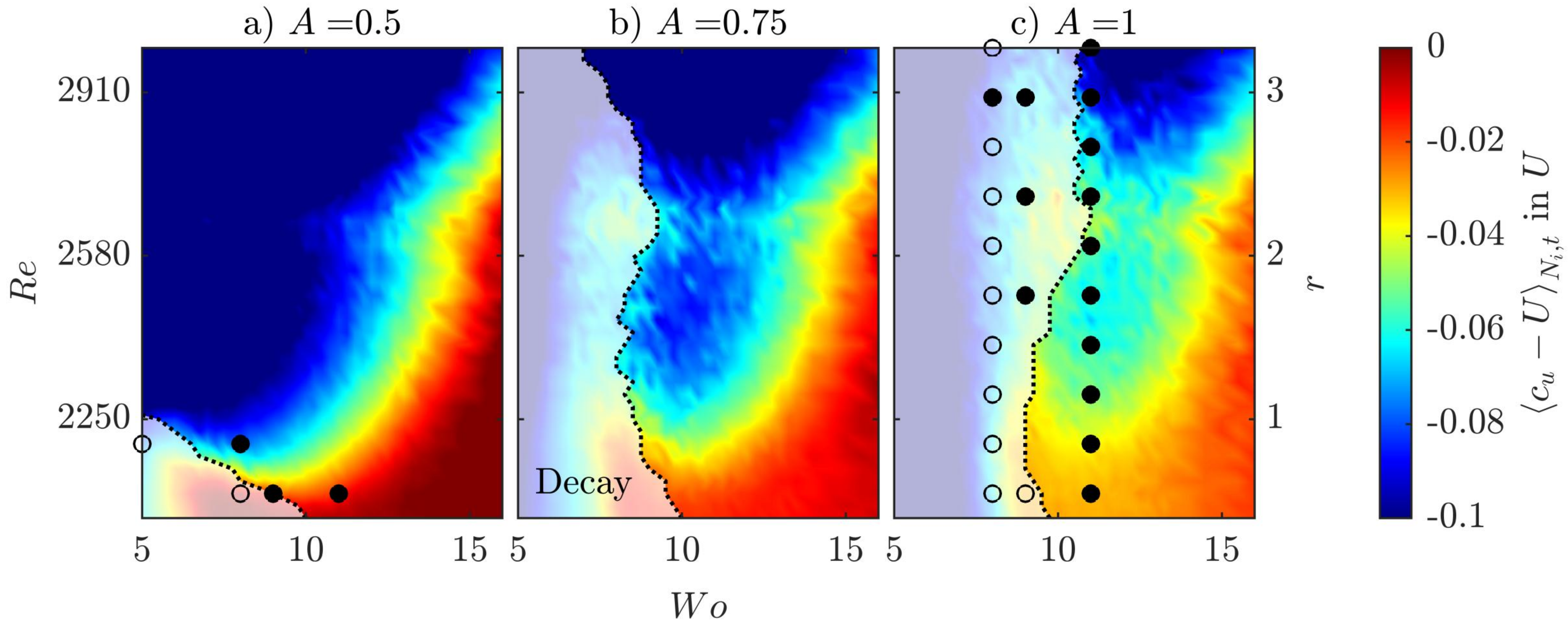

Appendix: spectral

$$f(x) = \sum_{k=-K/2}^{\frac{K}{2}-1} \hat{f}_k e^{ikx}$$

$$\int_0^{2\pi} f^2 dx = 2\pi \sum_k^{\infty} \hat{f}_k^2$$

$$2\pi \hat{f}_{N/2} = \int_0^{2\pi} f e^{-iNx/2} dx = \frac{-2}{iN}[f(2\pi) - f(0)] + \frac{2}{iN}\int_0^{2\pi} \frac{df}{dx} e^{-iNx/2} dx$$

$$\hat{f}_k = \frac{1}{K}\sum_{j=0}^{K-1}\left[\left(\sum_{m=-K/2}^{\frac{K}{2}-1} \hat{g}_m e^{imx_j}\right)\right]\left(\sum_{n=-K/2}^{\frac{K}{2}-1} \hat{h}_n e^{inx_j}\right) e^{-ikx_j} = \sum_{m+n=k} \hat{g}_m \hat{h}_n + \sum_{m+n=k\pm K} \hat{g}_m \hat{h}_n$$

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Appendix: LSA&TGA

$$u = \sum_{m=0}^{m} \sum_{\alpha=0}^{\alpha} \sum_{l=0}^{L} \left[a_l^{(1)} v_{l,m,\alpha}^{(1)}(r,t) + a_l^{(2)} v_{l,m,\alpha}^{(2)}(r,t) \right] e^{i(m\theta + \alpha x)}$$

Step 1: Pressure predictor

$$\nabla^2 \bar{p} = -\nabla \cdot [2N_u^n - N_u^{n-1}]$$

Step 2: Velocity predictor

$$\frac{3u^* - 4u^n + u^{n-1}}{2\Delta t} + 2N_u^n - N_u^{n-1} = -\nabla \cdot \bar{p} + \frac{1}{Re}\nabla^2 u^*$$

Step 3: Pseudo pressure

$$\nabla^2 \phi = -\nabla \cdot u^*$$

Step 4: Correct pressure and velocity

$$p^{n+1} = \bar{p} + \frac{3}{2\Delta t}\phi$$

$$u^{n+1} = u^* - \nabla\phi$$

Appendix: grid

$$\delta_\nu = \frac{\nu}{u_\tau} = \frac{D}{2Re_\tau}$$

$$u_\tau = \sqrt{\frac{\tau_w}{\rho}}$$

$$Re_\tau = \frac{Du_\tau}{2\nu}$$

Step 1: Estimate dissipation

$$\left\langle \frac{\partial p}{\partial x} \right\rangle_t U \approx \rho\varepsilon$$

$$\left\langle \frac{\partial p}{\partial x} \right\rangle_t \approx \left\langle \frac{4\tau_w}{D} \right\rangle_t$$

$$\varepsilon \approx \frac{4u_\tau^2 U}{D}$$

Step 2: Estimate wall-shear stress

$$\frac{\tau_w}{\rho U^2} = \frac{1}{8}\frac{0.316}{Re^{1/4}}$$

$$Re_\tau = \frac{u_\tau}{2U} Re = 0.099373\ Re^{7/8}$$

Step 3: Estimate Kolmogorov scale

$$\eta = \left(\frac{\nu^3}{\varepsilon}\right)^{1/4} \approx \left(\frac{D\nu^3}{4u_\tau^2 U}\right)^{1/4}$$

$$\eta^+ = \frac{\eta}{\delta_\nu} = \left(\frac{Du_\tau^2}{4U\nu}\right)^{1/4} = \left(\frac{u_\tau}{2U} Re_\tau\right)^{1/4} = \left(\frac{Re_\tau^2}{Re}\right)^{1/4}$$

Step 4: Estimate wall normal size

$$\Delta r^+ = \frac{Du_\tau}{2N_r\nu} \rightarrow N_r \geq \frac{Re_\tau}{\eta^+} \rightarrow N_r \geq 0.31523 Re^{11/16}$$

Appendix: Useful

Sexl-Womersley profile

$$U_{SW} = real\left\{\sum_{n=0}^{n} \frac{iP_n}{\rho n\omega}\left[1 - \frac{J_0\left(Wo\, n^{1/2} i^{3/2} r/R\right)}{J_0\left(Wo\, n^{1/2} i^{3/2}\right)}\right] e^{in\omega t}\right\}$$

Rayleigh criteria

$$v = \hat{v}e^{i(\alpha x - \alpha c t)} \rightarrow (U-c)(D^2-k^2)\hat{v} - U''\hat{v} = 0$$

$$\int_{-1}^{1} \frac{U'' c_i |\hat{v}|^2}{(U-c)^2} dy = 0$$

Fjørtoft criteria

$$U''(U-U_S) < 0$$

Cylindrical correction

$$Q = \frac{r}{m^2+\alpha^2 r^2} U'$$

Cell Peclet

$$Re_{\Delta x} = \frac{a\Delta t}{D}$$

Blood viscosity

$$v \sim 2.8 - 3.8\ 10^6 m^2/s$$

2D flow

$$\omega = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}, \qquad u = -\frac{\partial \Psi}{\partial y}, \qquad v = \frac{\partial \Psi}{\partial x}$$

BCon

$$f(r, \theta+\pi) = \pm f\ (-r, \theta)$$

$$u_{\pm} = u_r \pm i u_\theta$$

4.Outlook

1) Effect of **fluid-structure interaction**

2) Transition and turbulence in more **complex geometries**

3) **Non-Newtonian** effects

4) Use puffs in pulsatile pipe flow to study **puffs split and decay**

5) **Phase-dependent predictability** of extreme events

6) Use low-order model to design control laws